Fraunhofer
ITWM

T. Gornak, J. L. Guermond, O. Iliev, P. D. Minev

A direction splitting approach for incompressible Brinkmann flow

Berichte des Fraunhofer ITWM, Nr. 216 (2012)

ISSN 1434-9973

Bericht 216 (2012)

Die Veröffentlichungen in der Berichtsreihe des Fraunhofer ITWM können bezogen werden über:

Fraunhofer-Institut für Techno- und
Wirtschaftsmathematik ITWM
Fraunhofer-Platz 1

67663 Kaiserslautern
Germany

Telefon: +49(0)631/31600-4674
Telefax: +49(0)631/31600-5674
E-Mail: presse@itwm.fraunhofer.de
Internet: www.itwm.fraunhofer.de

Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe werden sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

A DIRECTION SPLITTING APPROACH FOR INCOMPRESSIBLE BRINKMAN FLOW

T.GORNAK[1,4], J.L. GUERMOND[2], O.ILIEV[1,4,5], P.D. MINEV[3]

[1]Department of Flows and Materials Simulation Fraunhofer Institute for Industrial Mathematics, Fraunhofer-Platz 1, D-67663 Kaiserslautern, Germany ({*gornak, iliev*}*@itwm.fhg.de*).

[2]Department of Mathematics, Texas A&M University, College Station, TX 77843-3368, USA, and LIMSI (CNRS-UPR 3152), BP 133, 91403, Orsay, France (*guermond@math.tamu.edu*).

[3]Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, Alberta, Canada T6G 2G1 (*minev@ualberta.ca*).

[4] Technical University of Kaiserslautern, Kaiserslautern, Germany

[5] Inst. of Mathematics, Bulgarian Academy of Science, Sofia. Bulgaria

Abstract. The direction splitting approach proposed earlier in [6], aiming at the efficient solution of Navier-Stokes equations, is extended and adopted here to solve the Navier-Stokes-Brinkman equations describing incompressible flows in plain and in porous media. The resulting pressure equation is a perturbation of the incompressibility constrained using a direction-wise factorized operator as proposed in [6]. We prove that this approach is unconditionally stable for the unsteady Navier-Stokes-Brinkman problem. We also provide numerical illustrations of the method's accuracy and efficiency.

1. Introduction

Flows in highly porous media occur often in industrial and scientific applications. Examples are the flows through various filters (air, oil, water filters, etc.) flow of helium in pebble-bed nuclear reactors, various physiological flows like the flow in the eye of glaucoma patients, flows in mangrove swamps etc. If the porosity of the media is high, such flows are usually modeled with the Brinkman equations. The Navier-Stokes-Brinkman model is positioned between the Darcy model for the flow in media with a very low porosity and the Navier-Stokes equations for flows with infinitely large porosity. Further on, prescribing properly the coefficients of the Navier-Stokes-Brinkman equations, one can use it to describe the flow in plain (pure fluid) region with or without embedded obstacles, coupled to the flow in highly porous media. In all these cases, the flow field is a subject to the incompressibility constraint. The imposition of this constraint is a well-known hurdle for any discretization algorithm for such problems. In the case of unsteady flows described by pure Navier-Stokes equations, probably the most popular algorithms for the imposition of incompressibility are the so-called projection methods. These methods are pioneered by Chorin [4] and Temam [12][1]. For a recent and comprehensive review on projection methods the reader is referred to [5]. All projection methods in fact are a discretization of a perturbation of the continuity equation which comes in the form of a pressure Poisson equation (L^2 projection onto a divergence-free subspace of the velocity space). The solution of this Poisson equation can often require the major computational effort in an incompressible flow solver. This difficulty was recently circumvented in [6] where it was shown that the Laplace operator is one of many possible perturbation operators which would yield a stable and accurate incompressible algorithm for unsteady flows. Also there, a direction-wise factorized operator of the form $\prod_{i=1}^{d}(I - \partial_{x_i x_i})$ (d being the dimension of the problem) was proposed instead of the classical Laplace operator. In the present article we extend this approach to the case of incompressible Brinkman flow and to the case of Navier-Stokes-Brinkman flow (the latter concerns coupled flow in plain and in porous media). We prove that if the momentum equation is not split direction-wise, the resulting algorithm is unconditionally stable. The direction splitting of the momentum equation, however, is more difficult than in the Navier-Stokes case because in general, the resulting one-dimensional operators do not commute unless the geometry of the entire domain and the geometry of the porous subdomain are simple rectangles/parallelepipeds.

2. The fictitious domain Brinkman equations

Consider the Brinkman equations in a domain $\tilde{\Omega} \subset \mathbb{R}^d$ ($d = 2, 3$) with a Lipschitz boundary $\Gamma = \partial\tilde{\Omega}$:

---

[1] The authors have recently discovered that a similar velocity-pressure decoupling approach was proposed earlier in the famous article of Harlow and Welch [8] which also proposed the MAC staggered grid setting for the Stokes problem with free boundaries. Thus, some credit for pioneering the projection methods should be given to this article.

$$\begin{cases} \partial_t \tilde{\mathbf{u}} - \tilde{\nu}\Delta\tilde{\mathbf{u}} + \nabla p + \frac{\tilde{\nu}}{\tilde{k}}\tilde{\mathbf{u}} = \tilde{\mathbf{f}} \quad \text{in } \tilde{\Omega}\times[0,T], \\ \nabla\cdot\tilde{\mathbf{u}} = 0 \quad \text{in } \tilde{\Omega}\times[0,T], \\ \tilde{\mathbf{u}}|_{\partial\tilde{\Omega}} = 0 \quad \text{in}[0,T], \qquad \text{and } \tilde{\mathbf{u}}|_{t=0} = \tilde{\mathbf{u}}_0 \quad \text{in} \quad \tilde{\Omega}, \end{cases} \tag{1}$$

where $\tilde{\nu}$ is the kinematic viscosity of the fluid and $\tilde{k}$ is a permeability that can vary throughout $\tilde{\Omega}$ representing the properties of the fluid and/or the porous media. Let Ω be a rectangle in 2D and parallelepiped in 3D such that $\tilde{\Omega} \subseteq \Omega$ and consider the following extension of the data

$$\nu = \tilde{\nu}, \quad \text{in} \quad \Omega, \tag{2}$$

$$\mathbf{f} = \begin{cases} \tilde{\mathbf{f}}, & \text{in} \quad \tilde{\Omega}, \\ 0, & \text{in} \quad \Omega\setminus\tilde{\Omega}, \end{cases} \tag{3}$$

$$\mathbf{u}_0 = \begin{cases} \tilde{\mathbf{u}}_0, & \text{in} \quad \tilde{\Omega}, \\ 0, & \text{in} \quad \Omega\setminus\tilde{\Omega}, \end{cases} \tag{4}$$

$$k(\mathbf{x}) = \begin{cases} \tilde{k}, & \text{in} \quad \tilde{\Omega}, \\ \nu\epsilon, & \text{in} \quad \Omega\setminus\tilde{\Omega}, \end{cases} \tag{5}$$

where $0 < \epsilon << 1$ is a penalty parameter used to enforce the boundary conditions on $\partial\tilde{\Omega}$. Then the L^2-penalty fictitious domain formulation of the problem in Ω reads

$$\begin{aligned} &\partial_t \mathbf{u}_\varepsilon - \nu\Delta\mathbf{u}_\varepsilon + \nabla p_\varepsilon + \frac{\nu}{k}\mathbf{u}_\varepsilon = \mathbf{f},\ x \in \Omega\times[0,T] \\ &\nabla\cdot\mathbf{u}_\varepsilon = 0 \text{ in } \Omega\times[0,T], \\ &\mathbf{u}_\varepsilon|_{\partial\Omega} = 0 \text{ in } [0,T],\ \text{and } \mathbf{u}_\varepsilon|_{t=0} = \mathbf{u}_0 \text{ in } \Omega. \end{aligned} \tag{6}$$

It is well known (see for example [2]) that the following result holds under sufficient regularity assumptions on the data and the domain:

$$u_\varepsilon(x) \xrightarrow[\varepsilon\to 0]{} u(x),\ x \in \tilde{\Omega}.$$

The order of convergence depends on the regularity of the data and the domain, but it is at least $O(\epsilon^{1/2})$.

3. Numerical algorithm

3.1. **Fractional time step discretization.**

It is not a new idea to use direction splitting techniques for the momentum equations in order to increase the computational efficiency. However, often the cost of computing the pressure, usually done via projection methods resulting in a Neumann problem for a Poisson equation, is dominant. Therefore, it is attractive to use a simpler perturbation of the incompressibility constraint which allows for a more efficient pressure computation. We discretize equations (6) using the splitting procedure introduced in [6]. Since the permeability can vary greatly throughout the computational domain, the direction splitting of the momentum equation, in case of an implicit treatment of the Brinkman term $\nu\mathbf{u}_\varepsilon/k$, is not very straightforward. To understand the problem, let us consider the Douglas splitting for the first equation in (6) (see [9]) in the following factorized form

(7)
$$\prod_{i=1}^{d}\left(b(\mathbf{x})I-\frac{\nu\Delta t}{2b(\mathbf{x})}\partial_{x_ix_i}\right)\frac{\mathbf{u}^{n+1}-\mathbf{u}^n}{\Delta t}-\nu\Delta\mathbf{u}^n+\frac{\nu}{k}\mathbf{u}^n+\nabla p^{*,n+1/2}=\mathbf{f}^{n+1/2},$$

with $b(\mathbf{x})=(1+\nu\Delta t/(2k))^{1/d}$ and I being the identity operator. It is clear that since $b(\mathbf{x})^{-1}\partial_{x_1x_1}$ and $b(\mathbf{x})^{-1}\partial_{x_2x_2}$ do not commute in general, the product operators of the type $b(\mathbf{x})^{-1}\partial_{x_1x_1}b(\mathbf{x})^{-1}\partial_{x_2x_2}$ do not necessarily generate a positive and self-adjoint bilinear form which affects the stability of the scheme. Therefore, in this paper we do not consider the possibility to split direction-wise the momentum equation. As a result, the full 2D/3D momentum problem must be solved by means of an iterative procedure. The reader is referred to [3] for some possibilities for a direction splitting of the momentum equation in the non-commutative case. In the sequel of the paper we will omit the subscript ε of the solution to the penalized problem (6) and abusing somewhat the notation will denote the solution of the semi-discrete (in time only) splitting scheme by $\mathbf{u}$ and p. Given some values $\mathbf{u}^n, p^{n-1/2}, \phi^{n-1/2}$ the splitting scheme proceeds as follows:

Pressure predictor:

$$p^{*,n+1/2}=p^{n-1/2}+\phi^{n-1/2}. \tag{8}$$

Velocity update:

$$\begin{aligned}\frac{\mathbf{u}^{n+1}-\mathbf{u}^n}{\Delta t}&-\frac{1}{2}\nu\Delta(\mathbf{u}^{n+1}+\mathbf{u}^n)+\nabla p^{*,n+1/2}+\\ &\frac{\nu}{2k}(\mathbf{u}^{n+1}+\mathbf{u}^n)=\mathbf{f}^{n+1/2},\qquad \mathbf{u}^{n+1}|_{\partial\Omega}=0.\end{aligned} \tag{9}$$

Pressure-corrector:

$$A\phi^{n+1/2}=-\tfrac{1}{\Delta t}\nabla\cdot\mathbf{u}^{n+1}, \tag{10}$$

where $A=\prod_{i=1}^{d}(I-\partial_{x_ix_i})$ together with homogeneous Neumann boundary conditions on $\partial\Omega$. Note that this operator is factorized alongside the spatial directions and is therefore much easier to invert than the usual for projection methods Laplace operator. It is also straightforward to prove (see [6]) that the bilinear form $a(p,q) := \int_\Omega qAp\mathrm{d}\mathbf{x}$ satisfies the following properties:

$$a \text{ is symmetric}, \qquad \text{and} \qquad \|\nabla q\|^2_{\mathbf{L}^2} \leq a(q,q), \quad \forall q \in D(A). \tag{11}$$

where $D(A)$ is the domain of A.

