Optimization Models to Enhance Resilience in Evacuation Planning*

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Abstract

We argue that the concepts of resilience in engineering science and robustness in mathematical optimization are strongly related. Using evacuation planning as an example application, we demonstrate optimization techniques to improve solution resilience. These include a direct modelling of the uncertainty for stochastic or robust optimization, as well as taking multiple objective functions into account.

Keywords: resilience; evacuation planning; disaster management; uncertainty; multiobjective optimization

1 Introduction

Resilience is a key aspect for many engineering and planning projects. Depending on the actual application, quite different definitions may apply. The one which we will adapt in this paper is “the ability of a system to cope with change” [TLO +13].

An especially dramatic change of environmental aspects occurs in human society during man-made or natural disasters, such as terrorist attacks, floods, hurricanes, or earthquakes. Therefore, there is an obvious need for evacuation planning to increase resilience to cope with such emergencies. We consider the field of disaster risk management (DRM) as a tool to increase resilience in dealing with disasters. DRM is an emerging area of research with several surveys (see, e.g., [HT01, AG06, YAM08]) and special issues (see, e.g., [DGVW11, She07]) on the topic.

In this paper, we aim at giving a short introduction to different ways of increasing the resilience using mathematical modeling and focus on the special

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case of evacuation planning. To this end, we review suitable techniques from the field of mathematical programming, such as stochastic, robust and multiobjective optimization. Throughout the paper we refer to results of the research projects REPKA and DSS-Evac-Logistic [REP14, DSS14] in which the methods presented in this paper are put to use. The intended audience are practitioners and planners with small background in mathematical theory.

The structure of this work is as follows: In Section 2 general modelling approaches, i.e., macro- and microscopic models and their combinations, are presented. The main part of this paper is Section 3, where we show how stochastic, robust, and multiobjective optimization techniques can be used to increase the resilience in evacuation planning. We conclude the discussion in Section 4.

2 Macroscopic and Microscopic Mathematical Models

Some of the main features of resilience in the case of a disaster are preparedness and efficient, well-designed crisis reaction measures. In both of these, mathematical models play a dominant role. In this section we sketch the idea of two meta models, the macroscopic and the microscopic one, and discuss their interdependence.

The macroscopic approach is characterized by a high level of problem abstraction to achieve a model that is solvable by efficient algorithms. Due to the model simplifications and the consideration of optimal, i.e., best possible solutions, macroscopic models are often used to produce lower bounds of objective values, e.g. the evacuation time, and to guide the evacuation planner in a what-if analysis.

A typical aspect of many macroscopic evacuation models is to use a graph to represent the actual street network. Figure 1 shows an example for this process: Street crossings are represented as vertices, and streets are modeled as edges between them. This removes aspects such as the curvature of a street. By aggregating nodes that are close to each other (as in the case for the roundabout in the lower right corner), the graph can be further simplified. When creating a macroscopic model, one has to carefully consider the pay-off between different

![Map data and Graph model](image-url)
degrees of simplifications: While a highly simplified data model is tractable by more involved optimization models, results may be meaningless, as crucial problem aspects are neglected. If, on the other hand, a detailed data model is used, the optimization model that can handle this data may be too simplistic to yield satisfying results.

Microscopic models work with a smaller degree of simplification than macroscopic ones. In evacuation, the models are—in principle—able to include individual properties, such as familiarity with the area from which the evacuation is taking place, the degree of fitness, age, etc. Obviously, the level of detail strongly influences the time it takes to solve the resulting models. Consequently, individual properties are often replaced by using distributions with regard to the individual properties (e.g., age distributions of the group of evacuees). The method of choice for macroscopic models is simulation (see, for instance, [DKSvS14]).

Macro- and microscopic models can be combined in an interactive solution process. The output of one type of model is used as input to the other until the output of both models is stable. The sandwich method, see [HHK+11], is an implementation of this idea. Its name can be attributed to evacuation models with the objective to minimize the evacuation time which is (provably) bounded from below using macroscopic optimization, and (experimentally) from above using microscopic simulation. One of the current research challenges is to establish probabilities for the validity of the upper bound. The fact that the joint output of macro+micro models is a time-window in which the actual evacuation time can be found is of particular importance to practitioners.

