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A. J. Parameswaran and Duco van Straten

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UNIVERSITÄT KAIERSLAUTERN
Fachbereich Mathematik
Erwin–Schrödinger–Straße
6750 Kaiserslautern

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A. J. Parameswaran and D. van Straten

1 Introduction

The following question has been posed in [P-S]:

**Question:** Let $R$ be a complete normal local ring with coefficient field $\mathbb{C}$. Does there exist a local ring $A$, essentially of finite type over $\mathbb{C}$, such that the class group $Cl(A)$ of $A$ is generated by the canonical module $\omega_A$ of $A$ and its completion $\hat{A} \cong \hat{R}$?

In general one knows that $Cl(A) \to Cl(R)$ is injective (see [Bo]) and the question arises how small one can make $CL(A)$. Srinivas has constructed UFD's (i.e., $Cl(A) = 0$) with arbitrary rational double point singularities in his study of the K-theory of these singularities (see [Sr]). Kollár conjectured that any isolated hypersurface singularity would have an UFD globalization and some partial results were obtained by Buium (see [Bu]). In [P-S] the first author and Srinivas settled the above question in the affirmative for isolated complete intersection singularities.

Recently, Heitmann (see [He]) has constructed for any complete local ring $R$ over $\mathbb{C}$ of depth at least two UFD's with completion $\hat{R}$. But these rings are not geometric in general and they do not have dualizing modules. Indeed, a theorem of Murthy asserts that a geometric Cohen-Macaulay UFD is Gorenstein (see [Mu]). So for non-Gorenstein $R$ it seems more natural to look for geometric $A$'s with class group generated by the canonical module.

In this note we will prove that the above question has an affirmative answer in the case of normal surface singularities:

**Theorem 1.1** Given an analytic normal surface singularity $Spec(R) = (X, 0)$, there is an affine algebraic surface $X = Spec(A)$ and a closed point $0 \in X$ which represents the same germ $(X, 0)$ and the class group of $X$ is generated by the canonical divisor $\omega_X$. 

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The proof follows by projecting the singularity into \((C^3, 0)\) and studying a suitable equisingular algebraic family via the monodromy of Lefschetz pencils and a variant of the Noether-Lefschetz theorem, as in [P-S].

2 Construction of the Family

Let \((X, 0) = \text{Spec}(R)\) be the given analytic germ and \((X, 0) \subset (C^N, 0)\) be an embedding of it. Let \(L : C^N \to C^3\) be a generic linear projection and \(\nu : (X, 0) \to (Y, 0)\) be the restriction of \(L\) to \(X\), and let \((Y, 0)\) be the image. Then the singular locus \((\Sigma, 0)\) of \((Y, 0)\) is a curve, possibly singular at 0 and generically \((Y, 0)\) has \(A_\infty\) singularities, i.e., locally defined by the equation \(x^2 + y^2 = 0\). Moreover, in this situation \(\nu : X \to Y\) can be identified with the normalization and hence one can reconstruct \((X, 0)\) out of \((Y, 0)\). As \((Y, 0)\) is a hypersurface germ in \((C^3, 0)\), it is defined by an analytic function \(f \in C\{x, y, z\}\). Let \(I\) be the reduced ideal of \((\Sigma, 0)\). Then we have the following:

**Theorem 2.1** (cf. [P]) The function \(f\) is finitely \(I\)-determined and is right-equivalent to a polynomial. Moreover given an integer \(r\), there exists an integer \(k = k(r, f)\) such that whenever \(g \in M^k \cap I^{(2)}\), then \(f\) and \(f + g\) are right-equivalent via an automorphism which is identity modulo \(M^r\), where \(M\) is the maximal ideal of \(C\{x, y, z\}\).

Though the second part of the theorem is not explicitly stated there, it can be easily obtained by multiplying by a power of the maximal ideal on both sides of the basic inclusion of Pellikaan \(M^{k+1} I^{(2)} \subset M^r \tau^*(F)\) (cf. [P], page 375, line 12).

Let \(\Sigma\) be a compactification of \((\Sigma, 0)\) in \(P^3\), which is smooth outside 0. Then by a result of de Jong (cf. [J]), there exists a homogeneous polynomial \(F \in C[x, y, z, t]\) such that \(\{F = 0, 0\} \cong (Y, 0)\) and \(Y := \{F = 0\}\) is smooth outside \(\Sigma\) and has only \(A_\infty\) and \(D_\infty\) points on \(\Sigma - 0\).