Pressure update:

$$p^{n+1/2} = p^{n-1/2} + \phi^{n+1/2} - \frac{1}{2}\chi\nu\nabla\cdot(\mathbf{u}^{n+1} + \mathbf{u}^n), \tag{12}$$

where $\chi \in [0,1]$.

The stability of the scheme is guaranteed by the following theorem.

Theorem 3.1. *Assume that the solution to* (6) *is smooth enough. There exist* c*, independent of* Δt*, such that for all* T *and all* $\chi \in [0,1]$ *the solution to (8-12), satisfies the following stability estimate:*

$$\begin{aligned} &\|\boldsymbol{u}\|^2_{\ell^\infty(0,T;\boldsymbol{L}^2)} + \frac{\Delta t}{2}\|\nu^{1/2}\nabla\boldsymbol{u}\|^2_{\ell^\infty(\Delta t,T;\boldsymbol{L}^2)} + \Delta t^2\|p\|_{\ell^\infty(\frac{\Delta t}{2},T-\frac{\Delta t}{2},D(A))} + \\ &\frac{\Delta t}{2}\|\left(\frac{\nu}{k}\right)^{1/2}\boldsymbol{u}\|^2_{\ell^\infty(\Delta t,T;\boldsymbol{L}^2)} \leq c(\|\boldsymbol{u}^0\|^2_{\boldsymbol{L}^2} + \Delta t^2\|p^{-1/2}\|^2_A + \\ &\frac{\Delta t}{2}\|\nu^{1/2}\nabla\boldsymbol{u}^0\|^2_{\boldsymbol{L}^2} + \frac{\Delta t}{2}\|\left(\frac{\nu}{k}\right)^{1/2}\boldsymbol{u}^0\|^2_{\boldsymbol{L}^2} + \nu^{-1}\|\mathbf{f}\|_{\ell^2(\frac{\Delta t}{2},T-\frac{\Delta t}{2};\boldsymbol{H}^{-1})}). \end{aligned} \tag{13}$$

Proof. The proof follows along the same lines as the stability of the direction splitting scheme in case of the Navier-Stokes equations given in [6]. Nevertheless, there are some differences and we provide a brief sketch here for the case $\chi = 0$ and $\mathbf{f} = 0$ only. This stability estimate can then be used to study the accuracy of the scheme similarly to the case of the rotational pressure correction method studied in [7]. We first multiply (9) by $2\Delta t\mathbf{u}^{n+1}$, integrate over Ω, and use the identity $2(a-b,a) = \|a\|^2 + \|a-b\|^2 - \|b\|^2$ and the Young's inequality to obtain

$$\begin{aligned} &\|\mathbf{u}^{n+1}\|^2_{\mathbf{L}^2} + \|\mathbf{u}^{n+1} - \mathbf{u}^n\|^2_{\mathbf{L}^2} + \frac{\Delta t}{2}\|\nu^{1/2}\nabla\mathbf{u}^{n+1}\|^2_{\mathbf{L}^2} \\ &+ 2\Delta t(\nabla p^{*,n+1/2}, \mathbf{u}^{n+1}) + \frac{\Delta t}{2}\|\left(\frac{\nu}{k}\right)^{1/2}\mathbf{u}^{n+1}\|^2_{\mathbf{L}^2} \leq \\ &\|\mathbf{u}^n\|^2_{\mathbf{L}^2} + \frac{\Delta t}{2}\|\nu^{1/2}\nabla\mathbf{u}^n\|^2_{\mathbf{L}^2} + \frac{\Delta t}{2}\|\left(\frac{\nu}{k}\right)^{1/2}\mathbf{u}^n\|^2_{\mathbf{L}^2}. \end{aligned} \tag{14}$$

Now we use the properties (11) of the operator A to deduce that the pressure correction $(p^{n+1/2} - p^{n-1/2}) \in D(A)$ solves the following problem for $n \geq 0$:

$$a(p^{n+1/2} - p^{n-1/2}, q) = -\Delta t^{-1}(\nabla\cdot\mathbf{u}^{n+1}, q), \qquad \forall q \in D(A). \tag{15}$$

Testing it with $2\Delta t^2 p^{*,n+1/2} := 2\Delta t^2(2p^{n-1/2} - p^{n-\frac{3}{2}})$ (in case $\chi = 0$) and using the symmetry and coercivity of $a(\cdot,\cdot)$, we obtain exactly as in the proof of Theorem 4.2 in [6], that

$$\begin{aligned} -2\Delta t(\nabla\cdot\mathbf{u}^{n+1}, p^{*,n+1/2}) = \Delta t^2 \big(\|p^{n+1/2}\|_A^2 - \\ \|p^{n-1/2}\|_A^2 + \|\delta p^{n-1/2}\|_A^2 - \|\delta^2 p^{n+1/2}\|_A^2\big)\,. \end{aligned} \tag{16}$$

where $\delta p^{n-1/2} = p^{n-1/2} - p^{n-3/2}$ is the usual difference operator. Again as in the proof of Theorem 4.2 in [6], the control on $\|\delta^2 p^{n+1/2}\|_A^2$ is obtained subtracting (15) at time t^n from (15) at time t^{n+1} and by testing the result with $\Delta t\delta^2 p^{n+1/2}$,

$$\begin{aligned} \Delta t\|\delta^2 p^{n+1/2}\|_A^2 = -(\nabla\cdot(\mathbf{u}^{n+1} - \mathbf{u}^n), \delta^2 p^{n+1/2}) = (\mathbf{u}^{n+1} - \mathbf{u}^n, \nabla\delta^2 p^{n+1/2}) \\ \le \|\mathbf{u}^{n+1} - \mathbf{u}^n\|_{\mathbf{L}^2}\|\nabla\delta^2 p^{n+1/2}\|_{\mathbf{L}^2}. \end{aligned}$$

Then the coercivity property of the bilinear form a implies that

$$\Delta t\|\nabla\delta^2 p^{n+1/2}\|_{\mathbf{L}^2}\|\delta^2 p^{n+1/2}\|_A \le \|\mathbf{u}^{n+1} - \mathbf{u}^n\|_{\mathbf{L}^2}\|\nabla\delta^2 p^{n+1/2}\|_{\mathbf{L}^2},$$

which yields the inequality $\Delta t^2\|\delta^2 p^{n+1/2}\|_A^2 \le \|\mathbf{u}^{n+1} - \mathbf{u}^n\|_{\mathbf{L}^2}^2$. This bound together with (16) gives the following bound on the pressure gradient term in (14)

$$\begin{aligned} \Delta t^2 \big(\|p^{n+1/2}\|_A^2 + \|\delta p^{n-1/2}\|_A^2 - \|p^{n-1/2}\|_A^2\big) \le \\ -2\Delta t(\nabla\cdot\mathbf{u}^{n+1}, p^{*,n+1/2}) + \|\mathbf{u}^{n+1} - \mathbf{u}^n\|_{\mathbf{L}^2}^2 \end{aligned} \tag{17}$$

which after summing with (14) gives

$$\begin{aligned} \|\mathbf{u}^{n+1}\|_{\mathbf{L}^2}^2 + \Delta t^2\|p^{n+1/2}\|_A^2 + \frac{\Delta t}{2}(\|\nu^{1/2}\nabla\mathbf{u}^{n+1}\|_{\mathbf{L}^2}^2 + \frac{\Delta t}{2}\|\left(\frac{\nu}{k}\right)^{1/2}\mathbf{u}^{n+1}\|_{\mathbf{L}^2}^2 \\ \le \|\mathbf{u}^n\|_{\mathbf{L}^2}^2 + \Delta t^2\|p^{n-1/2}\|_A^2 + \frac{\Delta t}{2}\|\nu^{1/2}\nabla\mathbf{u}^n\|_{\mathbf{L}^2}^2 + \frac{\Delta t}{2}\|\left(\frac{\nu}{k}\right)^{1/2}\mathbf{u}^n\|_{\mathbf{L}^2}^2. \end{aligned} \tag{18}$$

The final estimate is obtained by summing (18) over the time levels from 0 to $N - 1 = T/\Delta t - 1$. □

3.2. **Spatial discretization.** The equations (9–12) are discretized in space using the classical MAC stencil (see [8], figure 1). When discretizing the momentum equation in complex-shaped domains in the case of a MAC grid, it is necessary to take special care for computing the Brinkman term $\nu\mathbf{u}/k$, so that the accuracy is preserved. Essentially, such procedures compute some approximate or exact average of the coefficient ν/k over the MAC cells intersected by a boundary marking a jump in this quantity (see for example [11], section 3.2).

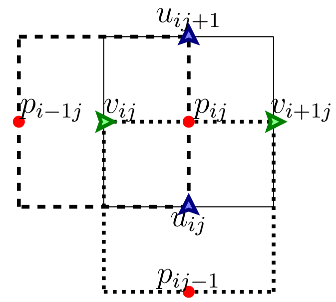


FIGURE 1. Control volumes associated with each node in case of MAC grid

The conservative finite volume approximation of the second derivative on a grid of a nonuniform grid size h_i is given by

(19)
$$D_x u^n_{i,j,k} = \frac{2}{h_{i+1}+h_i}\left(\bar{\nu}_{i-1/2}\frac{u^n_{i,j,k}-u^n_{i-1,j,k}}{h_i} - \bar{\nu}_{i+1/2}\frac{u^n_{i+1,j,k}-u^n_{i,j,k}}{h_{i+1}}\right),$$

where

$$\bar{\nu}_{i-1/2} = \left(\int_{-1}^{0}\frac{ds}{\nu(x_i+sh_i,y,z)}\right)^{-1} \tag{20}$$

is the average of ν on $[x_{i,j,k}, x_{i-1,j,k}]$, and $h_i = x_{i,j,k} - x_{i-1,j,k}, \quad i = 2,\dots,L$. Note that if the integral in (20) is approximated by means of the trapezoidal quadrature, $\bar{\nu}$ would be equal to the harmonic average of ν over a given discretization cell. The central difference in the y and z directions can be constructed in the same manner.

Similarly, the average Brinkman porosity at the point with x-coordinate x_i can be computed as:

$$\bar{k}_i = \left(\int_{0}^{1}\frac{ds}{k(x_{i-1/2}+0.5s(h_i+h_{i+1}),y,z)}\right)^{-1} \tag{21}$$

An alternative spatial approximation is provided by the stabilized central difference discretization on a collocated grid for the velocity and pressure proposed by Rhie and Chow [10]. All the test cases presented in the next section were also computed using such a collocated approximation and the results (not reported here) showed a very similar behaviour as the one computed on the MAC grid (presented below).

3.3. **Computational cost of the direction splitting operator vs. Laplace operator in the pressure correction equation.** For simplicity, let us consider a cubic domain covered with a grid containing $n \times n \times n$ nodes. Under the reasonable assumption that the cost for assembling the matrices is small compared to the cost of solving the systems of linear equations, let us evaluate the number of operations needed for computing the solution for the pressure *per time step.*

Direction splitting operator. In 3D, the Thomas algorithm for tri-diagonal matrices has to be employed $3n^2$ times (i.e., n^2 times in each direction). Each Thomas algorithm requires $5n$ multiplications/divisions and $3n$ summations/subtractions. So, the total of $15n^3 + 9n^3$ operations are required.

Laplace operator. Suppose now that we use an iterative method to solve the pressure Poisson equation [2]. Each iteration requires at least one matrix-vector multiplication, which for a seven-diagonal matrix will require $7n^3$ multiplications and $6n^3$ summations, thereby $7n^3 + 6n^3$ is the total number of operation for one matrix-vector multiplication.

These results clearly indicate that even if the iterative method converges with only a couple of iterations, the solution of pressure equation with a direction splitting operator and a direct tri-diagonal solver requires less operations than the iterative solution of the pressure Poisson equation.