3 Model Variations to Increase Resilience

In the following, we discuss how a given optimization problem may be modified to increase the resilience of a resulting optimal solution. We exemplify this process using a highly simplified macroscopic evacuation problem: Let a directed graph \( G = (V, E) \) be given, modeling a street network, along with arc lengths \( d_{ij} \) denoting the time to traverse edge \((i, j)\). Let a single node \( s \in V \) be the starting point of \( b \) evacuees, who need to travel to a subset of shelter nodes \( \{t_1, \ldots, t_k\} = T \subseteq V \). Each shelter \( t_i \in T \) has a capacity \( u_i \) of people it can accommodate. What is the best possible evacuation time, given as the sum of travel times of all evacuees?

The optimization problem can be modeled using the following network flow formulation:

\[
\begin{align*}
\text{(P)} \quad & \min \sum_{(i,j) \in E} d_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{(s,i) \in E} x_{si} - \sum_{(i,s) \in E} x_{is} = b \\
& \sum_{(i,j) \in E} x_{ij} - \sum_{(j,i) \in E} x_{ji} = 0 \quad \forall i \in V \setminus (T \cup \{s\}) \\
& \sum_{(j,t_i) \in E} x_{jt_i} - \sum_{(t_i,j) \in E} x_{t_ij} \leq u_i \quad \forall i = 1, \ldots, k \\
& x_{ij} \in \mathbb{R}_+^{|E|} 
\end{align*}
\]
where \( x_{ij} \) is a variable that denotes the amount of flow (=evacuees) along an edge \((i, j) \in E\). Constraint (2) ensures that all evacuees leave the starting node \( s \), while Constraints (3) model that an evacuee entering a node must also leave it. Finally, Constraints (4) include the shelter capacities. We shall write \( \mathcal{F} \) to denote the set of feasible solutions for \((P)\).

### 3.1 Uncertainty

The first step to include solution resilience is to identify the problem aspects that might not be known exactly, or that are likely to change over time. This step heavily depends on the actual problem application, and is a creative process.

In our problem example, many such uncertain aspects could be considered: What if there is a different number of evacuees? What if shelter capacities are not as expected, or shelters are damaged? What if streets are not safe to use, or take longer to traverse than expected?

Once it is decided what kind of uncertainty should be considered, it needs to be mathematically formalized. As an example, we might consider arc lengths that come from an interval, and write

\[
\mathcal{U} = \{d \in \mathbb{R}^{|E|} : d_{ij} \leq d_{ij} \leq \overline{d}_{ij} \ \forall (i, j) \in E\}.
\]

Simply speaking, it now depends on whether a probability distribution over this set is available, or not. In the former case, one might make use of stochastic programming methods, while for the second case, robust optimization is applicable. We discuss both approaches in the following. For the sake of clarity, the discussion is highly simplified. We refer to the respectively provided literature sources for an in-depth understanding of these concepts.

Making use of a known probability distribution of the set of scenarios \( \mathcal{U} \) amounts to **stochastic optimization models**. The topic is well-established with a wealth of textbooks, see, e.g., [KW94] and [BL97], and has been the approach of choice for several disaster management models, see, e.g., [RRHT14, SS09, KMS84, CAA+13, TS85].

There are plenty of different sides to stochastic optimization, which we cannot presume to explain in detail within the scope of this work. Thus, we highlight some key aspects used in evacuation planning.

Assuming we use a finite subset of important scenarios \( \mathcal{U}' = \{d^1, \ldots, d^N\} \subset \mathcal{U} \) along with probability values \( p_\ell, \ell = 1, \ldots, N \) with \( \sum_{\ell=1}^N p_\ell = 1 \), a natural approach is to optimize the expected performance of a solution, i.e., to solve

\[
\min \frac{1}{N} \sum_{\ell=1}^N \sum_{(i,j) \in E} d_{ij}^\ell x_{ij}
\]

\[
\text{ s.t. } x \in \mathcal{F}
\]

which amounts to solving an optimization problem of the original type with an average cost vector \( \frac{1}{N} \sum_{\ell=1}^N d^\ell \).

