



**Fraunhofer**

**ITWM**

S. Desmettre, A. Szimayer

Work effort, consumption, and portfolio selection: When the occupational choice matters

© Fraunhofer-Institut für Techno- und Wirtschaftsmathematik ITWM 2010

ISSN 1434-9973

Bericht 188 (2010)

Alle Rechte vorbehalten. Ohne ausdrückliche schriftliche Genehmigung des Herausgebers ist es nicht gestattet, das Buch oder Teile daraus in irgendeiner Form durch Fotokopie, Mikrofilm oder andere Verfahren zu reproduzieren oder in eine für Maschinen, insbesondere Datenverarbeitungsanlagen, verwendbare Sprache zu übertragen. Dasselbe gilt für das Recht der öffentlichen Wiedergabe.

Warennamen werden ohne Gewährleistung der freien Verwendbarkeit benutzt.

Die Veröffentlichungen in der Berichtsreihe des Fraunhofer ITWM können bezogen werden über:

Fraunhofer-Institut für Techno- und  
Wirtschaftsmathematik ITWM  
Fraunhofer-Platz 1

67663 Kaiserslautern  
Germany

Telefon: +49(0)631/3 1600-0  
Telefax: +49(0)631/3 1600-1099  
E-Mail: [info@itwm.fraunhofer.de](mailto:info@itwm.fraunhofer.de)  
Internet: [www.itwm.fraunhofer.de](http://www.itwm.fraunhofer.de)

# Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe werden sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.



Prof. Dr. Dieter Prätzel-Wolters  
Institutsleiter

Kaiserslautern, im Juni 2001



# Work Effort, Consumption, and Portfolio Selection: When the Occupational Choice Matters

Sascha Desmettre\* Alexander Szimayer†

July 5, 2010

## Abstract

We consider a highly-qualified individual with respect to her choice between two distinct career paths. She can choose between a mid-level management position in a large company and an executive position within a smaller listed company with the possibility to directly affect the company's share price. She invests in the financial market including the share of the smaller listed company. The utility maximizing strategy from consumption, investment, and work effort is derived in closed form for logarithmic utility. The power utility case is discussed as well. Conditions for the individual to pursue her career with the smaller listed company are obtained. The participation constraint is formulated in terms of the salary differential between the two positions. The smaller listed company can offer less salary. The salary shortfall is offset by the possibility to benefit from her work effort by acquiring own-company shares. This gives insight into aspects of optimal contract design. Our framework is applicable to the pharmaceutical and financial industry, and the IT sector.

*2000 MSC Subject Classification:* 49L20, 91B28, 93E20

*Key Words:* portfolio choice, work effort, consumption, occupational choice

---

\*Department of Financial Mathematics, Fraunhofer ITWM, Fraunhofer-Platz 1, 67663 Kaiserslautern, Germany; Center for Mathematical and Computational Modelling and Department of Mathematics, University of Kaiserslautern, Germany. Email: sascha.desmettre@itwm.fraunhofer.de. This paper is part of the PhD thesis of Sascha Desmettre.

†Faculty of Economics and Law, University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany. Email: szimayer@uni-bonn.de.

# 1 Introduction

It is widely supported that the remuneration of managers should be linked to performance, see, e.g., Ross (1973), Jensen and Meckling (1976), Holmstrom (1979) and others, for the fundamentals of agency theory, and the summaries of Murphy (1999) and Core, Guay and Larcker (2003). In contrast to past research, we investigate the motivation for an individual to voluntarily performance-link her private wealth by acquiring shares in the own-company. We consider a highly-qualified individual with respect to her choice between two distinct career paths. She can choose between a mid-level management position in a large company and an executive position in a smaller listed company with the possibility to directly affect the company's share price. The individual is assumed to be utility maximizing, deriving utility from terminal wealth and intertemporal consumption, and negative utility (disutility or cost) from work effort. The investment opportunities include the share of the smaller listed company and thus the individual can capitalize on her work effort by investing in own-company shares. Taking up the mid-level management position with the large company is the outside option in our setting. The outside option rules out the possibility to affect the share price of the smaller company. The individual is characterized by two time preference parameters ( $\rho$ , discount rate for utility from consumption, and  $\tilde{\rho}$ , discount rate for the disutility from work effort), and two work effectiveness parameters ( $\kappa$ , representing inverse work productivity, and  $\alpha$ , representing disutility stress).

First, we analyze the individual's optimal control problem under the assumption that she takes up the offer from the smaller listed company. The optimal investment strategy ( $\pi^*$ ), consumption ( $k^*$ ), and work effort ( $\lambda^*$ ), respectively, are derived in closed form in the log-utility setting using stochastic control theory and the corresponding Hamilton-Jacobi-Bellman equations. We demonstrate that an executive with higher work effectiveness (quality) undertakes more work effort. Additionally, the broader constant relative risk aversion setting is explored. By imposing a sensible parameter restriction we are able to reduce the problem to a Riccati equation which we can solve in closed form. As second step, we identify conditions for the individual to work for the smaller listed company. The participation constraint is given in terms of the salary differential of the two job alternatives. In particular, we derive the minimal required salary  $\delta^*$  that needs to be offered by the smaller company to attract the individual and thereby characterize the

participation constraint. In general, we find that a more talented individual requires a lower salary to be attracted to the smaller listed company. The salary shortfall is offset by the possibility to benefit from her work effort by acquiring shares of the company. This salary pattern can be observed in practice, e.g., in the pharmaceutical industry, the IT sector, and the financial industry. Other technical papers similarly concerned with dynamic principal-agent models include Cadenillas, Cvitanic and Zapatero (2004), Desmettre, Gould and Szimayer (2010), Korn and Kraft (2008) and Ou-Yang (2003), for example.

The paper is organized as follows. Section 2 introduces the notation and terminology. In Section 3 the Hamilton-Jacobi-Bellman equations characterizing the utility and consumption maximization problem are derived, and closed-form solutions for the log-utility case are established. The power utility case is discussed as well. The results are illustrated in Section 4. Section 5 concludes and gives an outlook for future research. Technical proofs are in the Appendix.

## 2 Notation and Setup

We consider an individual endowed with given initial wealth. She manages here financial objectives by investing in the financial market and choosing her instantaneous consumption. The individual can also choose the level of work effort she applies.

### 2.1 Financial Market

First we specify the financial market. We are given a filtered probability space  $(\Omega, \mathcal{F}, P, (\mathcal{F}_t)_{t \geq 0})$  satisfying the usual hypothesis and large enough to support two independent standard Brownian motions,  $W^P = (W_t^P)_{t \geq 0}$  and  $W = (W_t)_{t \geq 0}$ . The investment opportunities available are a money market account, a diversified market portfolio, and shares of a small listed company making a job offer to the individual.

The risk-free money market account has the price process  $B = (B_t)_{t \geq 0}$ , with dynamics

$$dB_t = r B_t dt, \quad B_0 = 1, \quad (1)$$

where  $r$  is the instantaneous risk-free rate of return, hence  $B_t = e^{rt}$ .

The price process of the market portfolio,  $P = (P_t)_{t \geq 0}$ , follows the stochastic differential equation (SDE)

$$dP_t = P_t (\mu_P dt + \sigma_P dW_t^P), \quad P_0 \in \mathbb{R}^+, \quad (2)$$

where  $\mu_P \in \mathbb{R}$  and  $\sigma_P > 0$  are respectively the expected return rate and volatility of the market portfolio. The corresponding Sharpe ratio is then  $\lambda_P = (\mu_P - r)/\sigma_P$ .

The company's stock price process,  $S^u = (S_t^u)_{t \geq 0}$ , is a controlled diffusion with SDE

$$dS_t^u = S_t^u \left( [r + \lambda_t \sigma] dt + \beta \left[ \frac{dP_t}{P_t} - r dt \right] + \sigma dW_t \right), \quad S_0^u \in \mathbb{R}^+, \quad (3)$$

where  $\mu = r + \lambda \sigma$  is the company's expected return rate in excess of the beta-adjusted market portfolio's expected excess return rate (i.e. the expected return compensation for non-systematic risk),  $\sigma$  is the company's non-systematic volatility, and  $\lambda = (\lambda_t)_{t \geq 0}$  is a control process collected in the control vector process  $u$  that will be specified below.

## 2.2 Controls and Wealth Process

The individual is endowed with the initial wealth  $V_0 > 0$ . She receives an instantaneous salary proportional to her current wealth at a relative rate  $\delta$ . For an exogenously given time horizon,  $T > 0$ , the individual seeks to maximize her total utility by controlling the portfolio holdings, consumption, and work effort.