4. SIMULATIONS

4.1. **Preliminaries.** The performance of the two approximations of the incompressibility constraint discussed here: the pressure Poisson equation, and the directionally factorized perturbation (10), is compared on two two-dimensional problems involving pure fluid and porous regions. The domain of the first problem is a channel with sudden contraction, containing a subdomain (called here porous subdomain) with a given permeability. If the permeability is very large, the Darcy term in the Brinkman equations tends to zero, and therefore the flow in the porous subdomain is unrestricted similarly to the rest of the fluid domain. For very small values of the permeability, the Darcy term dominates in the porous subdomain, and completely prevents the flow through it. In fact, in this case the Stokes-Brinkman equations become a Fictitious Domain formulation for the Stokes equations with no-slip

---

[2] Note that the Poisson equation can be solved using fast Fourier transform (FFT). The parallel performance of the FFT algorithm, however, is much worse than that of a Thomas-based Schur complement approach (see [6])

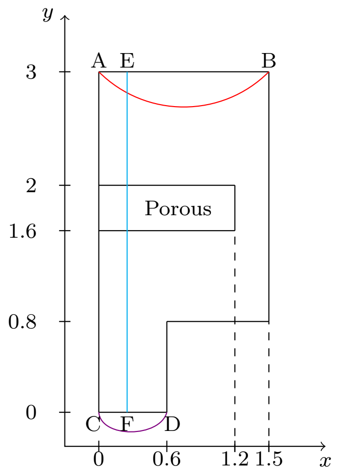


FIGURE 2. Sketch of a channel with a sudden contraction and a porous obstacle.

boundary conditions on the boundary of the ”porous” subdomain. The domain of the second problem is a channel with two porous subdomains. In this case time dependent boundary conditions are applied.

The main goal of these simulations is to compare the results for the velocity and pressure computed with the classical Poisson equation for the pressure increment (classical incremental projection scheme in a rotational form; see [5]), and the factorized operator A defined above. Therefore, we define $\mathbf{u}_\Delta, p_\Delta$ to be the velocity and pressure calculated using the classical projection scheme, and $\mathbf{u}_A, p_A$ – the velocity and pressure calculated using the scheme with the operator A in the pressure correction step.

In both test cases presented below the viscosity is set to $\nu = 10^{-6} m^2 \cdot s^{-1}$. The momentum equation for both schemes, and the pressure-Poisson equation in the classical projection scheme are solved by a generalized minimal residual method with ILU preconditioner (see [1] for implementation details). All simulations were performed on a machine with a dual core Intel Xeon 5148LV with 8 GB RAM .

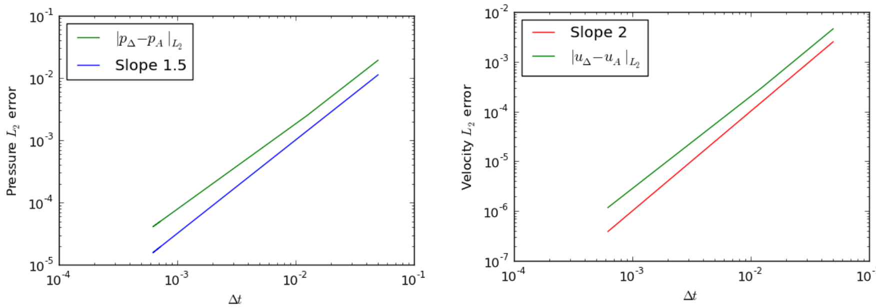
4.2. **Flow in a channel with a sudden contraction and a porous obstacle.** As a first test case, consider the flow in a channel with a sudden contraction and a porous obstacle (see figure 2). No-slip boundary conditions are prescribed on the solid walls, while parabolic profile for the velocity at the inlet and the outlet is prescribed. The usual zero Neumann boundary condition on the entire boundary is imposed on the pressure correction.

$$\begin{aligned} u_{inlet} = &\quad \alpha_1 * x * (x - 1.5), \quad 0 \leq x \leq 1.5, \quad y = 3 \\ u_{outlet} = &\quad \alpha_2 * x * (x - 0.6), \quad 0 \leq x \leq 1.5, \quad y = 0 \end{aligned}$$

Coefficients α_1 and α_2 are specified so that the flow rates at both ends of the channel are equal:

$$\int_{x1}^{x2} \alpha * (x - x_1) * (x - x_2) = Q, \text{ where Q is the flow rate}$$

We show in figure 3 the L^2 norm of the difference of the two velocities and the pressures, $\|u_\Delta - u_A\|_{L_2}$ and $\|p_\Delta - p_A\|_{L_2}$, as a function of Δt. Clearly, the rate of decrease of both errors is similar to the theoretical estimates for the convergence error of the classical incremental projection scheme in a rotational form (see [7]): second order for the velocity and order 3/2 for the pressure error in the L^2 norm. At the same time, the simulations with the directional splitting approach (using the factorized operator A) are significantly faster (see table 1).

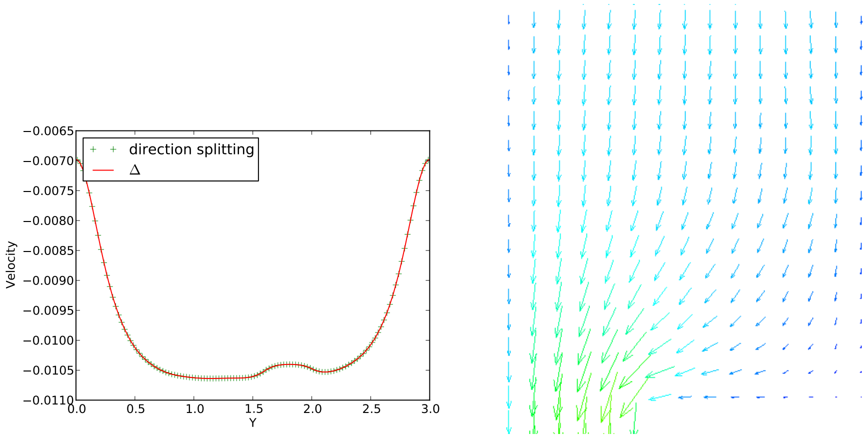


(A) Pressure L^2 error: $\|p_\Delta - p_A\|_{L_2}$

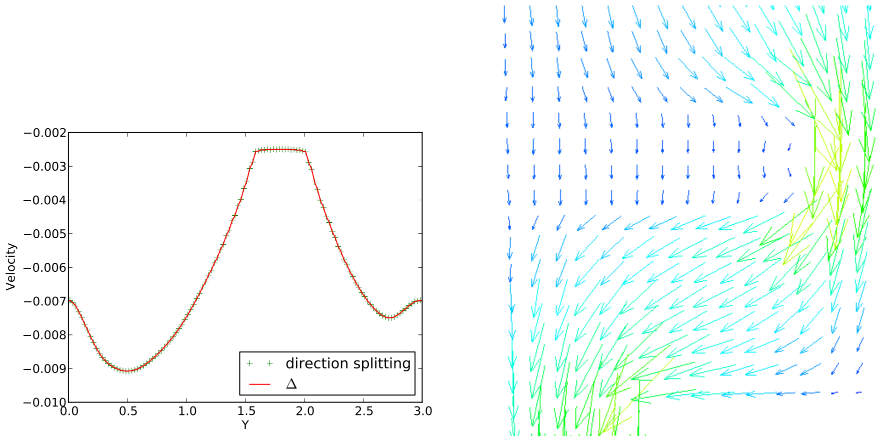
(B) Velocity L^2 error: $\|u_\Delta - u_A\|_{L_2}$

FIGURE 3. Pressure and velocity error; test case of section 4.2.

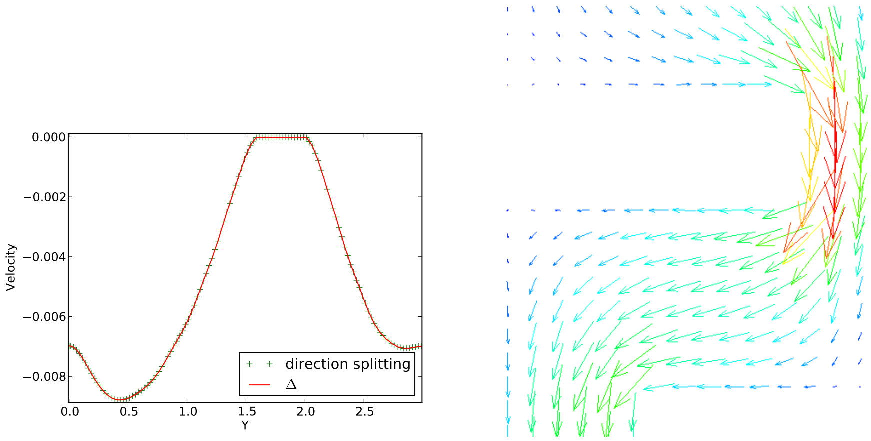
In the left panel of figure 4 we display the horizontal profiles of the velocity along the vertical segment EF (see figure 2) at various permeabilities. The corresponding velocity vectors are displayed in the right panel of figure 4.



High permeability ($k = 10^{-4}$).



Moderate permeability ($k = 10^{-6}$).



Extremely low permeability ($k = 10^{-8}$).

FIGURE 4. Velocity in the segment EF and the corresponding velocity field for different values of permeability; test case of section 4.2.

Table 1. CPU time comparison; test case of section 4.2

h/N	Δt	Tolerance for iterative solver	Time for Laplace	Time for DS operator	Speed up
0.01/37800	1e-5	1e-8	18.18	2.35	7.7
0.01/37800	1e-5	1e-12	34.12	2.35	14.52
0.01/37800	1e-7	1e-8	22.14	2.35	9.42
0.01/37800	1e-7	1e-12	28.89	2.35	12.3
0.005/151200	1e-5	1e-8	86.12	6.57	13.1
0.001/151200	1e-5	1e-12	97.43	6.57	14.8
0.001/151200	1e-7	1e-8	78.16	6.57	11.8
0.001/151200	1e-7	1e-12	84.25	6.57	12.8

4.3. **Flow in a channel with two porous obstacles and time dependent boundary conditions.** To further illustrate the properties of the factorized scheme, consider the flow in a vertical channel with two porous blocks (see figure 5). No-slip boundary conditions are prescribed on the solid walls, while time dependent profiles for the velocity at the inlet and the outlet are prescribed:

$$u_{inlet} = u_{outlet} = -(\pi + \sin(\alpha t)),$$
$$0 \le t \le T,\ 0 \le x \le 1.5,\ 0 \le y \le 1$$

where α is a frequency parameter.

In terms of figure 5, the segment AB is the inlet, and the segment CD is the outlet. In figure 6 we show the error on the velocity and the pressure as a function of Δt, and in figure 7 – the vertical profiles of the velocity along the vertical segment EF, as well as the velocity vectors in the entire domain. As in the previous test case, the velocity and pressure errors have a very similar convergence rate to the corresponding convergence errors of the classical incremental projection scheme in a rotational form.

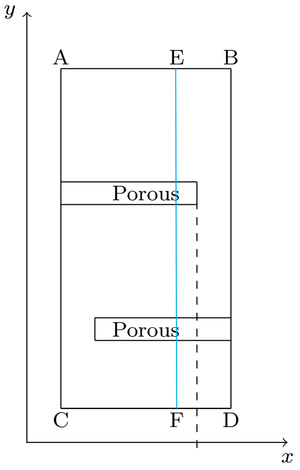
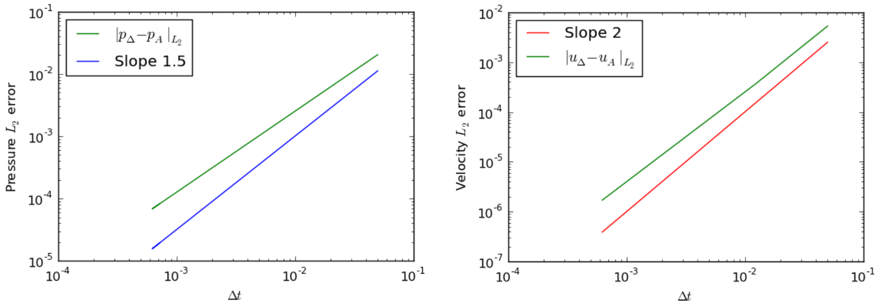


Figure 5. Sketch of the channel with two porous obstacles.