Let \(\pi : \widetilde{P}^3 \to P^3\) be an embedded resolution of \(Y\). Then it is the blow up of a coherent sheaf of ideals \(\mathcal{I}\) supported on \(\Sigma\) (cf. [H], Theorem 7.17). Generically \(\mathcal{I}\) can be assumed to be the reduced ideal \(I\) of \(\Sigma\). By the Artin-Rees lemma there is an integer \(r\) such that \(\mathcal{I} \cap M^r \subset M.I\). Let \(k = k(r, F)\)
be as in the theorem 2.1. Let \( d_0 \in \mathbb{N} \) be an integer such that \( d_0 > l := \text{deg } F \)
and for all \( d > d_0 \) we have

(i) The restriction map \( r : H^0(\mathbb{P}^3, \mathcal{M}^k \cap I^{(2)}(d)) \to H^0(\mathbb{P}^3, I^{(2)}/\mathcal{M}_p^k,I^{(2)}) \)
is surjective for all \( p \in \Sigma - 0 \) where \( \mathcal{M}_p \) is the maximal of \( O_{\mathbb{P}^3, p} \).

(ii) \( V := C.t^d I F + H^0(\mathbb{P}^3, \mathcal{M}^k \cap I^{(2)}(d)) \) is very ample on \( \mathbb{P}^3 - \Sigma \)

(iii) \( A_d := (d^3 - 6d^2 + 11d - 6)/6 > h^0(\mathcal{O}_{\Sigma}(d-4)) + p_g(X,0) \) where \( p_g(X,0) \)
is the geometric genus of the singularity \( (X,0) \) and \( h^0 \) of a sheaf is the dimension of \( H^1 \). This is possible because \( p_g(X,0) \) is a constant and \( h^0(\mathcal{O}_{\Sigma}(d-4)) \) is a linear function in \( d \) by the theorem of Riemann-Roch, while the left hand side is a cubic polynomial in \( d \).

Let \( P \subset P(V^*) \) be the hyperplane defined by the subspace \( \mathcal{M}^k \cap I^{(2)}(d) \)
and \( S \) be the complement of \( P \) in \( P(V^*) \). For each \( s \in S \) let \( Y_s \) denote the
subscheme of \( \mathbb{P}^3 \) defined by \( s = 0 \) and \( Z_s \) be the strict transform of \( Y_s \) in \( \widetilde{\mathbb{P}^3} \).
Consider the families,

\( \mathcal{Y} := \{(x,s) \in \mathbb{P}^3 \times S \mid x \in Y_s\} \)

\( \mathcal{Z} := \{(x,s) \in \widetilde{\mathbb{P}^3} \times S \mid x \in Z_s\} \)

Let \( f : \mathcal{Z} \to S \) be the second projection.

3 Elementary Properties of the Family

Recall that \( \pi : \widetilde{\mathbb{P}^3} \to \mathbb{P}^3 \) was the embedded resolution of \( Y \). Let \( \widetilde{Y} \) be the
strict transform of \( Y \) in \( \widetilde{\mathbb{P}^3} \) and put

\( E_0 := \pi^{-1}(0) \cap \widetilde{Y} \).

For each \( s \in S \), we let \( \Sigma_s \subset Z_s \) be the "strict transform" of \( \Sigma \), i.e.,

\( \Sigma_s := \pi^{-1}(\Sigma - 0) \cap Z_s \).

Lemma 3.1 In the above situation we have

(i) For every \( s \in S \), \( Z_s \cap \pi^{-1}(0) = E_0 \) and \( Z_s \) is non-singular along \( E_0 \).
(ii) $f : Z \to S$ is a submersion along $E_0 \times S \subset Z$

(iii) There exists a codimension 2 subset $T$ of $S$ such that for all $s \in S - T, Z_s$ is smooth along $\Sigma_s$

**Proof:** Fix an $s \in S$. By the theorem of Pellikaan there is an automorphism of $(C^3,0)$ which is identity modulo $\mathcal{M}'$ and defines an isomorphism of $(Y_s,0)$ with $(Y,0)$. Since $\mathcal{M}' \cap \mathcal{I} \subset \mathcal{M} \mathcal{I}$ it follows that this automorphism extends to the blow up of $\mathcal{I}$ in a neighbourhood of 0 and acts trivially on the fibre $\pi^{-1}(0)$, because it acts trivially on $\mathcal{I}/\mathcal{M}' \cap \mathcal{I}$ which maps onto $\mathcal{I}/\mathcal{M} \mathcal{I}$ and hence acts trivially on $\mathcal{N} \mathcal{I}/\mathcal{M} \mathcal{I}$. Hence it fixes $E_0$ and defines an isomorphism of $(Z_s,E_0)$ with $(\tilde{Y},E_0)$. This proves (i). As (ii) is a local assertion, it follows from (i).