The portfolio is determined by a self-financing trading strategy given by the bivariate control process  $\pi = (\pi^P, \pi^S)$ , where  $\pi^P = (\pi_t^P)_{t \geq 0}$  is the fraction of wealth invested in the market portfolio and  $\pi^S = (\pi_t^S)_{t \geq 0}$  is the fraction of wealth invested in the company's stock. The remainder in the risk-free account, that is, the strategy is self-financing. The individual consumes instantaneously at the relative rate  $k = (k_t)_{t \geq 0}$  proportional to the wealth  $V_t^\pi$  at time  $t$ , where  $k_t \geq 0$ , leading to a total consumption rate  $k_t V_t^\pi$ . Further, she influences the small company's stock price dynamics by choice of the control strategy  $\lambda = (\lambda_t)_{t \geq 0}$ , which is specified to be associated with work effort. The control strategy can be conceptualized as deriving from the individual's corporate investment. For example, identifying and initiating positive net present value projects. Value is added if  $\mu = r + \lambda \sigma$  is greater than  $r$ , indicating excess return compensation for non-systematic risk. To ensure sensible

solutions we require  $\lambda \geq 0$ , which effectively bars her from destroying company value ( $\lambda < 0$ ) and potentially profiting by shorting the company's stock. All controls are collected in the vector process  $u = (\pi^P, \pi^S, k, \lambda)$ .

For a fixed salary rate  $\delta$ , initial wealth  $V_0 > 0$ , and a control strategy  $u$ , the wealth process,  $V^u = (V_t^u)_{t \geq 0}$ , with starting value  $V_0^u = V_0$  is given by

$$dV_t^u = V_t^u \left( [1 - \pi_t^P - \pi_t^S] \frac{dB_t}{B_t} + \pi_t^P \frac{dP_t}{P_t} + \pi_t^S \frac{dS_t^u}{S_t^u} + \delta dt - k_t dt \right), t \geq 0. \quad (4)$$

The above equation can be rewritten as follows

$$\begin{aligned} dV_t^u = V_t^u & \left( [r + \delta - k_t + (\pi_t^P + \beta \pi_t^S) \lambda_P \sigma_P + \pi_t^S \lambda_t \sigma] dt \right. \\ & \left. + [\pi_t^P + \beta \pi_t^S] \sigma_P dW_t^P + \pi_t^S \sigma dW_t \right), t \geq 0. \end{aligned} \quad (5)$$

### 2.3 Stochastic Control Problem

The individual is assumed to maximize the expected value of the terminal utility of her wealth for time horizon  $T$ , subject to some utility function  $U_1$  and her consumption rate over the time period  $[t, T]$ , subject to some utility function  $U_2$ . The disutility for work effort is quantified by the cost function  $C$ . Both utility functions and the cost function will be specified when deriving closed-form solutions.

Assuming control of the company's stock price behavior  $\lambda$  is determined exogenously and comes at zero cost, the individual's *optimal investment and consumption decision* is then described by

$$\widehat{\Phi}(t, v) = \sup_{(\pi, k) \in \Pi(t, v)} \mathbb{E}^{t, v} \left[ U_1(V_T^{(\pi, k)}) + \int_t^T U_2(s, V_s^{(\pi, k)}, k_s) ds \right], \quad (6)$$

for  $(t, v) \in [0, T] \times \mathbb{R}^+$ , where  $\Pi(t, v)$  denotes the set of all admissible portfolio processes  $(\pi, k)$  at time  $t$  corresponding to portfolio value (i.e. wealth)  $v = V_t > 0$  (see for example Korn and Korn (2001)),  $U_1$  and  $U_2$  are utility functions, and  $\mathbb{E}^{t, v}$  denotes the expectation conditional on  $t$  and  $v$ .

The *optimal investment and consumption control decision including work effort* is then the solution of

$$\Phi(t, v) = \sup_{u \in A(t, v)} \mathbb{E}^{t, v} \left[ U_1(V_T^u) + \int_t^T U_2(s, V_s^u, k_s) ds - \int_t^T C(s, V_s^u, \lambda_s) ds \right], \quad (7)$$

for  $(t, v) \in [0, T] \times \mathbb{R}^+$ . The set of admissible strategies for the maximization  $A(t, v)$  problem is made precise in the following definition.

**Definition 2.1** Fix  $(t, v) \in [0, T] \times \mathbb{R}^+$ , then  $u = (\pi^P, \pi^S, k, \lambda)$  is in the set of admissible strategies  $A(t, v)$ , if and only if  $u$  is an  $\{\mathcal{F}_s; t \leq s \leq T\}$ -predictable processes, such that

(i) the stock price equation

$$dS_s^u = S_s^u \left( [r + \lambda_s \sigma] ds + \beta \left[ \frac{dP_s}{P_s} - r ds \right] + \sigma dW_s \right),$$

with initial condition  $S_t^u \in \mathbb{R}^+$  admits a non-negative solution and

$$\int_t^T (S_s^u)^2 (\sigma^2 + \beta^2 \sigma_P^2) ds < \infty \quad P - a.s.;$$

(ii) the wealth equation

$$dV_s^u = V_s^u \left( \left[ 1 - \pi_s^P - \pi_s^S \right] \frac{dB_s}{B_s} + \pi_s^P \frac{dP_s}{P_s} + \pi_s^S \frac{dS_s^u}{S_s^u} + \delta ds - k_s ds \right),$$

with initial condition  $V_t^u = v$  has a unique non-negative solution and

$$\int_t^T (V_s^u)^2 \left( ([\pi_s^P + \beta \pi_s^S] \sigma_P)^2 + (\pi_s^S \sigma)^2 \right) ds < \infty \quad P - a.s.;$$

(iii) and the utility of wealth and consumption, and the disutility of control satisfy

$$\mathbb{E} \left[ U_1(V_T^u)^- + \int_t^T U_2(s, V_s^u, k_s)^- ds + \int_t^T C(s, V_s^u, \lambda_s) ds \right] < \infty.$$

## 2.4 Outside Option

The individual can choose between two job offers at  $t = 0$ . As an alternative to taking on the executive position with the company with share price  $S^u$ , she can pursue her outside option and decide to work for a large company in a mid-management position paying a salary at rate  $\widehat{\delta}$ . In the latter case she cannot affect the stock price process any longer and hence  $\widehat{\lambda} = 0$ . The classical optimal investment and consumption decision applies.

Assume that portfolio process follows Eq. (5) where we set  $\delta = \widehat{\delta}$  and  $\lambda = \widehat{\lambda} = 0$ . Then the optimal investment decision problem in Equation (6) determines the value of the outside option  $\widehat{\Phi}(0, V_0)$  at time  $t = 0$  for initial wealth  $V_0 > 0$ .

### 3 Optimal Strategies

In this section we use stochastic control techniques to derive closed-form solutions to the control problem in (7). Our main focus is placed on the log utility specification for utility from terminal wealth and consumption and disutility that is a power function of work effort applied. In addition, we also discuss the general constant relative risk aversion specification.

#### 3.1 Hamilton-Jacobi-Bellman Equation

Having formulated the optimal investment and control decision problem including consumption with respect to the parameter set  $u = (\pi, k, \lambda)$  as given by (7), we can write down the corresponding Hamilton-Jacobi-Bellman equation. Note that we formulate this equation with respect to a general utility functions  $U_1$  and  $U_2$  and a general cost function  $C$ . For  $(t, v) \in [0, T] \times \mathbb{R}^+$  we have

$$\frac{\partial \Phi}{\partial t}(t, v) + \sup_{u \in \mathcal{U}} [(L^u \Phi)(t, v) + U_2(t, v, k) - C(t, v, \lambda)] = 0, \quad (8)$$

with terminal condition  $\Phi(T, v) = U_1(v)$ , for  $v \in \mathbb{R}^+$ , where  $\mathcal{U} = \mathbb{R}^2 \times [0, \infty)^2$  and the differential operator  $L^u$  is defined by

$$\begin{aligned} (L^u g)(t, v) = & \\ & \frac{\partial g}{\partial v}(t, v) v (r + \delta + \pi^S(t, v) \lambda(t, v) \sigma + [\pi^P(t, v) + \beta \pi^S(t, v)] \lambda_P \sigma_P - k(t, v)) \\ & + \frac{1}{2} \frac{\partial^2 g}{\partial v^2}(t, v) v^2 ([\pi^S(t, v) \sigma]^2 + [\pi^P(t, v) \sigma_P + \beta \pi^S(t, v) \sigma_P]^2). \end{aligned} \quad (9)$$