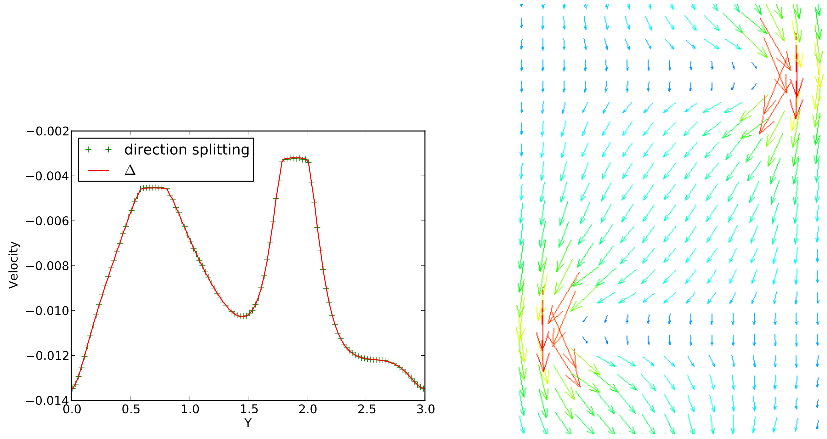


(A) Pressure L^2 error: $\|p_\Delta - p_A\|_{L_2}$

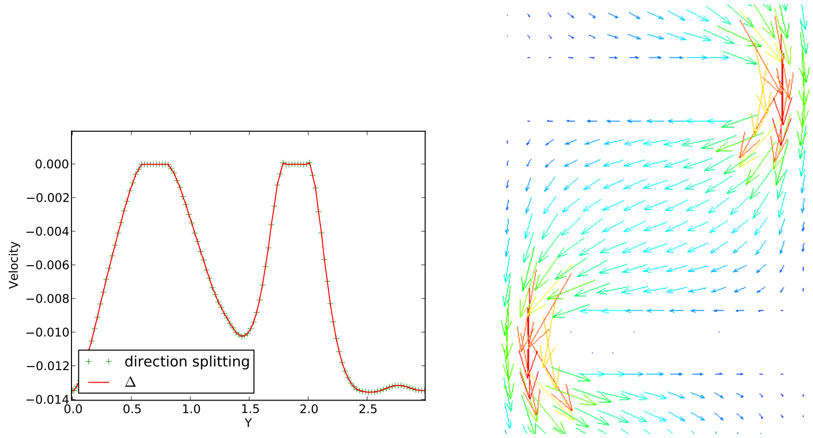
(B) Velocity L^2 error: $\|\mathbf{u_\Delta} - \mathbf{u_A}\|_{\mathbf{L_2}}$

Figure 6. Pressure and velocity error; test case of section 4.3.

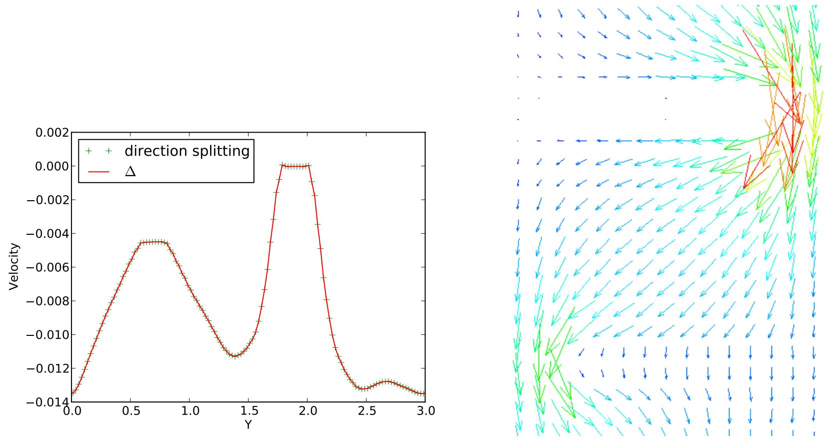
Table 2 shows the CPU time comparison for the two schemes, again the simulations with the directional splitting approach being significantly faster .



Moderate permeability in both porous areas ($k_{upper} = k_{lower} = 10^{-6}$).



Extremely low permeability in both porous areas ($k_{upper} = k_{lower} = 10^{-8}$).



Low permeability in the upper area ($k_{upper} = 10^{-8}$), and moderate permeability in the lower area ($k_{lower} = 10^{-6}$)

FIGURE 7. Velocity in the slice and corresponding velocity vectors for different values of permeability; test case of section 4.3.

Table 2. CPU time comparison; test case of section 4.3.

h/N	Δt	Tolerance for iterative solver	Time for Laplace	Time for DS operator	Speed up
0.01/45000	1e-5	1e-8	20.24	2.4	8.61
0.01/45000	1e-5	1e-12	38.12	2.4	16.2
0.01/45000	1e-7	1e-8	27.14	2.4	11.54
0.01/45000	1e-7	1e-12	33.89	2.4	14.42
0.005/180000	1e-5	1e-8	88.15	6.8	12.9
0.001/180000	1e-5	1e-12	99.4	6.8	14.6
0.001/180000	1e-7	1e-8	79.25	6.8	11.6
0.001/180000	1e-7	1e-12	85.13	6.8	12.5

5. Conclusions

The results presented in this article demonstrate that the direction-factorized perturbation of the incompressibility constraint in the Stokes equations proposed in [6] can be applied to the Stokes-Brinkman equations yielding an unconditionally stable scheme. We do not consider here the possibility for a direction splitting of the momentum equations since it would generally yield non-commutative one-dimensional operators which significantly complicates the stability estimate. Although some possibilities for such a splitting of the momentum equation in the non-commutative case are discussed in [3] the stability of of the overall algorithm in the Stokes or Stokes-Brinkman case is still an open problem.

The presented numerical results clearly demonstrate on two test cases involving fluid and porous areas, that the difference between the velocities and pressures computed with the current direction-slitting approach, and the classical incremental projection scheme in a rotational form decrease as $O(\Delta t^2)$ and $O(\Delta t^{3/2})$ respectively. In other words, the convergence rates in time of the two algorithms are close to the theoretical estimates for the incremental projection scheme in a rotational form.

Acknowledgements

The research of the first and the third authors (TG and OI) is supported by grant from German BMWi No. 11224. The second author

(JLG) is partially supported by Award No. KUS -C1-016-04, made by King Abdullah University of Science and Technology (KAUST). The fourth author (PM) acknowledges the support of an NSERC Discovery grant and the support of ITWM, Kaiserslautern.

References

[1] Scipy v0.11.dev reference guide.

[2] Ph. Angot. Analysis of singular perturbations on the Brinkman problem for fictitious domain models of viscous flows. *Math. Meth. Applied Sciences*, 22(16):1395–1412, 1999.

[3] Ph. Angot, J. Keating, and P.D. Minev. A direction splitting algorithm for incompressible flow in complex geometries. *Comp. Meth. Appl. Mech. Engng.* In print.

[4] A.J. Chorin. Numerical solution of the Navier-Stokes equations. *Math. Comp.*, 22:745–762, 1968.

[5] J.-L. Guermond, P. Minev, and J. Shen. An overview of projection methods for incompressible flows. *Comput. Methods Appl. Mech. Engrg.*, 195:6011–6054, 2006.

[6] J.-L. Guermond and P.D. Minev. A new class of splitting methods for the incompressible Navier-Stokes equations using direction splitting. *Comput. Methods Appl. Mech. Engrg.*, 200:2083–2093, 2011.

[7] J. L. Guermond and J. Shen. On the error estimates for the rotational pressure-correction projection methods. *Math. Comp.*, 73(248):1719–1737, 2004.

[8] F. Harlow and J.E. Welch. Numerical calculation of time-dependent viscous incompressible ow of uid with a free surface. *Phys. Fluids*, 8(2):2182–2189, 1965.

[9] J. Douglas Jr. Alternating direction methods for three space variables. *Numer. Math.*, 4:41–63, 1962.

[10] C.M. Rhie and W.L. Chow. Numerical study of the turbulent flow past an airfoil with trailing edge separation. *AIAA J.*, 21:1525–1532, 1983.

[11] A. Samarskii. *The theory of difference schemes*. Marcel Dekker, Inc., 2001.

[12] R. Temam. Sur l'approximation de la solution des équations de Navier-Stokes par la méthode des pas fractionnaires ii. *Arch. Rat. Mech. Anal.*, 33:377–385, 1969.

Published reports of the Fraunhofer ITWM

The PDF-files of the following reports are available under:
www.itwm.fraunhofer.de/de/ zentral__berichte/berichte

1. D. Hietel, K. Steiner, J. Struckmeier
A Finite - Volume Particle Method for Compressible Flows
(19 pages, 1998)

2. M. Feldmann, S. Seibold
Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 pages, 1998)

3. Y. Ben-Haim, S. Seibold
Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 pages, 1998)

4. F.-Th. Lentes, N. Siedow
Three-dimensional Radiative Heat Transfer in Glass Cooling Processes
(23 pages, 1998)

5. A. Klar, R. Wegener
A hierarchy of models for multilane vehicular traffic
Part I: Modeling
(23 pages, 1998)

Part II: Numerical and stochastic investigations
(17 pages, 1998)

6. A. Klar, N. Siedow
Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes
(24 pages, 1998)

7. I. Choquet
Heterogeneous catalysis modelling and numerical simulation in rarified gas flows Part I: Coverage locally at equilibrium
(24 pages, 1998)

8. J. Ohser, B. Steinbach, C. Lang
Efficient Texture Analysis of Binary Images
(17 pages, 1998)

9. J. Orlik
Homogenization for viscoelasticity of the integral type with aging and shrinkage
(20 pages, 1998)

10. J. Mohring
Helmholtz Resonators with Large Aperture
(21 pages, 1998)

11. H. W. Hamacher, A. Schöbel
On Center Cycles in Grid Graphs
(15 pages, 1998)

12. H. W. Hamacher, K.-H. Küfer
Inverse radiation therapy planning - a multiple objective optimisation approach
(14 pages, 1999)

13. C. Lang, J. Ohser, R. Hilfer
On the Analysis of Spatial Binary Images
(20 pages, 1999)

14. M. Junk
On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes
(24 pages, 1999)

15. M. Junk, S. V. Raghurame Rao
A new discrete velocity method for Navier-Stokes equations
(20 pages, 1999)

16. H. Neunzert
Mathematics as a Key to Key Technologies
(39 pages, 1999)

17. J. Ohser, K. Sandau
Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem
(18 pages, 1999)

18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel
Solving nonconvex planar location problems by finite dominating sets
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 pages, 2000)

19. A. Becker
A Review on Image Distortion Measures
Keywords: Distortion measure, human visual system
(26 pages, 2000)

20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn
Polyhedral Properties of the Uncapacitated Multiple Allocation Hub Location Problem
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 pages, 2000)

21. H. W. Hamacher, A. Schöbel
Design of Zone Tariff Systems in Public Transportation
(30 pages, 2001)

22. D. Hietel, M. Junk, R. Keck, D. Teleaga
The Finite-Volume-Particle Method for Conservation Laws
(16 pages, 2001)

23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel
Location Software and Interface with GIS and Supply Chain Management
Keywords: facility location, software development, geographical information systems, supply chain management
(48 pages, 2001)

24. H. W. Hamacher, S. A. Tjandra
Mathematical Modelling of Evacuation Problems: A State of Art
(44 pages, 2001)

25. J. Kuhnert, S. Tiwari
Grid free method for solving the Poisson equation
Keywords: Poisson equation, Least squares method, Grid free method
(19 pages, 2001)

26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier
Simulation of the fiber spinning process
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD
(19 pages, 2001)

27. A. Zemitis
On interaction of a liquid film with an obstacle
Keywords: impinging jets, liquid film, models, numerical solution, shape
(22 pages, 2001)