By the classification of line singularities (cf. [S] table on page 488), there is a subspace $T_\mathcal{P} \subset H^0(\mathbb{P}^3, I^{(2)}/\mathcal{M}_\mathcal{P}^3 I^{(2)})$ of codimension 3 for each $p \in \Sigma - 0$ such that all functions in $H^0(\mathbb{P}^3, \mathcal{M}_\mathcal{P}^3 \cap I^{(2)}(d)) - r^{-1}(T_\mathcal{P})$ has singularities of type $A_\infty$, $D_\infty$ or $J_\infty$ at $p$. By assumption (i) it follows that $r^{-1}(T_\mathcal{P})$ has codimension 3 in $H^0(\mathbb{P}^3, \mathcal{M}_\mathcal{P}^3 \cap I^{(2)}(d))$. Define $T'$ to be the closure of $\cup_{p \in \Sigma - r^{-1}(T_\mathcal{P})}$. Then $T'$ has codimension $\geq 2$ and is invariant under scalar multiplication. Let $T$ be the image of $T'$ in $S$. Then $T$ has codimension $\geq 2$ in $S$. Hence for every $s \in S - T$, $Y_s$ has only singularities of above mentioned types. It is easy to prove by local computations that $A_\infty$, $D_\infty$ and $J_\infty$ are resolved by the blow up of reduced singular locus. Hence $Z_s$ is smooth along $\Sigma_s$ for $s \in S - T$. This proves (iii). \hfill $\Box$

Let $C$ and $D$ be the critical and the discriminant locus of $f$, i.e., $C := \{(x,s) \in Z \mid Z_s \text{ is singular at } x\}$ and $D := f(C)$.

**Corollary 3.2** Outside the set $T \subset S$, we have

(i) $C - f^{-1}(T)$ is smooth and irreducible of dimension $\dim S - 1$.

(ii) $f : C - f^{-1}(T) \to D - T$ is birational.

(iii) For general $s \in D - T$, $Z_s$ has an ordinary double point.

(iv) $Y \to S$ is an admissible family of surfaces over $S - T$.
Proof: Since $V$ is very ample, it gives rise to an embedding of $\mathbb{P}^3 - \Sigma$ in $\mathbb{P}(V)$. Let $\pi': C \to \mathbb{P}^3$ be the projection. Then $C - \pi'^{-1}(\Sigma) \to \mathbb{P}^3 - \Sigma$ is the projective normal bundle of $\mathbb{P}^3 - \Sigma$ in $\mathbb{P}(V)$, by [La]. Moreover it is also proved there that $C - \pi'^{-1}(\Sigma) \to D$ is birational with the general point corresponds to an ordinary double point on $Z_s$. By lemma 3.1 $Z_s$ is non-singular along $Z_s \cap \pi^{-1}(\Sigma)$ for all $s \in S - T$. Hence the discriminant of $f: Z - f^{-1}(S - T) \to S - T$ is $D - T$. This proves (i), (ii) and (iii) of the corollary. The assertion (iv) follows from the definition of an admissible deformation (cf. [J-S]). Hence by normalizing $\mathcal{Y}|_{S-T}$ we obtain a family of normal surfaces $\mathcal{X} \to S - T$ with a section $\sigma$ such that each $X_s \to Y_s$ is the normalization and the singularities $(X_s, \sigma(s))$ are all isomorphic. \qed

Lemma 3.3 For general $s \in S - D$ one has,

(i) $\pi_1(Z_s) = 0$, hence $H^2(Z_s, \mathcal{Z})$ is torsion free

(ii) $H^2(Z_s, \mathcal{O}_{Z_s}) \neq 0$

Proof: (i) By stratified Morse theory (cf. [G-M], Part II, Theorem 1.1(1), page. 150-151) it follows that $Y_s - \Sigma$ is simply connected for general $s \in S$, because $V$ is very ample on $\mathbb{P}^3 - \Sigma$, which is simply connected. Since $Z_s$ is smooth and contains $Y_s - \Sigma$ as a dense open subset, it is simply connected.