Potential maximizers  $\pi^{P^*}$ ,  $\pi^{S^*}$ ,  $k^*$  and  $\lambda^*$  of the HJB (8) can be calculated by establishing the first order conditions:

$$\begin{aligned} \pi^{P^*}(t, v) &= -\frac{\lambda_P}{v \sigma^P} \frac{\Phi_v(t, v)}{\Phi_{vv}(t, v)} - \beta \pi^{S^*}(t, v), \\ \pi^{S^*}(t, v) &= -\frac{\lambda^*(t, v)}{v \sigma} \frac{\Phi_v(t, v)}{\Phi_{vv}(t, v)}, \end{aligned} \quad (10)$$

and  $\lambda^*$  is the solution of the implicit equation

$$\lambda \frac{\Phi_v^2(t, v)}{\Phi_{vv}(t, v)} + \frac{\partial C}{\partial \lambda}(t, v, \lambda) = 0, \quad \text{for all } (t, v) \in [0, T] \times \mathbb{R}^+, \quad (11)$$

where we have already used (10) to simplify the equation, and  $k^*$  is the solution of the equation

$$\frac{\partial U_2}{\partial k}(t, v, k) - v \Phi_v(t, v) = 0. \quad (12)$$

Substituting the maximizers (10) in the HJB (8) yields:

$$\begin{aligned} \Phi_t(t, v) + \Phi_v(t, v) v (r + \delta - k^*(t, v)) - \frac{1}{2} (\lambda^*(t, v))^2 \frac{\Phi_v^2(t, v)}{\Phi_{vv}(t, v)} \\ - \frac{1}{2} \lambda_P^2 \frac{\Phi_v^2(t, v)}{\Phi_{vv}(t, v)} + U_2(t, k^*(t, v)) - C(t, v, \lambda^*(t, v)) = 0. \end{aligned} \quad (13)$$

In the following we solve (13) with particular choices for the utility and disutility functions.

### 3.2 Closed-Form Solution for the Log-Utility Case

We specify the utility functions to be of log-utility type, belonging to the constant relative risk aversion class. The utility function of the final wealth  $U_1$  is

$$U_1(v) = K \log(v), \quad \text{for } v \in \mathbb{R}^+, \quad (14)$$

for a constant  $K > 0$ , the utility function of consumption  $U_2$  is

$$U_2(t, k, v) = e^{-\rho t} \log(v k), \quad \text{for } (t, v, k) \in [0, T] \times \mathbb{R}^+ \times \mathbb{R}_0^+, \quad (15)$$

where  $\rho \in \mathbb{R}$  parametrizes the time preference, and the cost function of work effort  $C$  is

$$C(t, v, \lambda) = e^{-\tilde{\rho} t} \kappa \frac{\lambda^\alpha}{\alpha}, \quad \text{for } (t, v, \lambda) \in [0, T] \times \mathbb{R}^+ \times \mathbb{R}_0^+, \quad (16)$$

where  $\kappa > 0$  and  $\alpha > 2$  are the individual's work effectiveness parameters, respectively termed 'inverse work productivity' and 'disutility stress', and  $\tilde{\rho} \in \mathbb{R}$  is a time preference parameter. The constant  $\kappa$  directly relates the her work effort disutility to the quality of his control decision as indicated by the non-systematic Sharpe ratio  $\lambda$ , and  $\alpha$  indicates how rapidly her work effort disutility will rise for the sake of an improved  $\lambda$ . The requirement  $\alpha > 2$  is a consequence of our set-up that ensures the executive's disutility grows with work effort, i.e.  $\lambda$ , at a rate that offsets (at some level of  $\lambda$ ) the rate

of her utility gain due to the flow-on from her work effort to the value of his own-company stockholding; this becomes evident with derivation of the solution to (7). A higher quality individual is able to achieve a given  $\lambda$  with lower disutility, and is able to improve  $\lambda$  with lower incremental disutility. That is, higher individual quality (i.e. higher work effectiveness) is implied by lower values of  $\kappa$  and  $\alpha$ .

For the remainder of the paper we assume that the optimal investment and control problem (7) admits a value function  $\Phi \in C^{1,2}$ . To guarantee that the candidates we will derive for the executive's optimal investment and control strategy (i.e. the choices for own-company stockholding, market portfolio holding and non-systematic Sharpe ratio) and value function are indeed optimal, we have to consider a more restrictive class of admissible strategies as follows.

**Definition 3.1** Fix  $(t, v) \in [0, T] \times \mathbb{R}^+$ . Then by  $A'(t, v)$  we denote the set of admissible strategies  $u \in A'(t, v)$ , such that  $u \in A(t, v)$  and

$$\mathbb{E} \left[ \int_t^T (\pi_s^P + \beta \pi_s^S)^2 (\sigma_P)^2 + (\pi_s^S \sigma)^2 ds \right] < \infty, \quad (17)$$

Restating the optimal investment and control problem:

$$\Phi(t, v) = \sup_{u \in A'(t, v)} \mathbb{E}^{t, v} \left[ U_1(V_T^u) + \int_t^T U_2(s, V_s^u, k_s) ds - \int_t^T C(s, V_s^u, \lambda_s) ds \right], \quad (18)$$

for  $(t, v) \in [0, T] \times \mathbb{R}^+$ .

A closed-form solution is obtained for the optimal investment and control problem in (18) using the utility and disutility functions (14), (15) and (16).

**Theorem 3.1** The full solution of the maximization problem (18) can be summarized by the strategy

$$\begin{aligned} \pi^{P^*}(t, v) &= \frac{\lambda_P}{\sigma_P} - \beta \pi^{S^*}(t, v), & \pi^{S^*}(t, v) &= \frac{\lambda^*(t, v)}{\sigma}, \\ \lambda^*(t, v) &= \left( \frac{e^{\tilde{\rho}t}}{\kappa} f(t) \right)^{\frac{1}{\alpha-2}}, & k^*(t, v) &= \frac{e^{-\rho t}}{f(t)}, \end{aligned} \quad (19)$$

and value function

$$\Phi(t, v) = f(t) \log(v) + g(t), \quad (20)$$

with

$$f(t) = \begin{cases} K + \frac{e^{-\rho t} - e^{-\rho T}}{\rho}, & \text{for } \rho \neq 0, \\ K + T - t, & \text{for } \rho = 0, \end{cases}$$

and

$$g(t) = \left( r + \delta + \frac{1}{2} \lambda_P^2 \right) \int_t^T f(s) ds + \frac{\alpha - 2}{2\alpha} \int_t^T \left( \frac{e^{\tilde{\rho}s}}{\kappa} \right)^{\frac{2}{\alpha-2}} f(s)^{\frac{\alpha}{\alpha-2}} ds \\ - \int_t^T (1 + \rho s) e^{-\rho s} ds - \int_t^T e^{-\rho s} \log(f(s)) ds.$$

*Proof.* First observe that a function  $F$  of the form  $F(\lambda) = a\lambda^2 - b\lambda^\alpha$ ,  $\lambda \geq 0$ , for given constants  $a, b > 0$  and  $\alpha > 2$ , has a unique maximizer  $\lambda^*$  and maximized value  $F(\lambda^*)$  given by

$$\lambda^* = \left( \frac{2a}{\alpha b} \right)^{\frac{1}{\alpha-2}}, \quad \text{and} \quad F(\lambda^*) = (\alpha - 2) \alpha^{-\frac{\alpha}{\alpha-2}} 2^{\frac{2}{\alpha-2}} a^{\frac{\alpha}{\alpha-2}} b^{-\frac{2}{\alpha-2}}. \quad (21)$$

Using this insight, the first order condition for  $\lambda^*$  in (11) is now solved. Set

$$a = \frac{1}{2} \frac{\Phi_v^2}{-\Phi_{vv}}, \quad \text{and} \quad b = e^{-\tilde{\rho}t} \frac{\kappa}{\alpha},$$

then (21) gives

$$\lambda^* = \left( \frac{e^{\tilde{\rho}t}}{\kappa} \frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{1}{\alpha-2}}, \quad \text{and} \quad F(\lambda^*) = \frac{\alpha - 2}{2\alpha} \left( \frac{e^{\tilde{\rho}t}}{\kappa} \right)^{\frac{2}{\alpha-2}} \left( \frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{\alpha}{\alpha-2}}. \quad (22)$$

Having specified the utility function  $U_2$  of the consumption rate as  $U_2(t, v, k) = e^{-\rho t} \log(vk)$ , we can also solve the first order condition for the optimal consumption rate. Equation (12) then gives:

$$k^* = \frac{e^{-\rho t}}{v \Phi_v}. \quad (23)$$

Substituting  $\lambda^*$  and  $k^*$  in equation (13) we get:

$$0 = \Phi_t + \Phi_v v (r + \delta) + \frac{1}{2} \lambda_P^2 \frac{\Phi_v^2}{-\Phi_{vv}} + \frac{\alpha - 2}{2\alpha} \left( \frac{e^{\tilde{\rho}t}}{\kappa} \right)^{\frac{2}{\alpha-2}} \left( \frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{\alpha}{\alpha-2}} \\ - e^{-\rho t} - \rho t e^{-\rho t} - e^{-\rho t} \log(\Phi_v). \quad (24)$$