28. I. Ginzburg, K. Steiner
Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models
(22 pages, 2001)

29. H. Neunzert
»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«
Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrskalenanalyse, Strömungsmechanik
(18 pages, 2001)

30. J. Kuhnert, S. Tiwari
Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation
AMS subject classification: 76D05, 76M28
(25 pages, 2001)

31. R. Korn, M. Krekel
Optimal Portfolios with Fixed Consumption or Income Streams
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems
(23 pages, 2002)

32. M. Krekel
Optimal portfolios with a loan dependent credit spread
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics
(25 pages, 2002)

33. J. Ohser, W. Nagel, K. Schladitz
The Euler number of discretized sets – on the choice of adjacency in homogeneous lattices
Keywords: image analysis, Euler number, neighborhod relationships, cuboidal lattice
(32 pages, 2002)

34. I. Ginzburg, K. Steiner
Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting
Keywords: Lattice Boltzmann models; free-surface phenomena; interface boundary conditions; filling processes; injection molding; volume of fluid method; interface boundary conditions; advection-schemes; upwind-schemes
(54 pages, 2002)

35. M. Günther, A. Klar, T. Materne, R. Wegener
Multivalued fundamental diagrams and stop and go waves for continuum traffic equations
Keywords: traffic flow, macroscopic equations, kinetic derivation, multivalued fundamental diagram, stop and go waves, phase transitions
(25 pages, 2002)

36. S. Feldmann, P. Lang, D. Prätzel-Wolters
Parameter influence on the zeros of network determinants
Keywords: Networks, Equicofactor matrix polynomials, Realization theory, Matrix perturbation theory
(30 pages, 2002)

37. K. Koch, J. Ohser, K. Schladitz
Spectral theory for random closed sets and estimating the covariance via frequency space
Keywords: Random set, Bartlett spectrum, fast Fourier transform, power spectrum
(28 pages, 2002)

38. D. d'Humières, I. Ginzburg
Multi-reflection boundary conditions for lattice Boltzmann models
Keywords: lattice Boltzmann equation, boudary condistions, bounce-back rule, Navier-Stokes equation
(72 pages, 2002)

39. R. Korn
Elementare Finanzmathematik
Keywords: Finanzmathematik, Aktien, Optionen, Portfolio-Optimierung, Börse, Lehrerweiterbildung, Mathematikunterricht
(98 pages, 2002)

40. J. Kallrath, M. C. Müller, S. Nickel
Batch Presorting Problems:
Models and Complexity Results
Keywords: Complexity theory, Integer programming, Assigment, Logistics
(19 pages, 2002)

41. J. Linn
On the frame-invariant description of the phase space of the Folgar-Tucker equation
Key words: fiber orientation, Folgar-Tucker equation, injection molding
(5 pages, 2003)

42. T. Hanne, S. Nickel
A Multi-Objective Evolutionary Algorithm for Scheduling and Inspection Planning in Software Development Projects
Key words: multiple objective programming, project management and scheduling, software development, evolutionary algorithms, efficient set
(29 pages, 2003)

43. T. Bortfeld , K.-H. Küfer, M. Monz, A. Scherrer, C. Thieke, H. Trinkaus
Intensity-Modulated Radiotherapy - A Large Scale Multi-Criteria Programming Problem
Keywords: multiple criteria optimization, representative systems of Pareto solutions, adaptive triangulation, clustering and disaggregation techniques, visualization of Pareto solutions, medical physics, external beam radiotherapy planning, intensity modulated radiotherapy
(31 pages, 2003)

44. T. Halfmann, T. Wichmann
Overview of Symbolic Methods in Industrial Analog Circuit Design
Keywords: CAD, automated analog circuit design, symbolic analysis, computer algebra, behavioral modeling, system simulation, circuit sizing, macro modeling, differential-algebraic equations, index
(17 pages, 2003)

45. S. E. Mikhailov, J. Orlik
Asymptotic Homogenisation in Strength and Fatigue Durability Analysis of Composites
Keywords: multiscale structures, asymptotic homogenization, strength, fatigue, singularity, non-local conditions
(14 pages, 2003)

46. P. Domínguez-Marín, P. Hansen, N. Mladenovic , S. Nickel
Heuristic Procedures for Solving the Discrete Ordered Median Problem
Keywords: genetic algorithms, variable neighborhood search, discrete facility location
(31 pages, 2003)

47. N. Boland, P. Domínguez-Marín, S. Nickel, J. Puerto
Exact Procedures for Solving the Discrete Ordered Median Problem
Keywords: discrete location, Integer programming
(41 pages, 2003)

48. S. Feldmann, P. Lang
Padé-like reduction of stable discrete linear systems preserving their stability
Keywords: Discrete linear systems, model reduction, stability, Hankel matrix, Stein equation
(16 pages, 2003)

49. J. Kallrath, S. Nickel
A Polynomial Case of the Batch Presorting Problem
Keywords: batch presorting problem, online optimization, competetive analysis, polynomial algorithms, logistics
(17 pages, 2003)

50. T. Hanne, H. L. Trinkaus
knowCube for MCDM –
Visual and Interactive Support for Multicriteria Decision Making
Key words: Multicriteria decision making, knowledge management, decision support systems, visual interfaces, interactive navigation, real-life applications.
(26 pages, 2003)

51. O. Iliev, V. Laptev
On Numerical Simulation of Flow Through Oil Filters
Keywords: oil filters, coupled flow in plain and porous media, Navier-Stokes, Brinkman, numerical simulation
(8 pages, 2003)

52. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva
On a Multigrid Adaptive Refinement Solver for Saturated Non-Newtonian Flow in Porous Media
Keywords: Nonlinear multigrid, adaptive refinement, non-Newtonian flow in porous media
(17 pages, 2003)

53. S. Kruse
On the Pricing of Forward Starting Options under Stochastic Volatility
Keywords: Option pricing, forward starting options, Heston model, stochastic volatility, cliquet options
(11 pages, 2003)

54. O. Iliev, D. Stoyanov
Multigrid – adaptive local refinement solver for incompressible flows
Keywords: Navier-Stokes equations, incompressible flow, projection-type splitting, SIMPLE, multigrid methods, adaptive local refinement, lid-driven flow in a cavity
(37 pages, 2003)

55. V. Starikovicius
The multiphase flow and heat transfer in porous media
Keywords: Two-phase flow in porous media, various formulations, global pressure, multiphase mixture model, numerical simulation
(30 pages, 2003)

56. P. Lang, A. Sarishvili, A. Wirsen
Blocked neural networks for knowledge extraction in the software development process
Keywords: Blocked Neural Networks, Nonlinear Regression, Knowledge Extraction, Code Inspection
(21 pages, 2003)

57. H. Knaf, P. Lang, S. Zeiser
Diagnosis aiding in Regulation
Thermography using Fuzzy Logic
Keywords: fuzzy logic,knowledge representation, expert system
(22 pages, 2003)

58. M. T. Melo, S. Nickel, F. Saldanha da Gama
Largescale models for dynamic multicommodity capacitated facility location
Keywords: supply chain management, strategic planning, dynamic location, modeling
(40 pages, 2003)

59. J. Orlik
Homogenization for contact problems with periodically rough surfaces
Keywords: asymptotic homogenization, contact problems
(28 pages, 2004)

60. A. Scherrer, K.-H. Küfer, M. Monz, F. Alonso, T. Bortfeld
IMRT planning on adaptive volume structures – a significant advance of computational complexity
Keywords: Intensity-modulated radiation therapy (IMRT), inverse treatment planning, adaptive volume structures, hierarchical clustering, local refinement, adaptive clustering, convex programming, mesh generation, multi-grid methods
(24 pages, 2004)

61. D. Kehrwald
Parallel lattice Boltzmann simulation of complex flows
Keywords: Lattice Boltzmann methods, parallel computing, microstructure simulation, virtual material design, pseudo-plastic fluids, liquid composite moulding
(12 pages, 2004)

62. O. Iliev, J. Linn, M. Moog, D. Niedziela, V. Starikovicius
On the Performance of Certain Iterative Solvers for Coupled Systems Arising in Discretization of Non-Newtonian Flow Equations

Keywords: Performance of iterative solvers, Preconditioners, Non-Newtonian flow
(17 pages, 2004)

63. R. Ciegis, O. Iliev, S. Rief, K. Steiner
On Modelling and Simulation of Different Regimes for Liquid Polymer Moulding
Keywords: Liquid Polymer Moulding, Modelling, Simulation, Infiltration, Front Propagation, non-Newtonian flow in porous media
(43 pages, 2004)

64. T. Hanne, H. Neu
Simulating Human Resources in Software Development Processes
Keywords: Human resource modeling, software process, productivity, human factors, learning curve
(14 pages, 2004)

65. O. Iliev, A. Mikelic, P. Popov
Fluid structure interaction problems in deformable porous media: Toward permeability of deformable porous media
Keywords: fluid-structure interaction, deformable porous media, upscaling, linear elasticity, stokes, finite elements
(28 pages, 2004)

66. F. Gaspar, O. Iliev, F. Lisbona, A. Naumovich, P. Vabishchevich
On numerical solution of 1-D poroelasticity equations in a multilayered domain
Keywords: poroelasticity, multilayered material, finite volume discretization, MAC type grid
(41 pages, 2004)

67. J. Ohser, K. Schladitz, K. Koch, M. Nöthe
Diffraction by image processing and its application in materials science
Keywords: porous microstructure, image analysis, random set, fast Fourier transform, power spectrum, Bartlett spectrum
(13 pages, 2004)

68. H. Neunzert
Mathematics as a Technology: Challenges for the next 10 Years
Keywords: applied mathematics, technology, modelling, simulation, visualization, optimization, glass processing, spinning processes, fiber-fluid interaction, trubulence effects, topological optimization, multicriteria optimization, Uncertainty and Risk, financial mathematics, Malliavin calculus, Monte-Carlo methods, virtual material design, filtration, bio-informatics, system biology
(29 pages, 2004)

69. R. Ewing, O. Iliev, R. Lazarov, A. Naumovich
On convergence of certain finite difference discretizations for 1D poroelasticity interface problems
Keywords: poroelasticity, multilayered material, finite volume discretizations, MAC type grid, error estimates
(26 pages,2004)

70. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva
On Efficient Simulation of Non-Newtonian Flow in Saturated Porous Media with a Multigrid Adaptive Refinement Solver
Keywords: Nonlinear multigrid, adaptive renement, non-Newtonian in porous media
(25 pages, 2004)

71. J. Kalcsics, S. Nickel, M. Schröder
Towards a Unified Territory Design Approach – Applications, Algorithms and GIS Integration
Keywords: territory desgin, political districting, sales territory alignment, optimization algorithms, Geographical Information Systems
(40 pages, 2005)

72. K. Schladitz, S. Peters, D. Reinel-Bitzer, A. Wiegmann, J. Ohser
Design of acoustic trim based on geometric modeling and flow simulation for non-woven
Keywords: random system of fibers, Poisson line process, flow resistivity, acoustic absorption, Lattice-Boltzmann method, non-woven
(21 pages, 2005)

73. V. Rutka, A. Wiegmann
Explicit Jump Immersed Interface Method for virtual material design of the effective elastic moduli of composite materials
Keywords: virtual material design, explicit jump immersed interface method, effective elastic moduli, composite materials
(22 pages, 2005)

74. T. Hanne
Eine Übersicht zum Scheduling von Baustellen
Keywords: Projektplanung, Scheduling, Bauplanung, Bauindustrie
(32 pages, 2005)

75. J. Linn
The Folgar-Tucker Model as a Differetial Algebraic System for Fiber Orientation Calculation
Keywords: fiber orientation, Folgar–Tucker model, invariants, algebraic constraints, phase space, trace stability
(15 pages, 2005)

76. M. Speckert, K. Dreßler, H. Mauch, A. Lion, G. J. Wierda
Simulation eines neuartigen Prüfsystems für Achserprobungen durch MKS-Modellierung einschließlich Regelung
Keywords: virtual test rig, suspension testing, multibody simulation, modeling hexapod test rig, optimization of test rig configuration
(20 pages, 2005)

