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ON A CHARACTERIZATION OF THE PREISACH MODEL FOR HYSTERESIS

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Abstract: A theorem due to Mayergoyz states that a hysteresis operator is a Preisach operator if and only if it has the congruency and wiping out property. We present a formal statement, proof and generalization of this result.

1. Introduction

In [6], Preisach formulated a mathematical model in order to describe hysteresis loops arising in ferromagnetism. It can be viewed as an operater W, which maps an input function $u:[0,1]\longrightarrow \mathbb{R}$, representing the (scalar) magnetic induction, to an output function $w:[0,1]\longrightarrow \mathbb{R}$, representing the magnetization. Usually one defines W by

$$(1) \quad (Wu)(t) = \int_P (W_r u)(t)\,d\mu(r),$$

where μ is a finite Borel measure on the Preisach plane

$$(2) \quad P = \left\{ r : r = (r_1, r_2),\ r_1 \leq r_2 \right\} \subset \mathbb{R}^2$$

and W_r denotes an elementary switch with hysteresis, switching to the value 1 when u(t) increases to the value r_2 and to the value -1 when u(t) decreases to r_1. In addition, an initial condition has to be specified for each elementary switch.

Since the action of an ideal switch is instantaneous, this model obviously is rate independent, i.e.

$$W(u \circ \Phi) = (Wu) \circ \Phi$$

for any (monotone) transformation Φ of the time scale. Also, any periodic input u(t) yields a periodic output w(t) with the same period. Thus, the map $t \longrightarrow (u(t), w(t))$ generates a hysteresis loop in the (u,w)-plane. Consider a periodic input (e.g. a sine function) oscillating between the values r_1 and and r_2 . The height of the corresponding hysteresis loop (assuming μ nonnegative) is given by

$$h(r_1, r_2) = 2\mu(\Delta),$$

Δ being the triangle $\left\{ (s_1, s_2) : r_1 \leq s_1 \leq s_2 \leq r_2 \right\}$ in the Preisach plane. Using this equality, one may determine the measure μ from experiment; on the other hand it shows that the height of the loop does not depend upon the past history. Moreover, the entire shape of the hysteresis loop is fixed by the measure μ independent from past history, and any change of input from

r_1 to r_2 and back to r_1 erases any memory due the previous input variation in the interval $[r_1, r_2]$.

Again, from the behaviour of individual switches it is obvious that the Preisach model has the properties stated above. It was Mayergoyz who pointed out in [4] that the latter two properties, which he calls congruency and wiping out property respectively, are also sufficient for a (nonanticipative and rate independent) operator to be a Preisach operator.
The aim of the present paper is to provide a formal statement and proof of this result. We try to clarify the role of the various assumptions; also, we admit general Borel measures μ. For more material on the Preisach operator, we refer to [1,2,3,7].

2. The characterization of the Preisach operator

Throughout this paper, we set $T = [0,1]$ and denote by $M(T)$ the set of all real valued functions on T.

<u>Definition 1</u>

Let $U \subset M(T)$. An operator $W: U \longrightarrow M(T)$ is called a hysteresis operator if it is rate independent and nonanticipative, i.e. if

$$W(u \circ \phi) = (Wu) \circ \phi \quad \forall u \in U$$

for any continuous nondecreasing $\phi: T \longrightarrow T$ with $\phi(0) = 0$, $\phi(1) = 1$ and $u \circ \phi \in U$;

$$u_1 = u_2 \text{ on } [0,t] \Rightarrow Wu_1 = Wu_2 \text{ on } [0,t]$$

for any u_1, $u_2 \in U$ and any $t \in T$.

□

Let $M_{pm}(T)$ resp. $C_{pm}(T)$ denote the set of all piecewise monotone continuous functions:

$$M_{pm}(T) = \Big\{ u \mid u: T \longrightarrow \mathbb{R},\ \exists\ 0 = t_1 < t_2 < \ldots < t_n = 1 \text{ such that } u \mid [t_i, t_{i+1}] \text{ is monotone for all } 1 \le i < n \Big\}$$

Definition 2

Let

$$X_o = \left\{x \mid x=(x_1,..,x_n),\ n\in N,\ x_i\in \mathbb{R}\right\} \cup \left\{\emptyset\right\}$$

be the set of all strings of real numbers including the empty string $\emptyset$, set

$$X = \left\{x \mid x\in X_o,\ \text{length}\ (x) \geq 2\right\}$$

Define a concatenation $x \cup y$ for $x,y,\in X_o$ by

$$x \cup y = (x_1,..,x_n,y_1,..,y_m)$$

and generate an equivalence relation $\sim$ on X from

$$(x_1,x_2,x_3) \sim (x_1,x_3) \quad \text{if} \quad x_1 \leq x_2 \leq x_3 \text{ or } x_1 \geq x_2 \geq x_3$$

$$x \sim \bar{x} \Rightarrow y \cup x \cup z \sim y \cup \bar{x} \cup z \quad \forall y,z\in X_o,$$

forming the reflexive, symmetric and transitive hull.

□

Definition 3

We define $p: M_{pm}(T) \longrightarrow X/\sim$ by

$$p(u) = (x_1,...,x_n)$$

where $x_i = u(t_i)$ and $0 = t_1 < ... < t_n = 1$ is a monotonicity partition for u such that $(u(t_{i+1}) - u(t_i))(u(t_i)-u(t_{i-1})) < 0$ for $1<i<n$. $U \subset M_{pm}(T)$ is called rich, if $p|U$ is surjective.

□