Other popular approaches include two-stage methods, where a solution can be modified once the scenario becomes known. Examples for two-stage methods are given subsequently, when we discuss robust optimization.
A more elaborate approach is the usage of chance constraints. In this case, we want to make sure that the evacuation can be accomplished within a given timebound with a desired probability, e.g., we ask for a solution of

$$\begin{align*}
\min \gamma \\
\text{s.t. } P(c^T x > \gamma) \leq \epsilon \\
x \in \mathcal{F}
\end{align*}$$

for a desired confidence level $1 \geq \epsilon \geq 0$. As chance constraints are in general very hard to solve, robust optimization has been developed as an alternative approach, which we will consider next.

Robust optimization has been receiving considerable attention from the research community as an alternative to stochastic models over the last fifteen years. We refer to the surveys [GS13, ABV09, BBC11] and textbooks [KY97, BTGN09] for a general overview on the topic.

As one generally does not assume the knowledge of a probability distribution over $\mathcal{U}$, a worst-case perspective is adopted. As such a point of view holds high appeal in situations where an expected value is not meaningful – for instance, if solutions are evaluated only once (e.g., for pension funds), or when a high degree of security is required (e.g., airplane design) – it is well suited for evacuation planning. This potential has been recognized by many recent publications on robust disaster management, see, e.g., [GG14, GDT13, NW10, CTC07, BTCMY11] to name just a few.

As is the case for stochastic optimization, there are many diverse approaches to formulate a robust counterpart of an uncertain problem. The simplest approach to our example problem (P) would be to assume the worst-case on each single edge, i.e., to consider the problem

$$\begin{align*}
\min \max_{d \in \mathcal{U}} \sum_{(i,j) \in E} d_{ij} x_{ij} = \sum_{(i,j) \in E} \bar{d}_{ij} x_{ij} \\
\text{s.t. } x \in \mathcal{F}
\end{align*}$$

The conservatism of such an approach can be reduced by ignoring too pessimistic scenarios, where all (or nearly all) edges have high travel times simultaneously. Following [BS03], let $\Gamma$ be a parameter we choose to regulate the degree of robustness we would like to achieve. The value of $\Gamma$ denotes the number of edges that may have their worst-case length at the same time. We then solve the less conservative problem

$$\begin{align*}
\min \max_{S \subseteq E, |S| = \Gamma} \left( \sum_{(i,j) \in S} \bar{d}_{ij} x_{ij} + \sum_{(i,j) \in E \setminus S} d_{ij} x_{ij} \right) \\
\text{s.t. } x \in \mathcal{F}
\end{align*}$$

which can be done using a dualization technique on the inner maximization problem.

Usually more elaborate to solve are two-stage approaches, where it is possible to adapt a solution once the scenario is revealed (see, e.g., [BTGGN03, LLMS09]). As an example, we consider only a finite subset of scenarios $\mathcal{U}' = \ldots$
\{d^1, \ldots, d^N\} \subset \mathcal{U}$, and assume that the total amount of flow we can change is restricted by a value $B$. Then the two-stage problem becomes

$$\min \max_{\ell = 1, \ldots, N} \sum_{(i,j) \in E} d^\ell_{ij} x^\ell_{ij}$$

s.t.

$$x \in \mathcal{F}$$

$$x^\ell \in \mathcal{F}, \quad \forall \ell = 1, \ldots, N$$

$$\sum_{(i,j) \in E} |x_{ij} - x^\ell_{ij}| \leq B, \quad \forall \ell = 1, \ldots, N$$

Depending on the uncertainty and the possible recovery action, there is a huge variety of possible two-stage counterparts to consider. The right choice depends both on the application and the computational solvability.

### 3.2 Conflicting Objectives

The second approach to enhance resilience in evacuation planning presented in this paper is to consider more than one objective in the optimization process.