77. K.-H. Küfer, M. Monz, A. Scherrer, P. Süss, F. Alonso, A. S. A. Sultan, Th. Bortfeld, D. Craft, Chr. Thieke
Multicriteria optimization in intensity modulated radiotherapy planning
Keywords: multicriteria optimization, extreme solutions, real-time decision making, adaptive approximation schemes, clustering methods, IMRT planning, reverse engineering
(51 pages, 2005)

78. S. Amstutz, H. Andrä
A new algorithm for topology optimization using a level-set method
Keywords: shape optimization, topology optimization, topological sensitivity, level-set
(22 pages, 2005)

79. N. Ettrich
Generation of surface elevation models for urban drainage simulation
Keywords: Flooding, simulation, urban elevation models, laser scanning
(22 pages, 2005)

80. H. Andrä, J. Linn, I. Matei, I. Shklyar, K. Steiner, E. Teichmann
OPTCAST – Entwicklung adäquater Strukturoptimierungsverfahren für Gießereien Technischer Bericht (KURZFASSUNG)
Keywords: Topologieoptimierung, Level-Set-Methode, Gießprozesssimulation, Gießtechnische Restriktionen, CAE-Kette zur Strukturoptimierung
(77 pages, 2005)

81. N. Marheineke, R. Wegener
Fiber Dynamics in Turbulent Flows
Part I: General Modeling Framework
Keywords: fiber-fluid interaction; Cosserat rod; turbulence modeling; Kolmogorov's energy spectrum; double-velocity correlations; differentiable Gaussian fields
(20 pages, 2005)

Part II: Specific Taylor Drag
Keywords: flexible fibers; k-ε *turbulence model; fiber-turbulence interaction scales; air drag; random Gaussian aerodynamic force; white noise; stochastic differential equations; ARMA process*
(18 pages, 2005)

82. C. H. Lampert, O. Wirjadi
An Optimal Non-Orthogonal Separation of the Anisotropic Gaussian Convolution Filter
Keywords: Anisotropic Gaussian filter, linear filtering, orientation space, nD image processing, separable filters
(25 pages, 2005)

83. H. Andrä, D. Stoyanov
Error indicators in the parallel finite element solver for linear elasticity DDFEM
Keywords: linear elasticity, finite element method, hierarchical shape functions, domain decom-position, parallel implementation, a posteriori error estimates
(21 pages, 2006)

84. M. Schröder, I. Solchenbach
Optimization of Transfer Quality in Regional Public Transit
Keywords: public transit, transfer quality, quadratic assignment problem
(16 pages, 2006)

85. A. Naumovich, F. J. Gaspar
On a multigrid solver for the three-dimensional Biot poroelasticity system in multilayered domains
Keywords: poroelasticity, interface problem, multigrid, operator-dependent prolongation
(11 pages, 2006)

86. S. Panda, R. Wegener, N. Marheineke
Slender Body Theory for the Dynamics of Curved Viscous Fibers
Keywords: curved viscous fibers; fluid dynamics; Navier-Stokes equations; free boundary value problem; asymptotic expansions; slender body theory
(14 pages, 2006)

87. E. Ivanov, H. Andrä, A. Kudryavtsev
Domain Decomposition Approach for Automatic Parallel Generation of Tetrahedral Grids
Key words: Grid Generation, Unstructured Grid, Delaunay Triangulation, Parallel Programming, Domain Decomposition, Load Balancing
(18 pages, 2006)

88. S. Tiwari, S. Antonov, D. Hietel, J. Kuhnert, R. Wegener
A Meshfree Method for Simulations of Interactions between Fluids and Flexible Structures
Key words: Meshfree Method, FPM, Fluid Structure Interaction, Sheet of Paper, Dynamical Coupling
(16 pages, 2006)

89. R. Ciegis , O. Iliev, V. Starikovicius, K. Steiner
Numerical Algorithms for Solving Problems of Multiphase Flows in Porous Media
Keywords: nonlinear algorithms, finite-volume method, software tools, porous media, flows
(16 pages, 2006)

90. D. Niedziela, O. Iliev, A. Latz
On 3D Numerical Simulations of Viscoelastic Fluids
Keywords: non-Newtonian fluids, anisotropic viscosity, integral constitutive equation
(18 pages, 2006)

91. A. Winterfeld
Application of general semi-infinite Programming to Lapidary Cutting Problems
Keywords: large scale optimization, nonlinear programming, general semi-infinite optimization, design centering, clustering
(26 pages, 2006)

92. J. Orlik, A. Ostrovska
Space-Time Finite Element Approximation and Numerical Solution of Hereditary Linear Viscoelasticity Problems
Keywords: hereditary viscoelasticity; kern approximation by interpolation; space-time finite element approximation, stability and a priori estimate
(24 pages, 2006)

93. V. Rutka, A. Wiegmann, H. Andrä
EJIIM for Calculation of effective Elastic Moduli in 3D Linear Elasticity
Keywords: Elliptic PDE, linear elasticity, irregular domain, finite differences, fast solvers, effective elastic moduli
(24 pages, 2006)

94. A. Wiegmann, A. Zemitis
EJ-HEAT: A Fast Explicit Jump Harmonic Averaging Solver for the Effective Heat Conductivity of Composite Materials
Keywords: Stationary heat equation, effective thermal conductivity, explicit jump, discontinuous coefficients, virtual material design, microstructure simulation, EJ-HEAT
(21 pages, 2006)

95. A. Naumovich
On a finite volume discretization of the three-dimensional Biot poroelasticity system in multilayered domains
Keywords: Biot poroelasticity system, interface problems, finite volume discretization, finite difference method
(21 pages, 2006)

96. M. Krekel, J. Wenzel
A unified approach to Credit Default Swaption and Constant Maturity Credit Default Swap valuation
Keywords: LIBOR market model, credit risk, Credit Default Swaption, Constant Maturity Credit Default Swap-method
(43 pages, 2006)

97. A. Dreyer
Interval Methods for Analog Circiuts
Keywords: interval arithmetic, analog circuits, tolerance analysis, parametric linear systems, frequency response, symbolic analysis, CAD, computer algebra
(36 pages, 2006)

98. N. Weigel, S. Weihe, G. Bitsch, K. Dreßler
Usage of Simulation for Design and Optimization of Testing
Keywords: Vehicle test rigs, MBS, control, hydraulics, testing philosophy
(14 pages, 2006)

99. H. Lang, G. Bitsch, K. Dreßler, M. Speckert
Comparison of the solutions of the elastic and elastoplastic boundary value problems
Keywords: Elastic BVP, elastoplastic BVP, variational inequalities, rate-independency, hysteresis, linear kinematic hardening, stop- and play-operator
(21 pages, 2006)

100. M. Speckert, K. Dreßler, H. Mauch
MBS Simulation of a hexapod based suspension test rig
Keywords: Test rig, MBS simulation, suspension, hydraulics, controlling, design optimization
(12 pages, 2006)

101. S. Azizi Sultan, K.-H. Küfer
A dynamic algorithm for beam orientations in multicriteria IMRT planning
Keywords: radiotherapy planning, beam orientation optimization, dynamic approach, evolutionary algorithm, global optimization
(14 pages, 2006)

102. T. Götz, A. Klar, N. Marheineke, R. Wegener
A Stochastic Model for the Fiber Lay-down Process in the Nonwoven Production
Keywords: fiber dynamics, stochastic Hamiltonian system, stochastic averaging
(17 pages, 2006)

103. Ph. Süss, K.-H. Küfer
Balancing control and simplicity: a variable aggregation method in intensity modulated radiation therapy planning
Keywords: IMRT planning, variable aggregation, clustering methods
(22 pages, 2006)

104. A. Beaudry, G. Laporte, T. Melo, S. Nickel
Dynamic transportation of patients in hospitals
Keywords: in-house hospital transportation, dial-a-ride, dynamic mode, tabu search
(37 pages, 2006)

105. Th. Hanne
Applying multiobjective evolutionary algorithms in industrial projects
Keywords: multiobjective evolutionary algorithms, discrete optimization, continuous optimization, electronic circuit design, semi-infinite programming, scheduling
(18 pages, 2006)

106. J. Franke, S. Halim
Wild bootstrap tests for comparing signals and images
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression
(13 pages, 2007)

107. Z. Drezner, S. Nickel
Solving the ordered one-median problem in the plane
Keywords: planar location, global optimization, ordered median, big triangle small triangle method, bounds, numerical experiments
(21 pages, 2007)

108. Th. Götz, A. Klar, A. Unterreiter,
R. Wegener
Numerical evidance for the non-existing of solutions of the equations desribing rotational fiber spinning
Keywords: rotational fiber spinning, viscous fibers, boundary value problem, existence of solutions
(11 pages, 2007)

109. Ph. Süss, K.-H. Küfer
Smooth intensity maps and the Bortfeld-Boyer sequencer
Keywords: probabilistic analysis, intensity modulated radiotherapy treatment (IMRT), IMRT plan application, step-and-shoot sequencing
(8 pages, 2007)

110. E. Ivanov, O. Gluchshenko, H. Andrä,
A. Kudryavtsev
Parallel software tool for decomposing and meshing of 3d structures
Keywords: a-priori domain decomposition, unstructured grid, Delaunay mesh generation
(14 pages, 2007)

111. O. Iliev, R. Lazarov, J. Willems
Numerical study of two-grid preconditioners for 1d elliptic problems with highly oscillating discontinuous coefficients
Keywords: two-grid algorithm, oscillating coefficients, preconditioner
(20 pages, 2007)

112. L. Bonilla, T. Götz, A. Klar, N. Marheineke,
R. Wegener
Hydrodynamic limit of the Fokker-Planck-equation describing fiber lay-down processes
Keywords: stochastic differential equations, Fokker-Planck equation, asymptotic expansion, Ornstein-Uhlenbeck process
(17 pages, 2007)

113. S. Rief
Modeling and simulation of the pressing section of a paper machine
Keywords: paper machine, computational fluid dynamics, porous media
(41 pages, 2007)

114. R. Ciegis, O. Iliev, Z. Lakdawala
On parallel numerical algorithms for simulating industrial filtration problems
Keywords: Navier-Stokes-Brinkmann equations, finite volume discretization method, SIMPLE, parallel computing, data decomposition method
(24 pages, 2007)

115. N. Marheineke, R. Wegener
Dynamics of curved viscous fibers with surface tension
Keywords: Slender body theory, curved viscous fibers with surface tension, free boundary value problem
(25 pages, 2007)

116. S. Feth, J. Franke, M. Speckert
Resampling-Methoden zur mse-Korrektur und Anwendungen in der Betriebsfestigkeit
Keywords: Weibull, Bootstrap, Maximum-Likelihood, Betriebsfestigkeit
(16 pages, 2007)

117. H. Knaf
Kernel Fisher discriminant functions – a concise and rigorous introduction
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression
(30 pages, 2007)

118. O. Iliev, I. Rybak
On numerical upscaling for flows in heterogeneous porous media

Keywords: numerical upscaling, heterogeneous porous media, single phase flow, Darcy's law, multiscale problem, effective permeability, multipoint flux approximation, anisotropy
(17 pages, 2007)

119. O. Iliev, I. Rybak
On approximation property of multipoint flux approximation method
Keywords: Multipoint flux approximation, finite volume method, elliptic equation, discontinuous tensor coefficients, anisotropy
(15 pages, 2007)

120. O. Iliev, I. Rybak, J. Willems
On upscaling heat conductivity for a class of industrial problems
Keywords: Multiscale problems, effective heat conductivity, numerical upscaling, domain decomposition
(21 pages, 2007)

121. R. Ewing, O. Iliev, R. Lazarov, I. Rybak
On two-level preconditioners for flow in porous media
Keywords: Multiscale problem, Darcy's law, single phase flow, anisotropic heterogeneous porous media, numerical upscaling, multigrid, domain decomposition, efficient preconditioner
(18 pages, 2007)