Using the ansatz  $\Phi(t, v) = \log(v) f(t) + g(t)$  with  $f(T) = K$  and  $g(T) = 0$  results in

$$\begin{aligned}\Phi_t &= \log(v) \dot{f}(t) + \dot{g}(t), & \Phi_v &= \frac{1}{v} f(t), & \Phi_{vv} &= -\frac{1}{v^2} f(t), \text{ and} \\ \Phi(T, v) &= K \log(v) = U_1(v).\end{aligned}$$

Then (24) reduces to

$$\begin{aligned}0 &= \log(v) \dot{f}(t) + \dot{g}(t) + f(t) \left( r + \delta + \frac{1}{2} \lambda_P^2 \right) + \frac{\alpha - 2}{2\alpha} \left( \frac{e^{\tilde{\rho}t}}{\kappa} \right)^{\frac{2}{\alpha-2}} f(t)^{\frac{\alpha}{\alpha-2}} \\ &\quad - e^{-\rho t} - \rho t e^{-\rho t} + e^{-\rho t} \log(v) - e^{-\rho t} \log(f(t)).\end{aligned}\tag{25}$$

Taking the derivative of this equation w.r.t.  $v$  gives:

$$\frac{1}{v} \dot{f}(t) + \frac{1}{v} e^{-\rho t} = 0 \quad \iff \quad \dot{f}(t) = -e^{-\rho t}.$$

Using the condition  $f(T) = K$  we then get by integration

$$f(t) = \begin{cases} K + \frac{e^{-\rho t} - e^{-\rho T}}{\rho}, & \text{for } \rho \neq 0, \\ K + T - t, & \text{for } \rho = 0. \end{cases}$$

Following the derivation of  $f$  we can eliminate the  $\log(v)$  in (25)

$$\begin{aligned}-\dot{g}(t) &= f(t) \left( r + \delta + \frac{1}{2} \lambda_P^2 \right) + \frac{\alpha - 2}{2\alpha} \left( \frac{e^{\tilde{\rho}t}}{\kappa} \right)^{\frac{2}{\alpha-2}} f(t)^{\frac{\alpha}{\alpha-2}} \\ &\quad - e^{-\rho t} - \rho t e^{-\rho t} - e^{-\rho t} \log(f(t)), \quad \text{and } g(T) = 0.\end{aligned}\tag{26}$$

Equation (26) can now be solved by simple integration:

$$\begin{aligned}g(t) &= \left( r + \delta + \frac{1}{2} \lambda_P^2 \right) \int_t^T f(s) ds + \frac{\alpha - 2}{2\alpha} \int_t^T \left( \frac{e^{\tilde{\rho}s}}{\kappa} \right)^{\frac{2}{\alpha-2}} f(s)^{\frac{\alpha}{\alpha-2}} ds \\ &\quad - \int_t^T (1 + \rho s) e^{-\rho s} ds - \int_t^T e^{-\rho s} \log f(s) ds,\end{aligned}$$

where  $f(t)$  is given as above.

Combining the results for the functions  $f$  and  $g$  we then get the claimed result

for the value function. Noting that  $\Phi_v/\Phi_{vv} = -v$  and using the first order conditions in (10) establishes the claimed optimal strategies  $\pi^{P^*}$  and  $\pi^{S^*}$ . Finally noting that  $\Phi_v^2/\Phi_{vv} = -f(t)$  and using the solved first order condition (22), we get the desired result for the optimal sharpe ratio  $\lambda^*$  and plugging in  $v\Phi_v = f$  in (23) we get the claimed result for the optimal consumption rate  $k^*$ . The claimed optimal investment and control choices are deterministic and the optimal consumption rate are continuous on a compact support, so they are uniformly bounded implying  $u^* = (\pi^{S^*}, \pi^{P^*}, \lambda^*, k^*) \in A(t, v)$ .  $\square$

**Remark 3.1** *The expression for  $g$  in Theorem 3.1 can be partially calculated fairly explicitly. For  $\rho \neq 0$  we obtain*

$$\begin{aligned} g(t) = & \left( r + \delta + \frac{1}{2} \lambda_P^2 \right) \left( K [T - t] + \frac{1}{\rho^2} [e^{-\rho t} - e^{-\rho T} (1 + \rho [T - t])] \right) \\ & - \frac{1}{\rho} (e^{-\rho t} - e^{-\rho T}) - t e^{-\rho t} + T e^{-\rho T} + K \log(K) \\ & - \log \left( K + \frac{1}{\rho} [e^{-\rho t} - e^{-\rho T}] \right) \left( K + \frac{1}{\rho} [e^{-\rho t} - e^{-\rho T}] \right) \\ & + \frac{\alpha - 2}{2\alpha} \int_t^T \left( \frac{e^{\tilde{\rho}s}}{\kappa} \right)^{\frac{2}{\alpha-2}} f(s)^{\frac{\alpha}{\alpha-2}} ds. \end{aligned}$$

*The integral in the last line can in general not be computed in closed form. However, it can be expressed as a hypergeometric function. For  $\rho = 0$ , the function  $g$  can be obtained by continuity in  $\rho$ , i.e. fix  $t$  and then compute the limit for  $\rho \rightarrow 0$ .*

The solutions of the maximization problems given in Theorem 3.1 are candidates for the optimal investment and control choices as well as for the optimal consumption rate for the problem in (18). In the following theorem we verify that under sufficient assumptions these solutions are indeed optimal.

**Theorem 3.2 (Verification)** *Let  $\kappa > 0$  and  $\alpha > 2$ . Assume the executive's utility function of wealth, the utility function of the consumption rate as well as the cost function are given by (14), (15) and (16). Then the candidates given in (19) and (20) are the optimal investment and control strategy (i.e. own-company stockholding, market portfolio holding and non-systematic Sharpe ratio strategy), the optimal consumption rate and value function of the optimal control problem (18).*

*Proof.* Define the performance functional of our optimal investment, consumption and control decision by

$$J'(t, v; \pi, k, \lambda) := \mathbb{E}^{t,v} \left[ U_1(V_T^u) + \int_t^T U_2(s, V_s^u, k_s) ds - \int_t^T C(s, V_s^u, \lambda_s) ds \right]. \quad (27)$$

Our candidates are optimal if we have

$$\begin{aligned} J'(t, v; \pi^*, k^*, \lambda^*) &= \Phi(t, v) \text{ and} \\ J'(t, v; \pi, k, \lambda) &\leq \Phi(t, v), \text{ for all } (\pi, k, \lambda) \in A'_1(t, v). \end{aligned} \quad (28)$$

Let  $u \in A'_1(t, v)$ . Since  $\Phi \in C^{1,2}$ , we obtain by Ito's formula:

$$\begin{aligned} &\Phi(T, V_T^u) + \int_t^T e^{-\rho s} \log(V_s^u k_s) ds - \int_t^T e^{-\tilde{\rho} s} \kappa \frac{\lambda_s^\alpha}{\alpha} ds \\ &= \Phi(t, v) + \int_t^T \left( \Phi_t(s, V_s^u) + e^{-\rho s} \log(V_s^u k_s) - e^{-\tilde{\rho} s} \kappa \frac{\lambda_s^\alpha}{\alpha} \right) ds \\ &\quad + \int_t^T \Phi_v(s, V_s^u) V_s^u (r + \delta - k_s + [\pi_s^P + \beta \pi_s^S] \lambda_P \sigma_P + \pi_s^S \lambda_s \sigma) ds \\ &\quad + \frac{1}{2} \int_t^T \Phi_{vv}(s, V_s^u) (V_s^u)^2 ([(\pi_s^P + \beta \pi_s^S) \sigma_P]^2 + [\pi_s^S \sigma]^2) ds \\ &\quad + \int_t^T \Phi_v(s, V_s^u) V_s^u (\pi_s^P + \beta \pi_s^S) \sigma_P dW_s^P + \int_t^T \Phi_v(s, V_s^u) V_s^u \pi_s^S \sigma dW_s. \end{aligned} \quad (29)$$

First, we investigate the optimal control  $u^* = (\pi^{P^*}, \pi^{S^*}, \lambda^*, k^*)$  given in (19). To show that the local martingale component in (29) vanishes in expectation we check the sufficient integrability condition

$$\mathbb{E} \left[ \int_t^T (\Phi_v(s, V_s^{u^*}) V_s^{u^*})^2 \left( [\pi_s^{P^*} + \beta \pi_s^{S^*}]^2 \sigma_P^2 + [\pi_s^{S^*}]^2 \sigma^2 \right) ds \right] < \infty. \quad (30)$$

From (19) and (20) we obtain

$$(\Phi_v(s, V_s^{u^*}) V_s^{u^*})^2 \left( [\pi_s^{P^*} + \beta \pi_s^{S^*}]^2 \sigma_P^2 + [\pi_s^{S^*}]^2 \sigma^2 \right) = (f(s))^2 (\lambda_P^2 + [\lambda^*(s)]^2).$$

Now,  $f$  and  $\lambda^*$  are deterministic continuous functions on the compact  $[0, T]$ , and thus the above expression is uniformly bounded. Accordingly the expectation in (30) is finite, and the Wiener integrals in (29) vanish in expectation.