(ii) Choose an $s \in S - D$ and write $X$, $Y$ and $Z$ for $X|_{Y_s}$, $Y_s$ and $Z_s$ respectively. Then we have the following exact sequence:

$$0 \to C \to \mathcal{O}_Y \to \mathcal{O}_S \to 0$$

Here $C = I$ is the conductor $\text{Hom}_{\mathcal{O}_Y}(\mathcal{O}_X, \mathcal{O}_Y)$ and is also the ideal sheaf of $\Sigma$. Also note that $\nu_* \mathcal{O}_X = \text{Hom}_{\mathcal{O}_Y}(\mathcal{C}, \mathcal{O}_Y)$, as $\mathcal{O}_Y$-modules. If we take $\text{Hom}_{\mathcal{O}_Y}(-, \mathcal{O}_Y)$ to the above exact sequence, we get

$$0 \to \text{Hom}_{\mathcal{O}_Y}(\mathcal{O}_Y, \mathcal{O}_Y) \to \text{Hom}_{\mathcal{O}_Y}(\mathcal{C}, \mathcal{O}_Y) \to \text{Ext}^1_{\mathcal{O}_Y}(\mathcal{O}_S, \mathcal{O}_Y) \to 0$$

Hence we obtain an exact sequence:

$$0 \to \mathcal{O}_Y \to \nu_* \mathcal{O}_X \to \text{Ext}^1_{\mathcal{O}_Y}(\mathcal{O}_S, \mathcal{O}_Y) \to 0$$

because the map $\text{Hom}_{\mathcal{O}_Y}(\mathcal{O}_Y, \mathcal{O}_Y) \to \text{Hom}_{\mathcal{O}_Y}(\mathcal{C}, \mathcal{O}_Y)$ is the natural map $\mathcal{O}_Y \to \nu_* \mathcal{O}_X$. From the long exact cohomology sequence of this exact sequence, we obtain,

$$H^1(Y, \text{Ext}^1_{\mathcal{O}_Y}(\mathcal{O}_S, \mathcal{O}_Y)) \to H^2(Y, \mathcal{O}_Y) \to H^2(Y, \nu_* \mathcal{O}_X) \to 0$$

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Also note that,

$$\text{Ext}^1_{\mathcal{O}_Y}(\mathcal{O}_Z, \mathcal{O}_Y) = \omega_\Sigma \otimes \omega_Y^{-1} = \omega_\Sigma(4 - d)$$

Hence $h^2(Y, \nu, \mathcal{O}_X) = h^2(X, \mathcal{O}_X) \geq h^2(Y, \mathcal{O}_Y) - h^1(Y, \text{Ext}^1_{\mathcal{O}_Y}(\mathcal{O}_Z, \mathcal{O}_Y)) = A_d - h^1(\Sigma, \text{Ext}^1_{\mathcal{O}_Y}(\mathcal{O}_Z, \mathcal{O}_Y)) = A_d - h^1(\Sigma, \omega_\Sigma(4 - d)) = A_d - h^0(\Sigma, \mathcal{O}_\Sigma(d - 4)) > p_g(X, 0).

From the Leray spectral sequence applied to $p : Z \to X$, we obtain an exact sequence,

$$H^0(X, R^1p_*\mathcal{O}_Z) \to H^2(X, p_*\mathcal{O}_Z) \to H^2(Z, \mathcal{O}_Z)$$

Hence $h^2(Z, \mathcal{O}_Z) \geq h^2(X, p_*\mathcal{O}_Z) - p_g(X, 0) = h^2(X, \mathcal{O}_X) - p_g(X, 0) > 0.$

Hence $H^2(Z, \mathcal{O}_Z) \neq 0$, which proves (ii). $\square$

4 A Noether-Lefschetz theorem

Here we prove that for a general $s \in S$, $Z_s$ has Pic generated by the exceptional cycles (the reduced irreducible components of $E_0$, $f^*\mathcal{O}_Y(1)$ and $\Sigma$). The representation of $\pi_1(S - D, s)$ on $H^2(Z_s, \mathbb{Q})$ gives rise to a local system $\mathcal{H}$ on $S - D$. Let $\mathcal{H}^*$ be the space of invariants of this representation and $\mathcal{P}$ be its orthogonal complement with respect to the intersection form. Note that the restriction of the intersection form is non-degenerate on $\mathcal{H}^*$ as it contains an ample divisor. Hence we have an orthogonal direct sum decomposition,

$$\mathcal{H} = \mathcal{H}^* \otimes \mathcal{P}$$

Let us denote by $A_s \subset H^2(Z_s, \mathbb{Z})$ the subgroup generated by $E_0$, $\Sigma_s$, and $f^*(\mathcal{O}_Y(1))$.