122. M. Brickenstein, A. Dreyer
POLYBORI: A Gröbner basis framework for Boolean polynomials
Keywords: Gröbner basis, formal verification, Boolean polynomials, algebraic cryptoanalysis, satisfiability
(23 pages, 2007)

123. O. Wirjadi
Survey of 3d image segmentation methods
Keywords: image processing, 3d, image segmentation, binarization
(20 pages, 2007)

124. S. Zeytun, A. Gupta
A Comparative Study of the Vasicek and the CIR Model of the Short Rate
Keywords: interest rates, Vasicek model, CIR-model, calibration, parameter estimation
(17 pages, 2007)

125. G. Hanselmann, A. Sarishvili
Heterogeneous redundancy in software quality prediction using a hybrid Bayesian approach
Keywords: reliability prediction, fault prediction, non-homogeneous poisson process, Bayesian model averaging
(17 pages, 2007)

126. V. Maag, M. Berger, A. Winterfeld, K.-H. Küfer
A novel non-linear approach to minimal area rectangular packing
Keywords: rectangular packing, non-overlapping constraints, non-linear optimization, regularization, relaxation
(18 pages, 2007)

127. M. Monz, K.-H. Küfer, T. Bortfeld, C. Thieke
Pareto navigation – systematic multi-criteria-based IMRT treatment plan determination
Keywords: convex, interactive multi-objective optimization, intensity modulated radiotherapy planning
(15 pages, 2007)

128. M. Krause, A. Scherrer
On the role of modeling parameters in IMRT plan optimization
Keywords: intensity-modulated radiotherapy (IMRT), inverse IMRT planning, convex optimization, sensitivity analysis, elasticity, modeling parameters, equivalent uniform dose (EUD)
(18 pages, 2007)

129. A. Wiegmann
Computation of the permeability of porous materials from their microstructure by FFF-Stokes
Keywords: permeability, numerical homogenization, fast Stokes solver
(24 pages, 2007)

130. T. Melo, S. Nickel, F. Saldanha da Gama
Facility Location and Supply Chain Management – A comprehensive review
Keywords: facility location, supply chain management, network design
(54 pages, 2007)

131. T. Hanne, T. Melo, S. Nickel
Bringing robustness to patient flow management through optimized patient transports in hospitals
Keywords: Dial-a-Ride problem, online problem, case study, tabu search, hospital logistics
(23 pages, 2007)

132. R. Ewing, O. Iliev, R. Lazarov, I. Rybak, J. Willems
An efficient approach for upscaling properties of composite materials with high contrast of coefficients
Keywords: effective heat conductivity, permeability of fractured porous media, numerical upscaling, fibrous insulation materials, metal foams
(16 pages, 2008)

133. S. Gelareh, S. Nickel
New approaches to hub location problems in public transport planning
Keywords: integer programming, hub location, transportation, decomposition, heuristic
(25 pages, 2008)

134. G. Thömmes, J. Becker, M. Junk, A. K. Vaikuntam, D. Kehrwald, A. Klar, K. Steiner, A. Wiegmann
A Lattice Boltzmann Method for immiscible multiphase flow simulations using the Level Set Method
Keywords: Lattice Boltzmann method, Level Set method, free surface, multiphase flow
(28 pages, 2008)

135. J. Orlik
Homogenization in elasto-plasticity
Keywords: multiscale structures, asymptotic homogenization, nonlinear energy
(40 pages, 2008)

136. J. Almquist, H. Schmidt, P. Lang, J. Deitmer, M. Jirstrand, D. Prätzel-Wolters, H. Becker
Determination of interaction between MCT1 and CAII via a mathematical and physiological approach
Keywords: mathematical modeling; model reduction; electrophysiology; pH-sensitive microelectrodes; proton antenna
(20 pages, 2008)

137. E. Savenkov, H. Andrä, O. Iliev
An analysis of one regularization approach for solution of pure Neumann problem
Keywords: pure Neumann problem, elasticity, regularization, finite element method, condition number
(27 pages, 2008)

138. O. Berman, J. Kalcsics, D. Krass, S. Nickel
The ordered gradual covering location problem on a network
Keywords: gradual covering, ordered median function, network location
(32 pages, 2008)

139. S. Gelareh, S. Nickel
Multi-period public transport design: A novel model and solution approaches
Keywords: Integer programming, hub location, public transport, multi-period planning, heuristics
(31 pages, 2008)

140. T. Melo, S. Nickel, F. Saldanha-da-Gama
Network design decisions in supply chain planning
Keywords: supply chain design, integer programming models, location models, heuristics
(20 pages, 2008)

141. C. Lautensack, A. Särkkä, J. Freitag, K. Schladitz
Anisotropy analysis of pressed point processes
Keywords: estimation of compression, isotropy test, nearest neighbour distance, orientation analysis, polar ice, Ripley's K function
(35 pages, 2008)

142. O. Iliev, R. Lazarov, J. Willems
A Graph-Laplacian approach for calculating the effective thermal conductivity of complicated fiber geometries
Keywords: graph laplacian, effective heat conductivity, numerical upscaling, fibrous materials
(14 pages, 2008)

143. J. Linn, T. Stephan, J. Carlsson, R. Bohlin
Fast simulation of quasistatic rod deformations for VR applications
Keywords: quasistatic deformations, geometrically exact rod models, variational formulation, energy minimization, finite differences, nonlinear conjugate gradients
(7 pages, 2008)

144. J. Linn, T. Stephan
Simulation of quasistatic deformations using discrete rod models
Keywords: quasistatic deformations, geometrically exact rod models, variational formulation, energy minimization, finite differences, nonlinear conjugate gradients
(9 pages, 2008)

145. J. Marburger, N. Marheineke, R. Pinnau
Adjoint based optimal control using mesh-less discretizations
Keywords: Mesh-less methods, particle methods, Eulerian-Lagrangian formulation, optimization strategies, adjoint method, hyperbolic equations
(14 pages, 2008

146. S. Desmettre, J. Gould, A. Szimayer
Own-company stockholding and work effort preferences of an unconstrained executive
Keywords: optimal portfolio choice, executive compensation
(33 pages, 2008)

147. M. Berger, M. Schröder, K.-H. Küfer
A constraint programming approach for the two-dimensional rectangular packing problem with orthogonal orientations
Keywords: rectangular packing, orthogonal orientations non-overlapping constraints, constraint propagation
(13 pages, 2008)

148. K. Schladitz, C. Redenbach, T. Sych, M. Godehardt
Microstructural characterisation of open foams using 3d images
Keywords: virtual material design, image analysis, open foams
(30 pages, 2008)

149. E. Fernández, J. Kalcsics, S. Nickel, R. Ríos-Mercado
A novel territory design model arising in the implementation of the WEEE-Directive
Keywords: heuristics, optimization, logistics, recycling
(28 pages, 2008)

150. H. Lang, J. Linn
Lagrangian field theory in space-time for geometrically exact Cosserat rods
Keywords: Cosserat rods, geometrically exact rods, small strain, large deformation, deformable bodies, Lagrangian field theory, variational calculus
(19 pages, 2009)

151. K. Dreßler, M. Speckert, R. Müller, Ch. Weber
Customer loads correlation in truck engineering
Keywords: Customer distribution, safety critical components, quantile estimation, Monte-Carlo methods
(11 pages, 2009)

152. H. Lang, K. Dreßler
An improved multiaxial stress-strain correction model for elastic FE postprocessing
Keywords: Jiang's model of elastoplasticity, stress-strain correction, parameter identification, automatic differentiation, least-squares optimization, Coleman-Li algorithm
(6 pages, 2009)

153. J. Kalcsics, S. Nickel, M. Schröder
A generic geometric approach to territory design and districting
Keywords: Territory design, districting, combinatorial optimization, heuristics, computational geometry
(32 pages, 2009)

154. Th. Fütterer, A. Klar, R. Wegener
An energy conserving numerical scheme for the dynamics of hyperelastic rods
Keywords: Cosserat rod, hyperealstic, energy conservation, finite differences
(16 pages, 2009)

155. A. Wiegmann, L. Cheng, E. Glatt, O. Iliev, S. Rief
Design of pleated filters by computer simulations
Keywords: Solid-gas separation, solid-liquid separation, pleated filter, design, simulation
(21 pages, 2009)

156. A. Klar, N. Marheineke, R. Wegener
Hierarchy of mathematical models for production processes of technical textiles
Keywords: Fiber-fluid interaction, slender-body theory, turbulence modeling, model reduction, stochastic differential equations, Fokker-Planck equation, asymptotic expansions, parameter identification
(21 pages, 2009)

157. E. Glatt, S. Rief, A. Wiegmann, M. Knefel, E. Wegenke
Structure and pressure drop of real and virtual metal wire meshes
Keywords: metal wire mesh, structure simulation, model calibration, CFD simulation, pressure loss
(7 pages, 2009)

158. S. Kruse, M. Müller
Pricing American call options under the assumption of stochastic dividends – An application of the Korn-Rogers model
Keywords: option pricing, American options, dividends, dividend discount model, Black-Scholes model
(22 pages, 2009)

159. H. Lang, J. Linn, M. Arnold
Multibody dynamics simulation of geometrically exact Cosserat rods
Keywords: flexible multibody dynamics, large deformations, finite rotations, constrained mechanical systems, structural dynamics
(20 pages, 2009)

160. P. Jung, S. Leyendecker, J. Linn, M. Ortiz
Discrete Lagrangian mechanics and geometrically exact Cosserat rods
Keywords: special Cosserat rods, Lagrangian mechanics, Noether's theorem, discrete mechanics, frame-indifference, holonomic constraints
(14 pages, 2009)

161. M. Burger, K. Dreßler, A. Marquardt, M. Speckert
Calculating invariant loads for system simulation in vehicle engineering
Keywords: iterative learning control, optimal control theory, differential algebraic equations (DAEs)
(18 pages, 2009)

162. M. Speckert, N. Ruf, K. Dreßler
Undesired drift of multibody models excited by measured accelerations or forces
Keywords: multibody simulation, full vehicle model, force-based simulation, drift due to noise
(19 pages, 2009)

163. A. Streit, K. Dreßler, M. Speckert, J. Lichter, T. Zenner, P. Bach
Anwendung statistischer Methoden zur Erstellung von Nutzungsprofilen für die Auslegung von Mobilbaggern
Keywords: Nutzungsvielfalt, Kundenbeanspruchung, Bemessungsgrundlagen
(13 pages, 2009)

164. I. Correia, S. Nickel, F. Saldanha-da-Gama
The capacitated single-allocation hub location problem revisited: A note on a classical formulation
Keywords: Capacitated Hub Location, MIP formulations
(10 pages, 2009)

165. F. Yaneva, T. Grebe, A. Scherrer
An alternative view on global radiotherapy optimization problems
Keywords: radiotherapy planning, path-connected sublevelsets, modified gradient projection method, improving and feasible directions
(14 pages, 2009)

166. J. I. Serna, M. Monz, K.-H. Küfer, C. Thieke
Trade-off bounds and their effect in multi-criteria IMRT planning
Keywords: trade-off bounds, multi-criteria optimization, IMRT, Pareto surface
(15 pages, 2009)

167. W. Arne, N. Marheineke, A. Meister, R. Wegener
Numerical analysis of Cosserat rod and string models for viscous jets in rotational spinning processes
Keywords: Rotational spinning process, curved viscous fibers, asymptotic Cosserat models, boundary value problem, existence of numerical solutions
(18 pages, 2009)

168. T. Melo, S. Nickel, F. Saldanha-da-Gama
An LP-rounding heuristic to solve a multi-period facility relocation problem
Keywords: supply chain design, heuristic, linear programming, rounding
(37 pages, 2009)

169. I. Correia, S. Nickel, F. Saldanha-da-Gama
Single-allocation hub location problems with capacity choices
Keywords: hub location, capacity decisions, MILP formulations
(27 pages, 2009)