Furthermore,  $\Phi$  satisfies the HJB equation (8) implying

$$\begin{aligned} 0 &= \Phi_v(s, V_s^{u^*}) V_s^{u^*} (r + \delta - k_s + [\pi_s^{P^*} + \beta \pi_s^{S^*}] \lambda_P \sigma_P + \pi_s^{S^*} \lambda_s^* \sigma) \\ &\quad + \frac{1}{2} \Phi_{vv}(s, V_s^{u^*}) (V_s^{u^*})^2 ([(\pi_s^{P^*} + \beta \pi_s^{S^*}) \sigma_P]^2 + [\pi_s^{S^*} \sigma]^2) \\ &\quad + \Phi_t(s, V_s^{u^*}) + e^{-\rho s} \log(V_s^{u^*} k_s^*) - e^{-\tilde{\rho} s} \kappa \frac{(\lambda_s^*)^\alpha}{\alpha}, \quad \text{for } t \leq s \leq T. \end{aligned}$$

Then, using that  $\Phi(T, v) = U_1(v)$  the expectation of (29) is:

$$\begin{aligned} \Phi(t, v) &= \mathbb{E}^{t,v} \left[ \Phi(T, V_T^{u^*}) + \int_t^T e^{-\rho s} \log(V_s^{u^*} k_s^*) ds - \int_t^T e^{-\tilde{\rho} s} \kappa \frac{(\lambda_s^*)^\alpha}{\alpha} ds \right] \\ &= \mathbb{E}^{t,v} \left[ U_1(V_T^{u^*}) + \int_t^T U_2(s, V_s^{u^*}, k_s^*) ds - \int_t^T C(s, V_s^{u^*}, \lambda_s^*) ds \right] \\ &= J'(t, v; \pi^*, \lambda^*, k^*). \end{aligned}$$

Thus we have verified the first part of (28).

Next, fix  $u \in A'(t, v)$ . By the HJB equation (8), we have

$$\begin{aligned} 0 &\geq \Phi_t(s, V_s^u) + \Phi_v(s, V_s^u) V_s^u (r + \delta - k_s + [\pi_s^P + \beta \pi_s^S] \lambda_P \sigma_P + \pi_s^S \lambda_s \sigma) \\ &\quad + \frac{1}{2} \Phi_{vv}(s, V_s^u) (V_s^u)^2 ([(\pi_s^P + \beta \pi_s^S) \sigma_P]^2 + [\pi_s^S \sigma]^2) \\ &\quad + e^{-\rho s} \log(V_s^u k_s) - e^{-\tilde{\rho} s} \kappa \frac{\lambda_s^\alpha}{\alpha}, \quad \text{for } t \leq s \leq T. \end{aligned}$$

Substituting this in (29) and recalling that  $\Phi_v(t, v) = \frac{1}{v} f(t)$  we get:

$$\begin{aligned} \Phi(T, V_T^\pi) &+ \int_t^T e^{-\rho s} \log(V_s^\pi k_s) ds - \int_t^T e^{-\tilde{\rho} s} \kappa \frac{\lambda_s^\alpha}{\alpha} ds \\ &\leq \Phi(t, v) + \int_t^T f(s) (\pi_s^P + \beta \pi_s^S) \sigma_P dW_s^P + \int_t^T f(s) \pi_s^S \sigma dW_s. \end{aligned}$$

Taking the expectation on both sides and keeping in mind that  $\Phi(T, v) =$

$U_1(v)$  then yields

$$\begin{aligned}
& J'(t, v; \pi, \lambda, k) \\
&= \mathbb{E}^{t, v} \left[ U_1(V_T^u) + \int_t^T U_2(s, V_s^u, k_s) ds - \int_t^T C(s, V_s^u, \lambda_s) ds \right] \\
&= \mathbb{E}^{t, v} \left[ \Phi(T, V_T^u) + \int_t^T e^{-\rho s} \log(V_s^u k_s) ds - \int_t^T e^{-\hat{\rho} s} \kappa \frac{\lambda_s^\alpha}{\alpha} ds \right] \\
&\leq \Phi(t, v) + \underbrace{\mathbb{E}^{t, v} \left[ \int_t^T f(s) (\pi_s^P + \beta \pi_s^S) \sigma_P dW_s^P + \int_t^T f(s) \pi_s^S \sigma dW_s \right]}_{=0, \text{ by (17)}}.
\end{aligned}$$

The Wiener integral vanishes in expectation since the corresponding integrand is square integrable, since  $f$  is uniformly bounded and (17).  $\square$

### 3.3 Participation Constraint for the Log-Utility Case

The optimal strategies in Theorem 3.1 and Theorem 3.2 above apply in case the individual decides to work for the smaller listed company. However, she has the opportunity to take up an outside option, that is, working for a larger company in a mid-level management position. The outside option offers a contract that differs in the salary rate and foregoes the possibility of controlling the stock price of the smaller listed company. Next, we calculate the value of the outside option and derive the participation constraint.

The outside option pays a salary rate  $\hat{\delta}$ . Taking on the position results in the loss of the ability to influence the stock price of the smaller listed company and therefore  $\hat{\lambda} = 0$ . She can invest in the financial market. The classical optimal investment and consumption decision applies. For the remainder of this subsection we assume that the portfolio process follows Eq. (5) where we set  $\delta = \hat{\delta}$  and  $\lambda = \hat{\lambda} = 0$ . Then the optimal investment decision problem in Equation (6) determines the value of the outside option  $\hat{\Phi}(0, V_0)$  at time  $t = 0$  for initial wealth  $V_0 > 0$ . The solution  $\hat{\Phi}$  can be obtained as a simplification of the results in Theorem 3.1 and Theorem 3.2, i.e.  $\hat{\Phi}(t, v) = \hat{f}(t) \log(v) + \hat{g}(t)$  with

$$\hat{f}(t) = \begin{cases} K + \frac{e^{-\rho t} - e^{-\rho T}}{\rho}, & \text{for } \rho \neq 0, \\ K + T - t, & \text{for } \rho = 0, \end{cases}$$

and

$$\begin{aligned}\widehat{g}(t) &= \left( r + \widehat{\delta} + \frac{1}{2} \lambda_P^2 \right) \int_t^T \widehat{f}(s) \, ds \\ &\quad - \int_t^T (1 + \rho s) e^{-\rho s} \, ds - \int_t^T e^{-\rho s} \log(\widehat{f}(s)) \, ds.\end{aligned}$$

Observe that  $\widehat{f} = f$  and

$$g(t) - \widehat{g}(t) = (\delta - \widehat{\delta}) \int_t^T f(s) \, ds + \frac{\alpha - 2}{2\alpha} \int_t^T \left( \frac{e^{\widehat{\rho}s}}{\kappa} \right)^{\frac{2}{\alpha-2}} f(s)^{\frac{\alpha}{\alpha-2}} \, ds. \quad (31)$$

Based on the discussion above we can state the participation constraint.

**Theorem 3.3** *Let  $\widehat{\delta}$  be the salary rate of the outside option. Then the value of the outside option is the solution  $\widehat{\Phi}$  to optimal investment and consumption problem in (6) with dynamics (5) where we set  $\delta = \widehat{\delta}$  and  $\lambda = \widehat{\lambda} = 0$ . The participation constraint for the individual is*

$$\delta \geq \widehat{\delta} - \frac{(\alpha - 2)}{2\alpha} \frac{\int_0^T \left( \frac{e^{\widehat{\rho}t}}{\kappa} \right)^{\frac{2}{\alpha-2}} f(t)^{\frac{\alpha}{\alpha-2}} \, dt}{\int_0^T f(t) \, dt}, \quad (32)$$

where  $f$  is given in Theorem 3.1.