Lemma 4.1 In the above situation we have

(i) $\mathcal{H}^*_s = A_s \otimes \mathbb{Q}$.

(ii) The local sub-system $\mathcal{P}$ is irreducible.
Proof: The theorem of the fixed part of Deligne (cf. [D]) states that

\[ \mathcal{H}_s^* = \text{Im}(H^2(Z, \mathbb{Q}) \to H^2(Z_s, \mathbb{Q})) \]

Now choose a line \( L \subset S \) that intersects \( D \) transversely. Then \( L \cap D \subset D - T \) and \( Z_s \) has exactly one quadratic singularity for each \( s \in L \cap D \). Let \( L' = L - D \) and \( Z_{L'} = f^{-1}(L') \). By a theorem of Zariski \( \pi_1(L', s) \to \pi_1(S - D, s) \) is surjective. Hence

\[ \text{Im}(H^2(Z, \mathbb{Q}) \to H^2(Z_s, \mathbb{Q})) = \text{Im}(H^2(Z_{L'}, \mathbb{Q}) \to H^2(Z_s, \mathbb{Q})) = \mathcal{H}_s^* \]

Since \( Z_{L'} \) is a pencil of hypersurfaces in \( P^3 \), it is smooth and rational. Let \( \overline{Z}_{L'} \) be a smooth compactification of \( Z_{L'} \) with a morphism \( \overline{\pi} : \overline{Z}_{L'} \to P^3 \) which restrict to \( \pi \) on \( Z_{L'} \). Then \( \overline{Z}_{L'} \) is also smooth rational and complete, hence the cycle map \( \overline{\pi} : \text{Pic}(\overline{Z}_{L'}) \to H^2(\overline{Z}_{L'}, Z) \) is an isomorphism. Now look at the diagram (with exact top row):

\[
\begin{array}{ccc}
\text{Pic}(\overline{Z}_{L'}) & \longrightarrow & \text{Pic}(Z_{L'}) \\
\overline{\pi} \downarrow & & \downarrow c \\
H^2(\overline{Z}_{L'}, Z) & \longrightarrow & H^2(Z_{L'}, Z) \quad \longrightarrow & H^2(Z_s, Z)
\end{array}
\]

Hence it suffices to compute the image of \( \text{Pic}(\overline{Z}_{L'}) \). Since \( E_0 \times S \subset Z \), it follows that each reduced irreducible component of \( E_0 \) is in the image. The complement of all irreducible components of the exceptional divisor of \( \overline{Z}_{L'} \to P^3 \) is isomorphic to \( P^3 - \Sigma \). Hence its Picard group is generated by \( \overline{\pi}^* \mathcal{O}_{P^3}(1) \). It is also clear that \( \Sigma \) is in the image as it is \( Z_s \cap \pi^{-1}(\Sigma - 0) \). This proves (i).

Now each point of \( L \cap D \) defines a vanishing cycle and the space of invariants is precisely the orthogonal complement to the span of vanishing cycles, by the Picard-Lefschetz formula (cf. [La]). Hence the span of vanishing cycles is the stalk \( P_s \) of \( P \) at each point. Since the smooth points of \( D \) form a connected subset of \( S \), it follows that the vanishing cycles form a single conjugacy class and hence \( P \) is irreducible. This proves (ii).

\[ \text{Lemma 4.2} \quad \text{For } s \in S_U, \text{ where } S_U \text{ is the complement of countably many analytic subsets in } S, \text{ we have:} \]

\[ NS(Z_s) \otimes \mathbb{Q} = A_s \otimes \mathbb{Q} \]