170. S. Acar, K. Natcheva-Acar
A guide on the implementation of the Heath-Jarrow-Morton Two-Factor Gaussian Short Rate Model (HJM-G2++)
Keywords: short rate model, two factor Gaussian, G2++, option pricing, calibration
(30 pages, 2009)

171. A. Szimayer, G. Dimitroff, S. Lorenz
A parsimonious multi-asset Heston model: calibration and derivative pricing
Keywords: Heston model, multi-asset, option pricing, calibration, correlation
(28 pages, 2009)

172. N. Marheineke, R. Wegener
Modeling and validation of a stochastic drag for fibers in turbulent flows
Keywords: fiber-fluid interactions, long slender fibers, turbulence modelling, aerodynamic drag, dimensional analysis, data interpolation, stochastic partial differential algebraic equation, numerical simulations, experimental validations
(19 pages, 2009)

173. S. Nickel, M. Schröder, J. Steeg
Planning for home health care services
Keywords: home health care, route planning, metaheuristics, constraint programming
(23 pages, 2009)

174. G. Dimitroff, A. Szimayer, A. Wagner
Quanto option pricing in the parsimonious Heston model
Keywords: Heston model, multi asset, quanto options, option pricing
(14 pages, 2009) 174. G. Dimitroff, A. Szimayer, A. Wagner

175. S. Herkt, K. Dreßler, R. Pinnau
Model reduction of nonlinear problems in structural mechanics
Keywords: flexible bodies, FEM, nonlinear model reduction, POD
(13 pages, 2009)

176. M. K. Ahmad, S. Didas, J. Iqbal
Using the Sharp Operator for edge detection and nonlinear diffusion
Keywords: maximal function, sharp function,image processing, edge detection, nonlinear diffusion
(17 pages, 2009)

177. M. Speckert, N. Ruf, K. Dreßler, R. Müller, C. Weber, S. Weihe
Ein neuer Ansatz zur Ermittlung von Erprobungslasten für sicherheitsrelevante Bauteile
Keywords: sicherheitsrelevante Bauteile, Kundenbeanspruchung, Festigkeitsverteilung, Ausfallwahrscheinlichkeit, Konfidenz, statistische Unsicherheit, Sicherheitsfaktoren
(16 pages, 2009)

178. J. Jegorovs
Wave based method: new applicability areas
Keywords: Elliptic boundary value problems, inhomogeneous Helmholtz type differential equations in bounded domains, numerical methods, wave based method, uniform B-splines
(10 pages, 2009)

179. H. Lang, M. Arnold
Numerical aspects in the dynamic simulation of geometrically exact rods
Keywords: Kirchhoff and Cosserat rods, geometrically exact rods, deformable bodies, multibody dynamics,artial differential algebraic equations, method of lines, time integration
(21 pages, 2009)

180. H. Lang
Comparison of quaternionic and rotation-free null space formalisms for multibody dynamics
Keywords: Parametrisation of rotations, differential-algebraic equations, multibody dynamics, constrained mechanical systems, Lagrangian mechanics
(40 pages, 2010)

181. S. Nickel, F. Saldanha-da-Gama, H.-P. Ziegler
Stochastic programming approaches for risk aware supply chain network design problems
Keywords: Supply Chain Management, multi-stage stochastic programming, financial decisions, risk
(37 pages, 2010)

182. P. Ruckdeschel, N. Horbenko
Robustness properties of estimators in generalized Pareto Models
Keywords: global robustness, local robustness, finite sample breakdown point, generalized Pareto distribution
(58 pages, 2010)

183. P. Jung, S. Leyendecker, J. Linn, M. Ortiz
A discrete mechanics approach to Cosserat rod theory – Part 1: static equilibria
Keywords: Special Cosserat rods; Lagrangian mechanics; Noether's theorem; discrete mechanics; frame-indifference; holonomic constraints; variational formulation
(35 pages, 2010)

184. R. Eymard, G. Printsypar
A proof of convergence of a finite volume scheme for modified steady Richards' equation describing transport processes in the pressing section of a paper machine
Keywords: flow in porous media, steady Richards' equation, finite volume methods, convergence of approximate solution
(14 pages, 2010)

185. P. Ruckdeschel
Optimally Robust Kalman Filtering
Keywords: robustness, Kalman Filter, innovation outlier, additive outlier
(42 pages, 2010)

186. S. Repke, N. Marheineke, R. Pinnau
On adjoint-based optimization of a free surface Stokes flow
Keywords: film casting process, thin films, free surface Stokes flow, optimal control, Lagrange formalism
(13 pages, 2010)

187. O. Iliev, R. Lazarov, J. Willems
Variational multiscale Finite Element Method for flows in highly porous media
Keywords: numerical upscaling, flow in heterogeneous porous media, Brinkman equations, Darcy's law, subgrid approximation, discontinuous Galerkin mixed FEM
(21 pages, 2010)

188. S. Desmettre, A. Szimayer
Work effort, consumption, and portfolio selection: When the occupational choice matters
Keywords: portfolio choice, work effort, consumption, occupational choice
(34 pages, 2010)

189. O. Iliev, Z. Lakdawala, V. Starikovicius
On a numerical subgrid upscaling algorithm for Stokes-Brinkman equations
Keywords: Stokes-Brinkman equations, subgrid approach, multiscale problems, numerical upscaling
(27 pages, 2010)

190. A. Latz, J. Zausch, O. Iliev
Modeling of species and charge transport in Li-Ion Batteries based on non-equilibrium thermodynamics
Keywords: lithium-ion battery, battery modeling, electrochemical simulation, concentrated electrolyte, ion transport
(8 pages, 2010)

191. P. Popov, Y. Vutov, S. Margenov, O. Iliev
Finite volume discretization of equations describing nonlinear diffusion in Li-Ion batteries
Keywords: nonlinear diffusion, finite volume discretization, Newton method, Li-Ion batteries
(9 pages, 2010)

192. W. Arne, N. Marheineke, R. Wegener
Asymptotic transition from Cosserat rod to string models for curved viscous inertial jets
Keywords: rotational spinning processes; inertial and viscous-inertial fiber regimes; asymptotic limits; slender-body theory; boundary value problems
(23 pages, 2010)

193. L. Engelhardt, M. Burger, G. Bitsch
Real-time simulation of multibody-systems for on-board applications
Keywords: multibody system simulation, real-time simulation, on-board simulation, Rosenbrock methods
(10 pages, 2010)

194. M. Burger, M. Speckert, K. Dreßler
Optimal control methods for the calculation of invariant excitation signals for multibody systems
Keywords: optimal control, optimization, mbs simulation, invariant excitation
(9 pages, 2010)

195. A. Latz, J. Zausch
Thermodynamic consistent transport theory of Li-Ion batteries
Keywords: Li-Ion batteries, nonequilibrium thermodynamics, thermal transport, modeling
(18 pages, 2010)

196. S. Desmettre
Optimal investment for executive stockholders with exponential utility
Keywords: portfolio choice, executive stockholder, work effort, exponential utility
(24 pages, 2010)

197. W. Arne, N. Marheineke, J. Schnebele, R. Wegener
Fluid-fiber-interactions in rotational spinning process of glass wool production
Keywords: Rotational spinning process, viscous thermal jets, fluid-fiber-interactions, two-way coupling, slender-body theory, Cosserat rods, drag models, boundary value problem, continuation method
(20 pages, 2010)

198. A. Klar, J. Maringer, R. Wegener
A 3d model for fiber lay-down in nonwoven production processes
Keywords: fiber dynamics, Fokker-Planck equations, diffusion limits
(15 pages, 2010)

199. Ch. Erlwein, M. Müller
A regime-switching regression model for hedge funds
Keywords: switching regression model, Hedge funds, optimal parameter estimation, filtering
(26 pages, 2011)

200. M. Dalheimer
Power to the people – Das Stromnetz der Zukunft
Keywords: Smart Grid, Stromnetz, Erneuerbare Energien, Demand-Side Management
(27 pages, 2011)

201. D. Stahl, J. Hauth
PF-MPC: Particle Filter-Model Predictive Control
Keywords: Model Predictive Control, Particle Filter, CSTR, Inverted Pendulum, Nonlinear Systems, Sequential Monte Carlo
(40 pages, 2011)

202. G. Dimitroff, J. de Kock
Calibrating and completing the volatility cube in the SABR Model
Keywords: stochastic volatility, SABR, volatility cube, swaption
(12 pages, 2011)

203. J.-P. Kreiss, T. Zangmeister
Quantification of the effectiveness of a safety function in passenger vehicles on the basis of real-world accident data
Keywords: logistic regression, safety function, real-world accident data, statistical modeling
(23 pages, 2011)

204. P. Ruckdeschel, T. Sayer, A. Szimayer
Pricing American options in the Heston model: a close look on incorporating correlation
Keywords: Heston model, American options, moment matching, correlation, tree method
(30 pages, 2011)

205. H. Ackermann, H. Ewe, K.-H. Küfer, M. Schröder
Modeling profit sharing in combinatorial exchanges by network flows
Keywords: Algorithmic game theory, profit sharing, combinatorial exchange, network flows, budget balance, core
(17 pages, 2011)

206. O. Iliev, G. Printsypar, S. Rief
A one-dimensional model of the pressing section of a paper machine including dynamic capillary effects
Keywords: steady modified Richards' equation, finite volume method, dynamic capillary pressure, pressing section of a paper machine
(29 pages, 2011)

207. I. Vecchio, K. Schladitz, M. Godehardt, M. J. Heneka
Geometric characterization of particles in 3d with an application to technical cleanliness
Keywords: intrinsic volumes, isoperimetric shape factors, bounding box, elongation, geodesic distance, technical cleanliness
(21 pages, 2011)

208. M. Burger, K. Dreßler, M. Speckert
Invariant input loads for full vehicle multibody system simulation
Keywords: multibody systems, full-vehicle simulation, optimal control
(8 pages, 2011)

209. H. Lang, J. Linn, M. Arnold
Multibody dynamics simulation of geometrically exact Cosserat rods
Keywords: flexible multibody dynamics, large deformations, finite rotations, constrained mechanical systems, structural dynamics
(28 pages, 2011)

210. G. Printsypar, R. Ciegis
On convergence of a discrete problem describing transport processes in the pressing section of a paper machine including dynamic capillary effects: one-dimensional case
Keywords: saturated and unsaturated fluid flow in porous media, Richards' approach, dynamic capillary pressure, finite volume methods, convergence of approximate solution
(24 pages, 2011)

211. O. Iliev, G. Printsypar, S. Rief
A two-cimensional model of the pressing section of a paper machine including dynamic capillary effects
Keywords: two-phase flow in porous media, steady modified Richards' equation, finite volume method, dynamic capillary pressure, pressing section of a paper machine, multipoint flux approximation
(44 pages, 2012)

212. M. Buck, O. Iliev, H. Andrä
Multiscale finite element coarse spaces for the analysis of linear elastic composites
Keywords: linear elasticity, domain decomposition, multiscale finite elements, robust coarse spaces, rigid body modes, discontinuous coefficients
(31 pages, 2012)

213. A. Wagner
Residual demand modeling and application to electricity pricing
Keywords: residual demand modeling, renewable infeed, wind infeed, solar infeed, electricity demand, German power market, merit-order effect
(28 pages, 2012)

214. O. Iliev, A. Latz, J. Zausch, S. Zhang
An overview on the usage of some model reduction approaches for simulations of Li-ion transport in batteries
Keywords: Li-ion batteries, porous electrode model, model reduction
(21 pages, 2012)

215. C. Zémerli, A. Latz, H. Andrä
Constitutive models for static granular systems and focus to the Jiang-Liu hyperelastic law
Keywords: granular elasticity, constitutive modelling, non-linear finite element method
(33 pages, 2012)

216. T. Gornak, J. L. Guermond, O. Iliev, P. D. Minev
A direction splitting approach for incompressible Brinkmann flow
Keywords: unsteady Navier-Stokes-Brinkman equations, direction splitting algorithms, nuclear reactors safety simulations
(16 pages, 2012)

Status quo: June 2012