*Proof.* The value function is of the form  $\widehat{\Phi} = \widehat{f}(t) \log(v) + \widehat{g}(t)$  with  $\widehat{f} = f$  and  $\widehat{g} - g$  given in (31). Then we have of course  $\Phi(t, v) - \widehat{\Phi}(t, v) = g(t) - \widehat{g}(t)$  and the participation constraint  $\Phi(0, V_0) \geq \widehat{\Phi}(0, V_0)$  follows as stated in (32).  $\square$

### 3.4 Discussion of the Power-Utility Case

In this subsection, we derive a closed-form solution for the case of power utility. In particular, we specify a constant relative risk aversion utility-disutility set-up. For the relative risk aversion parameter  $\gamma > 1$ , the utility function of the final wealth  $U_1$  is

$$U_1(v) = \frac{v^{1-\gamma}}{1-\gamma}, \quad \text{for } \gamma > 1, \quad (33)$$

the utility function of the consumption  $U_2$  is

$$U_2(k, v) = \frac{(vk)^{1-\gamma}}{1-\gamma}, \quad \text{for } \gamma > 1, \quad (34)$$

and the disutility of control (i.e. work effort)  $C$  is

$$C(v, \lambda) = \kappa v^{1-\gamma} \frac{\lambda^\alpha}{\alpha}, \quad \text{for } \gamma > 1, \quad (35)$$

where  $\kappa > 0$  and  $\alpha > 2$  are as in the log-utility part.

Compared to the log-utility setup we have made the simplifying assumption that utility from consumption in (34) and the cost from work effort in (35) are not depending on time, see (15) and (16) for time preferences in the log-utility setup. These assumption enable us to obtain a tractable formulation of the problem. However, we require a further structural assumption linking the cost function parameter  $\alpha$  to the relative risk aversion  $\gamma$ . The following condition is assumed to hold:

$$\alpha = 2\gamma + 2. \quad (36)$$

Condition (36) enables us to reduce an ODE of inhomogeneous Bernoulli type that appears in the HJB equation to an ODE of Riccati type, which we are able to solve in closed-form. This restriction is however not counterintuitive. A more risk averse individual is implicitly assumed to be more sensitive towards work. When focusing on the optimal work effort  $\lambda^*$  as a main result we can rely on Desmettre et al. (2010) discussing a related framework although without consumption and salary. Their results indicate that  $\lambda^*$  decreases with increasing risk aversion as well as with increasing disutility stress. So by relating those two parameters via (36) we do not change the qualitative behavior of the optimal work effort.

Analogously to the log-utility case, to guarantee indeed the optimality of the candidates we will derive for the executive's optimal investment and control strategy and value function, we consider again a more restrictive class of admissible strategies as follows.

**Definition 3.2** *Fix  $(t, v) \in [0, T] \times \mathbb{R}^+$ . Then for  $\gamma > 1$ , we denote by  $A'_\gamma(t, v)$  the set of admissible strategies  $u \in A'_\gamma(t, v)$ , such that  $u \in A_\gamma(t, v)$*

and

$$\int_t^T (\pi_s^P + \beta \pi_u^S)^4 (\sigma^P)^4 + (\pi_u^S \sigma)^4 du \leq C_1 < \infty, \quad \text{for some } C_1 \in \mathbb{R}_0^+, \quad (37)$$

$$\int_t^T \pi_u^S \sigma \lambda_u du \geq C_2 > -\infty, \quad \text{for some } C_2 \in \mathbb{R}_0^+, \quad (38)$$

$$\int_t^T k_u du \leq C_3 < \infty, \quad \text{for some } C_3 \in \mathbb{R}_0^+. \quad (39)$$

**Theorem 3.4 (The power-utility case:  $\gamma > 1$ )** *Suppose that the relative risk aversion parameter  $\gamma$  and the disutility stress parameter  $\alpha$  are connected via the relation (36), then the full solution of the maximization problem (18) can be summarized by the strategy*

$$\begin{aligned} \pi^{P^*}(t, v) &= \frac{\mu^P - r}{\gamma (\sigma^P)^2} - \beta \pi^{S^*}(t, v), \quad \pi^{S^*}(t, v) = \frac{\lambda^*(t, v)}{\gamma \sigma^*(t, v, \lambda^*(t, v))}, \\ \lambda^*(t, v) &= \left( \frac{1}{\kappa \gamma} f(t) \right)^{\frac{1}{2\gamma}}, \quad k^*(t, v) = (f(t))^{-\frac{1}{\gamma}}, \end{aligned} \quad (40)$$

and value function

$$\Phi(t, v) = \frac{v^{1-\gamma}}{1-\gamma} f(t), \quad (41)$$

where

$$f(t) = \left( \frac{2(1-g_P)\sqrt{C_0}}{2\sqrt{C_0}e^{-2\sqrt{C_0}(T-t)} + (1-g_P)(e^{-2\sqrt{C_0}(T-t)} - 1)} + g_P \right)^{-\gamma}, \quad (42)$$

with

$$C_0 = \frac{(\gamma-1)^2}{4\gamma^2} \left( r + \delta + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right)^2 - \frac{\kappa(1-\gamma)}{2(1+\gamma)} \left( \frac{1}{\kappa\gamma} \right)^{\frac{\gamma+1}{\gamma}}, \quad (43)$$

and

$$g_P = -\frac{1-\gamma}{2\gamma} \left( r + \delta + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right) + \sqrt{C_0}. \quad (44)$$

*Proof.* First observe that a function  $F$  of the form  $F(\lambda) = a\lambda^2 - b\lambda^\alpha$ ,  $\lambda \geq 0$ , for given constants  $a, b > 0$  and  $\alpha > 2$ , has a unique maximizer  $\lambda^*$  and maximized value  $F(\lambda^*)$  given by

$$\lambda^* = \left( \frac{2a}{\alpha b} \right)^{\frac{1}{\alpha-2}}, \quad \text{and} \quad F(\lambda^*) = (\alpha - 2) \alpha^{-\frac{\alpha}{\alpha-2}} 2^{\frac{2}{\alpha-2}} a^{\frac{\alpha}{\alpha-2}} b^{-\frac{2}{\alpha-2}}. \quad (45)$$

Using this insight the first order condition for  $\lambda^*$  in (11) is now solved. Set

$$a = \frac{1}{2} \frac{\Phi_v^2}{-\Phi_{vv}}, \quad \text{and} \quad b = \frac{\kappa}{\alpha} v^{1-\gamma},$$

then (45) gives

$$\lambda^* = \left( \frac{1}{\kappa v^{1-\gamma}} \frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{1}{\alpha-2}}, \quad F(\lambda^*) = \frac{\alpha - 2}{2\alpha} (\kappa v^{1-\gamma})^{-\frac{2}{\alpha-2}} \left( \frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{\alpha}{\alpha-2}}.$$

Having specified the utility function as  $U_2(t, k_t) = \frac{(vk)^{1-\gamma}}{1-\gamma}$ , the first order condition (12) for the optimal consumption rate becomes:

$$k^* = \frac{1}{v} (\Phi_v)^{-\frac{1}{\gamma}}.$$

Substituting  $\lambda^*$  and  $k^*$  in (13) then yields:

$$\begin{aligned} 0 = & \Phi_t + \Phi_v v (r + \delta) + \frac{1}{2} \frac{\Phi_v^2}{-\Phi_{vv}} (\lambda^P)^2 \\ & + \frac{\alpha - 2}{2\alpha} (\kappa v^{1-\gamma})^{-\frac{2}{\alpha-2}} \left( \frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{\alpha}{\alpha-2}} + \frac{\gamma}{1-\gamma} (\Phi_v)^{\frac{\gamma-1}{\gamma}}. \end{aligned} \quad (46)$$

Using the separation ansatz  $\Phi(t, v) = f(t) \frac{v^{1-\gamma}}{1-\gamma}$  results in

$$\Phi_t = \dot{f} \frac{v^{1-\gamma}}{1-\gamma}, \quad \Phi_v = f v^{-\gamma}, \quad \Phi_{vv} = -\gamma f v^{-\gamma-1}, \quad \text{and} \quad f(T) = 1. \quad (47)$$

Thus (46) becomes

$$\begin{aligned} 0 = & \dot{f} \frac{v^{1-\gamma}}{1-\gamma} + f v^{1-\gamma} (r + \delta) + \frac{1}{2} \frac{f v^{1-\gamma}}{\gamma} (\lambda^P)^2 \\ & + \frac{\alpha - 2}{2\alpha} (\kappa v^{1-\gamma})^{-\frac{2}{\alpha-2}} \left( \frac{f v^{1-\gamma}}{\gamma} \right)^{\frac{\alpha}{\alpha-2}} + \frac{\gamma}{1-\gamma} v^{1-\gamma} f^{\frac{\gamma-1}{\gamma}}. \end{aligned}$$