In the evacuation model (P), minimization of the evacuation time is the only objective under consideration and the capacity of the shelters is treated in Constraint (4) as input. This objective can be improved by investing in additional capacities. In the process of designing resilient evacuation plans, the question will be asked to minimize the cost of such investments in addition to minimize the evacuation time. In the resulting bi-objective optimization model we minimize simultaneously the evacuation time and the investment cost for additional capacities in the shelters:

(P-BOM):

Minimize $f(x) = \left( \sum_{(i,j) \in E} d_{ij} x_{ij} \right)$

s.t.

$$\sum_{(s,i) \in E} x_{si} - \sum_{(i,s) \in E} x_{is} = b$$

$$\forall i \in V \setminus (T \cup \{s\})$$

$$\sum_{(i,j) \in E} x_{ij} - \sum_{(j,i) \in E} x_{ji} = 0$$

$$\forall i \in V \setminus (T \cup \{s\})$$

$$\sum_{(j,t_i) \in E} x_{jti} - \sum_{(t_i,j) \in E} x_{t_ij} \leq u_i + y_i$$

$$\forall i = 1, \ldots, k$$

$$x_{ij} \in \mathbb{R}^{|E|}_+, y_i \in \mathbb{R}^k_+$$

Here, $y_i$ and $c_i$ is for $i = 1, \ldots, k$ the additional capacity created in shelter $t_i$ and its non-negative unit cost for generating this additional capacity, respectively.

Obviously, the previous example of two objectives can be further extended to include more objectives which are relevant in evacuation planning, like the risk, the number of evacuation personnel, etc., yielding the following multiobjective optimization model.
Multiobjective optimization models in emergency planning have been presented in [SMT09], [TCH07], [GPKM10], [AAW10], [CRTAA12] and in combination with robust optimization in [NED13]. An introduction to general multiobjective optimization can be found in [Ehr05].

In order to find minimizers of the vector-valued objective (11), an ordering of the \( Q \)-dimensional space \( \mathbb{R}^Q \) has to be given. The simplest one is the lexicographical ordering, where we define for two vectors \( a = (a_1, \ldots, a_Q), b = (b_1, \ldots, b_Q) \in \mathbb{R}^Q \) the relation \( a \leq_{\text{lex}} b \) iff \( a = b \) or \( a_q < b_q \) for the smallest index \( q \in \{1, \ldots, Q\} \) such that \( a_q \neq b_q \). Accordingly, the objectives \( f_1(x), \ldots, f_Q(x) \) are sorted according to their importance. An optimal lexicographical solution \( x^* \) is optimal with respect to the first objective \( f_1 \), and – if there is a choice, among all optimal solutions one is found minimizing the second objective, etc.

There are two general approaches to solve lexicographical optimization problems: In the integrated approach, algorithms are modified internally replacing every scalar-valued comparison by a vector-valued one using the lexicographical ordering. In the sequential approach, one iteratively solves restricted optimization problems, starting with the single-objective problem \( \min \{ f_1(x) : x \in F \} \) and output \( X_1 \). In the \( q \)-th iteration, \( q = 2, \ldots, Q \) the single-objective problem \( \min \{ f_q(x) : x \in X_{q-1} \} \) is solved with output \( X_q \). By the definition of the lexicographical ordering, the set \( X_Q \) is the set of optimal lexicographical solutions.

In the special case of the bi-objective model (P-BOM), this generic idea can be implemented by first solving a single-objective evacuation problem of type (P) with unlimited shelter capacities \( u_i = \infty \), \( i = 1, \ldots, k \). The output is an optimal evacuation time \( ET_{\text{opt}} \) and \( X_1 \) is the set of all \( x \in F \) such that \( \sum_{(i,j) \in E} d_{ij} x_{ij} = ET_{\text{opt}} \). In the second – and, due to \( Q = 2 \), final – iteration we solve an additional single-objective evacuation problem of type (P) with modified (scalar!) objective function \( \sum_{i=1, \ldots, k} c_i y_i \) and additional constraint \( \sum_{(i,j) \in E} d_{ij} x_{ij} = ET_{\text{opt}} \).

Any P-MOM will have a unique optimal objective vector (albeit with, in general, many lexicographical optimal solutions), since the lexicographical ordering is complete, i.e. any pair of vectors can be compared with each other.