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Proof: By Hodge theory, the map of sheaves $P \to H^2 f_* \mathcal{O}_Z |_{s-D}$ is surjective after tensoring with $\mathbb{C}$. Since $P$ is irreducible and the kernel of this map is a local sub-system, it has to be injective. If $s \in S - D$, then in some open neighbourhood $U$ of $s$ (in the Euclidean topology), $P$ can be trivialised as a local system, and $H^2 f_* \mathcal{O}_Z |_{s-D}$ as an $\mathcal{O}_U$-module, so that a non-zero element $v \in P_s$ yields a holomorphic function of several variables on $U$ which is not identically zero. Hence the zero set of this function is a closed analytic subset $Z_u \subset U$ of smaller dimension, and the collection of such $v$ is countable as $P_s \subset H^2(Z_s, \mathbb{Q})$ is countable. Hence for $s \in \{U - \cup_{v \neq 0} Z_v\}$, the map $P_s \to H^2(Z_s, \mathcal{O}_{Z_s})$ is injective. But $S - D$ can be covered by a countable collection of such sets $U$. Then for any $s \in S_U := \cup_U \{U - \cup_{v \neq 0} Z_v\}$ the map $P_s \to H^2(Z_s, \mathcal{O}_{Z_s})$ is injective. By the exponential sequence and GAGA we have, $$NS(Z_s) \otimes \mathbb{Q} = \text{Ker}(H^2(Z_s, \mathbb{Q}) \to H^2(Z_s, \mathcal{O}_{Z_s}))$$ Hence $P_s$ is orthogonal to $NS(Z_s) \otimes \mathbb{Q}$, i.e., the cycles representing $P_s$ are not algebraic. Hence the statement. \qed

Corollary 4.3 For $s \in S_U$ one has:

$$NS(Z_s) = A_s$$

Proof: One clearly has:

$$A_s \subset NS(Z_s) \subset H^2(Z_s, \mathbb{Z})$$

As by lemma 4.2 the result is true over $\mathbb{Q}$ and by lemma 3.3 (i) we know that $H^2(Z_s)$ is torsion free, it is sufficient to show that $A_s$ is a primitive lattice in $H^2(Z_s, \mathbb{Z})$. Now take a look at the diagram used in the proof of lemma 4.1. From the Leray spectral sequence of the map $Z_{L'} \to L'$ one gets that the map $j$ is injective, with as image the invariants of the monodromy. Hence $H^2(Z_{L'}, \mathbb{Z})$ is primitive in $H^2(Z_s, \mathbb{Z})$: It follows from the exponential sequence that the cokernel of $c : \text{Pic}(Z_{L'}) \to H^2(Z_{L'}, \mathbb{Z})$ injects into a $\mathbb{C}$-vectorspace, hence its image also must be primitive. Hence, $A_s$ as the image of $\text{Pic}(Z_{L'})$ in $H^2(Z_s, \mathbb{Z})$ is primitive. \qed

Proof of theorem 1.1: Choose an $s \in SU$ and write $X, Y$ and $Z$ for $X_s$, $Y_s$ and $Z_s$ respectively. Let $\Sigma' \subset X$ be the inverse image $\nu^{-1}(\Sigma)$ of
By corollary 4.3, it follows that the class group of $X$ is generated by $\Sigma'$ and $\nu^*\mathcal{O}_Y(1)$. So it only remains to prove that the class of $\Sigma'$ represents the dualizing module of $(X,0)$. Duality for finite maps applied to $\nu$ gives:

$$\nu_*\text{Hom}_{\mathcal{O}_X}(\mathcal{O}_X,\omega_X) = \text{Hom}_{\mathcal{O}_Y}(\nu_*\mathcal{O}_X,\omega_Y)$$

Since $\omega_Y$ is locally free as $Y$ is a hypersurface, we have

$$\nu_*\omega_X = \text{Hom}_{\mathcal{O}_Y}(\nu_*\mathcal{O}_X,\omega_Y) = \text{Hom}_{\mathcal{O}_Y}(\nu_*\mathcal{O}_X,\mathcal{O}_Y) \otimes \omega_Y = \mathcal{C} \otimes \omega_Y$$

Hence the class of $\mathcal{C}$ represents the dualizing module as $\omega_Y = \mathcal{O}_Y(d-4)$ is locally free.

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References


School of Mathematics, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400 005, India.

Fachbereich Mathematik, Universitat Kaiserslautern, Erwin-Schroedinger-Strasse, D-6750 Kaiserslautern.