Dividing by  $\frac{v^{1-\gamma}}{1-\gamma}$  and then defining

$$\begin{aligned} a_1 &= (1-\gamma) \left( r + \delta + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right), \quad a_n = (1-\gamma) \frac{\kappa}{2} \frac{\alpha-2}{\alpha} \left( \frac{1}{\kappa\gamma} \right)^{\frac{\alpha}{\alpha-2}}, \\ a_m &= \gamma, \quad n = \frac{\alpha}{\alpha-2}, \quad \text{and} \quad m = \frac{\gamma-1}{\gamma}. \end{aligned} \quad (48)$$

results in an ordinary differential equation of the form

$$\dot{f} + a_1 f + a_n f^n + a_m f^m = 0. \quad (49)$$

The ansatz  $g = f^{1-n}$  yields  $\dot{g} = \frac{1-n}{f^n} \dot{f}$  and thus

$$\dot{g} + a_1 (1-n) g + a_m (1-n) g^{\frac{m-n}{1-n}} = -a_n (1-n), \quad g(T) = 1.$$

Using (36), i.e.  $\alpha = 2 + 2\gamma$ , and plugging in the coefficients in (48) we obtain the following ODE of Riccati type

$$\dot{g} - \frac{1-\gamma}{\gamma} \left( r + \delta + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right) g - g^2 = \frac{\kappa}{2} \frac{1-\gamma}{1+\gamma} \left( \frac{1}{\kappa\gamma} \right)^{\frac{\gamma+1}{\gamma}}. \quad (50)$$

This ODE can be solved if we know a particular solution  $g_P$ , since then we can reduce this ODE by using the standard ansatz

$$h = 1/(g - g_P)$$

to the following linear form:

$$\dot{h} + \left[ 2g_P + \frac{\gamma-1}{\gamma} \left( r + \delta + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right) \right] h + 1 = 0, \quad h(T) = \frac{1}{1 - g_P}.$$

This equation can now be solved by variation of constants. A nonnegative particular solution of (50) is

$$\begin{aligned} g_P &= -\frac{1-\gamma}{2\gamma} \left( r + \delta + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right) \\ &\quad + \sqrt{\frac{(\gamma-1)^2}{4\gamma^2} \left( r + \delta + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right)^2 - \frac{\kappa(1-\gamma)}{2(1+\gamma)} \left( \frac{1}{\kappa\gamma} \right)^{\frac{\gamma+1}{\gamma}}}, \end{aligned}$$

which means that we have to solve the following inhomogeneous linear ODE

$$\dot{h} + \left[ 2\sqrt{\frac{(\gamma-1)^2}{4\gamma^2} \left( r + \delta + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right)^2 - \frac{\kappa(1-\gamma)}{2(1+\gamma)} \left( \frac{1}{\kappa\gamma} \right)^{\frac{\gamma+1}{\gamma}}} \right] h + 1 = 0. \quad (51)$$

Now applying variation of constants and using that  $h(T) = 1/(1-g_P)$ , the solution of this ODE is

$$h(t) = \frac{1}{1-g_P} e^{2\sqrt{C_0}(T-t)} + \frac{1}{2\sqrt{C_0}} \left( e^{2\sqrt{C_0}(T-t)} - 1 \right), \quad (52)$$

where

$$C_0 = \frac{(\gamma-1)^2}{4\gamma^2} \left( r + \delta + \frac{1}{2} \frac{\lambda_P^2}{\gamma} \right)^2 - \frac{\kappa(1-\gamma)}{2(1+\gamma)} \left( \frac{1}{\kappa\gamma} \right)^{\frac{\gamma+1}{\gamma}}.$$

Transforming the result back to the function  $f$  we get

$$f(t) = \left( g_P + \frac{2(1-g_P)\sqrt{C_0}}{2\sqrt{C_0}e^{2\sqrt{C_0}(T-t)} + (1-g_P)(e^{2\sqrt{C_0}(T-t)} - 1)} \right)^{-\gamma}. \quad (53)$$

Using the representations (47) we get

$$\lambda^*(t, v) = \left( \frac{1}{\kappa v^{1-\gamma}} \frac{\Phi_v^2}{-\Phi_{vv}} \right)^{\frac{1}{\alpha-2}} = \left( \frac{1}{\kappa\gamma} f(t) \right)^{\frac{1}{\alpha-2}} = \left( \frac{1}{\kappa\gamma} f(t) \right)^{\frac{1}{2\gamma}},$$

and

$$\begin{aligned} \pi^{P^*}(t, v) &= -\frac{(\mu^P - r)}{v(\sigma^P)^2} \frac{\Phi_v(t, v)}{\Phi_{vv}(t, v)} - \beta \pi^{S^*}(t, v) = \frac{\mu^P - r}{\gamma(\sigma^P)^2} - \beta \pi^{S^*}(t, v), \\ \pi^{S^*}(t, v) &= -\frac{\lambda^*(t, v)}{v\sigma^*(t, v, \lambda^*(t, v))} \frac{\Phi_v(t, v)}{\Phi_{vv}(t, v)} = \frac{\lambda^*(t, v)}{\gamma\sigma^*(t, v, \lambda^*(t, v))}, \end{aligned}$$

as well as

$$k^*(t, v) = \frac{1}{v} (\phi_v(t, v))^{-\frac{1}{\gamma}} = \frac{1}{v} (f(t) v^{-\gamma})^{-\frac{1}{\gamma}} = (f(t))^{-\frac{1}{\gamma}}.$$

And the proof is finished.  $\square$

**Remark 3.2** *Establishing the solution is based on the function  $f$  in (49). The transformation  $g = f^{-1/\gamma}$  is applied and requires  $f$  to be nonnegative. Accordingly, the function  $g$  satisfies the Riccati ODE in (50) and lives also on  $\mathbb{R}^+$ . As a solution strategy we identify a particular solution  $g_P$ . This works for  $\gamma > 1$ , since then  $g_P > 0$ , i.e. the particular solution is in the region where  $g$  is specified on. However, the solution strategy breaks down for  $0 < \gamma < 1$ . Then we would have  $g_P < 0$  and this candidate is not an admissible solution. This explains why we cannot provide a solution for the case  $0 < \gamma < 1$ , at least, with our methods at hand.*

Again, we need to show that the candidates derived in Theorem (3.4) are indeed optimal. This is done in the following verification theorem. The proof is provided in the Appendix.

**Theorem 3.5 (Verification Result for the Case  $\gamma > 1$ )** *Let  $\kappa > 0$  and  $\alpha > 2$  and  $\alpha = 2\gamma + 2$ . Assume the utility function of wealth, the utility function of the consumption rate and the disutility function are given by (33), (34) and (35), respectively. Then the candidates given via (40) - (44) are the optimal investment and control strategy (i.e. own-company stockholding, market portfolio holding and non-systematic Sharpe ratio strategy), the optimal consumption rate and value function of the optimal control problem (18) for the case  $\gamma > 1$ .*

## 4 Discussion and Implications of Results

The previous section established results on the optimal behavior of the individual and derived the participation constraint, i.e. conditions for her to accept the offer by the smaller listed company. In the following we discuss the results by investigating the sensitivities of the optimal strategies and the participation constraint when varying model parameters.

### 4.1 Optimal Work Effort

Theorems 3.1 and 3.2 indicate the individual's maximized utility and associated optimal behavior in terms of personal portfolio selection, consumption and work effort decision, given that she accepts to job offer by the smaller listed company, all subject to the log utility set-up. We now investigate the sensitivity of the optimal work effort to variations of the risk aversion and

work effectiveness characteristics and time preference. Note that the portfolio selection and consumption are in line with standard results in the log utility setup and are here of limited interest.

The individual is characterized by the work effectiveness parameters work productivity ( $1/\kappa$ , with  $\kappa > 0$ ), and disutility stress ( $\alpha > 2$ ) and the time preferences of consumption from work effort ( $\rho \in \mathbb{R}$ ) and disutility ( $\tilde{\rho} \in \mathbb{R}$ ), respectively. To produce results that have relativity to a base-level of work effort, as indicated by a base-level non-systematic Sharpe ratio control decision  $\lambda_0 > 0$ , the disutility  $C$  given by (16) is reparameterized to

$$C(t, v, \lambda) = e^{-\tilde{\rho}t} \frac{\tilde{\kappa}}{\alpha} \left( \frac{\lambda}{\lambda_0} \right)^\alpha, \quad \text{for } \lambda \geq 0, \quad \gamma > 0,$$

and the utility of wealth  $U_1$  and the utility of consumption  $U_2$  remain unchanged.

The individual's optimal work effort for the new disutility parametrization is  $\lambda^*(t, v) = \lambda_0^{\frac{\alpha}{\alpha-2}} \left( \frac{e^{\tilde{\rho}t}}{\tilde{\kappa}} f(t) \right)^{\frac{1}{\alpha-2}}$  (see Theorem 3.1 for the optimal choice under the original parametrization). Assuming that the inverse work productivity satisfies  $1/\tilde{\kappa} > \lambda_0^{-2} e^{|\tilde{\rho}|T}/K$  we guarantee that the optimal work effort  $\lambda^*$  is not less than the base level  $\lambda_0$ , i.e.  $\lambda^* \geq \lambda_0 > 0$ . If not stated otherwise, the default values for the parameters are  $\alpha = 5$ ,  $1/\tilde{\kappa} = 1000$ ,  $r = 0.05$ ,  $\lambda_P = 0.20$ ,  $\lambda_0 = 0.10$ ,  $\rho = 0.10$ ,  $\tilde{\rho} = -0.10$ ,  $K = 1$ ,  $\hat{\delta} = 0.20$ , and  $T = 10$ .