This is no longer true for the second ordering considered in this paper, the component-wise ordering, defined for all \( a = (a_1, \ldots, a_Q), b = (b_1, \ldots, b_Q) \in \mathbb{R}^Q \) by \( a \leq b \) iff \( a \neq b \) and \( a_q < b_q \) for all \( q \in \{1, \ldots, Q\} \) with \( a_q \neq b_q \). As the example \( a = (4, 1) \) and \( b = (2, 3) \) shows, two vectors may be non-comparable, i.e. neither \( a \leq b \) nor \( b \leq a \) holds.

The non-comparability is in the application of (P-MOM) to evacuation planning a desired property to reflect reality. It is, for instance, not decidable by a mathematical model, which of the two vectors \( a = (4 \text{ hours}, 10k \text{ Euro}) \) or \( b = (2 \text{ hours}, 30k \text{ Euro}) \) for the vector objective (evacuation time, capacity
increment cost) is preferable. In this context, resilient evacuation plans correspond to solutions \( x \in \mathcal{F} \) of our model (P-MOM) which are contained in the set \( \mathcal{X}_{Par} \) of Pareto solutions, i.e.

there does not exist another \( y \in \mathcal{F} \) such that \( f(y) \leq f(x) \). The set \( \{ f(x) : x \in \mathcal{X}_{Par} \} \) of corresponding objective vectors is called the non-dominated set \( N_{nd} \) of (P-MOM).

![Diagram](image)

Figure 2: Example of non-dominated set (thick black line) with box representation (red boxes) and representative system \( \mathcal{R} = \{ y^1, \ldots, y^4 \} \) (black dots).

There are numerous ways to compute \( \mathcal{X}_{Par} \) and \( N_{nd} \). It is beyond the scope of this short paper to even sketch all of these methods and we refer to [Ehr05] for details. In the following we will concentrate on an idea which is currently used in the decision support system DSS-Evac-Logistic developed by a German-French research consortium [DSS14].

The non-dominated set \( N_{nd} \) (which is in general of infinite cardinality) is approximated by a finite representative system \( \mathcal{R} \) using the box algorithm of [HPR07] and \( \mathcal{R} \) is visualized in such a way that non-specialist can use the information to make well-founded decisions towards resilient evacuation plans.

The idea of the box algorithm can easily be described using Figure 2 and the case of bi-criteria problems. We start by solving two lexicographical problems with vector-valued objective functions \( (f_1, f_2) \) and \( (f_2, f_1) \), respectively, and compute lexicographical optimal solutions \( x^1, x^2 \) and their vector-valued objectives \( y^1 = f_1(x^1) \) and \( y^2 = f_2(x^2) \), respectively. The rectangle with corner points \( y^1 \) and \( y^2 \) is the starting box of the algorithm and the points \( y^1 \) and \( y^2 \) are the first ones to be included in the representative system \( \mathcal{R} \). At this point and at any time of the algorithm the non-dominated set is covered by a set of boxes, i.e. \( N_{nd} \) is a subset of the union of these boxes, and in each box a non-dominated solution is included in the representative system \( \mathcal{R} \). The algorithm stops as soon as all boxes have a size smaller than a given threshold defined at the beginning of the algorithm.

For any box not satisfying this criterion \( \{ y : y = f_1(x) \leq \epsilon \} \) halving this box is added (see Figure 2(a)) and the corresponding constraint \( f_1(x) \leq \epsilon \) is added. After solving the resulting (scalar) optimization problem \( \min \{ f_2(x) : x \in \mathcal{F} \text{ and } f_1(x) \leq \epsilon \} \), the original box decomposes into two subboxes with cumulative size at most half of the original box. Using this iterative process, it is possible to approximate the complete non-dominated set by a representative system \( \mathcal{R} \) with any required accuracy and with a worst-case complexity which can be determined a-priori (see [HPR07] for details).
In order to make use of this idea for non-specialists, the representative system is fed into a database and stored in a spider diagram, see Figure 3 for an example. Each vector-valued objective is represented as a polygon and users can search among the solutions stored in $\mathcal{R}$ to find the most suitable one.

![Spider diagram example](image)

**Figure 3**: Example of a spider diagram.

## 4 Conclusion

We presented a short introduction to optimization methods that increase resilience. Using evacuation planning as an application scenario, we discussed approaches that take uncertainty directly into account (using stochastic or robust optimization), as well as multiobjective optimization.

More details can be found in the reference section.

### References