The individual's optimal work effort choice is positively related to her work productivity and negatively related to her disutility stress. This result is illustrated by Figures 1 and 2, which graph the optimal work effort  $\lambda^*$  versus time  $t$  and work productivity  $1/\tilde{\kappa}$  and, time  $t$  and disutility stress  $\alpha$ , respectively. Both figures indicate that the individual's optimal work effort is negatively related to time, i.e.  $\lambda^*$  is decreasing over time. The individual spends in general more work effort at the beginning of the time horizon. Note that the monotonicity of the optimal work effort depends on the sign of  $\rho$ , see discussion of Figure 4 below.

Figure 3 shows the optimal work effort choice  $\lambda^*$  w.r.t. the time preference of consumption  $\rho$  and time  $t$ . The figure indicates that with increasing time the optimal work effort decreases as already observed above. This implies that the individual is more productive at the beginning of her career path. The optimal work effort is also decreasing for increasing time preference of consumption  $\rho$ . An individual which has a higher consumption preference will deliver a lower work effort, especially at the beginning of the time horizon.

Figure 4 graphs the optimal work effort choice  $\lambda^*$  w.r.t. the time preference of disutility  $\tilde{\rho}$  and time  $t$ . The optimal work effort is positively related to the time preference of work related disutility  $\tilde{\rho}$ , i.e. with increasing value of  $\tilde{\rho}$  the individual is becoming more productive and delivers a higher level of the optimal work effort indicating a reasonable behavior: The higher the cost for spending work effort the lower is the optimal work effort. Note that positive values of  $\tilde{\rho}$  are associated with work effort becoming cheaper over time. For this parameter set, we first observe over time an increase of work effort and then a decrease at the end of the time horizon. However, typically we expect  $\tilde{\rho}$  to be negative, i.e., work effort becomes more expensive with the passing of time.

## 4.2 Participation Constraint

The participation constraint is given in Theorem 3.3. Denote  $\delta^*$  the minimal salary rate such that the participation constraint holds, i.e.  $\delta^* = \inf\{\delta \in \mathbb{R} : \delta \text{ satisfies (32)}\}$ . Taking account of the reparametrization gives

$$\delta^* = \begin{cases} \delta_0 - \frac{(\alpha - 2)}{2\alpha} \lambda_0^{\frac{2\alpha}{\alpha-2}} \frac{\int_0^T \left(\frac{e^{\tilde{\rho}s}}{\tilde{\kappa}}\right)^{\frac{2}{\alpha-2}} f(s)^{\frac{\alpha}{\alpha-2}} ds}{KT + \frac{1}{\rho^2} [1 - e^{-\rho T}(1 + \rho T)]}, & \text{for } \rho \neq 0, \\ \delta_0 - \frac{(\alpha - 2)}{2\alpha} \lambda_0^{\frac{2\alpha}{\alpha-2}} \frac{\int_0^T \left(\frac{e^{\tilde{\rho}s}}{\tilde{\kappa}}\right)^{\frac{2}{\alpha-2}} f(s)^{\frac{\alpha}{\alpha-2}} ds}{KT + \frac{1}{2}T^2}, & \text{for } \rho = 0. \end{cases} \quad (54)$$

Now,  $\alpha > 2$  by assumption and  $f > 0$  by Theorem 3.1. And the minimal salary rate of the smaller listed company that is satisfying the participation constraint is always below the salary rate of the outside option, i.e.  $\delta^* < \delta$ . The salary rate discount can be explained by the fact that the smaller company is offering in return for the reduced salary the possibility to affect the share price by work effort and thereby to increase the utility derived from the individual's investment. In the following we investigate the minimal required salary rate  $\delta^*$  depending on the individual's parameters (work productivity  $1/\tilde{\kappa}$ , disutility stress  $\alpha$ , time preference of consumption  $\rho$  and time preference for work effort  $\tilde{\rho}$ ) to characterize individuals that are attracted by an offer of the smaller listed company.

Figure 5 displays the minimal required salary rate  $\delta^*$  w.r.t. disutility stress  $\alpha$  and work productivity  $1/\tilde{\kappa}$ . The minimal required salary rate is

decreasing with increasing work productivity and increasing with increasing disutility stress. This means that a more productive individual is willing to accept a lower salary rate because she can compensate the loss of utility by the ability to improve the unsystematic Sharpe ratio  $\lambda$ . On the other hand, an individual with a higher disutility stress requires a higher salary rate to accept the contract from the smaller listed company.

The effect of the time preferences is shown in Figure 6. The required minimal salary rate  $\delta^*$  is graphed against the time preference of consumption  $\rho$  and the time preference of disutility from work effort  $\tilde{\rho}$ , respectively. Increasing the time preference parameter for consumption increases the minimal required salary rate. In contrast, the required minimal salary rate decreases with increasing time preference of disutility. This is attributed to the average disutility from work effort being lower for a higher value of  $\tilde{\rho}$ . The individual will deliver a higher work effort, see also Figure 4.

We summarize that the offered salary rate  $\delta$  can act as a selection device for the smaller listed company. Under the assumption that potential job candidates have an identical outside option, the group of individuals satisfying a more restrictive participation constraint is in general more talented, i.e. the individuals exhibit a lower disutility stress  $\alpha$ , a higher productivity  $1/\tilde{\kappa}$ , a lower time preference for consumption  $\rho$ , and a higher time preference for disutility from work effort  $\tilde{\rho}$ . Viewing the holdings in the own-company shares ( $\pi^{S^*}(t) = \lambda^*(t)/\sigma$ ) as a way of voluntarily linking the pay to performance, our results reflect common practice in executive remuneration. A more talented manager is in general attracted by a lower fixed salary component and a higher performance linked salary component.

## 5 Conclusion and Outlook

We establish a model framework that gives insight into an individual's occupational decision when she can choose between two different positions. She is offered an executive position in smaller listed company where she can affect the company's share price by work effort. Alternatively, she can take up a mid-level management position with a larger company but then forgoes the possibility to affect the other company's share price. We identify conditions for the individual to work for the smaller listed company where the participation constraint is given in terms of the salary differential of the two job alternatives. In particular, we derive the minimal required salary  $\delta^*$  that

needs to be offered by the smaller company to attract the individual and thereby characterize the participation constraint. In general, we find that a more talented individual requires a lower salary to be attracted to the smaller listed company. This salary pattern can be observed in practice, e.g., in the pharmaceutical industry, the IT sector, and the financial industry.

Given that the participation constraint holds, we give explicit solutions for the individual's utility maximizing behavior in terms of the investment strategy ( $\pi = (\pi^P, \pi^S)$ ), consumption ( $k$ ), and work effort ( $\lambda$ ). Overall, our results depend sensibly on her characteristics, work productivity  $1/\kappa$ , disutility stress  $\alpha$ , time preference of consumption  $\rho$ , and time preference of work effort  $\tilde{\rho}$ . We demonstrate that an executive with higher work effectiveness (quality) undertakes more work effort, which is associated with a lower minimal required salary  $\delta^*$ . The main analysis is performed in the log-utility setting. However, we also explore the broader setup of constant relative risk aversion.

A future development of this work is to extend the semi-static game between the individual and the smaller listed company to a stochastic differential game. The aim of the company is then to maximize share holder value. The additional control available to the company is the quantity of share-based payments granted to the individual that affect her holdings in the company's shares. The stochastic differential game can then be investigated for equilibria. This setup is likely to provide more insight into the design of optimal share-based payments.

## Appendix

*Proof of Theorem 3.5.* Define the performance functional of our optimal investment, consumption and control decision again by (27). Our candidates are optimal if we have

$$\begin{aligned} J'(t, v; \pi^*, \lambda^*, k^*) &= \Phi(t, v) \text{ and} \\ J'(t, v; \pi, \lambda, k) &\leq \Phi(t, v), \text{ for all } (\pi, \lambda, k) \in A'_\gamma(t, v). \end{aligned}$$

Let  $u \in A'_\gamma(t, v)$ . Since  $\Phi \in C^{1,2}$ , we obtain by Ito's formula:

$$\begin{aligned} \Phi(T, V_T^u) - \int_t^T \kappa(V_s^u)^{1-\gamma} \frac{\lambda_s^\alpha}{\alpha} ds + \int_t^T \frac{(V_s^u k)^{1-\gamma}}{1-\gamma} ds = \Phi(t, v) + \\ \int_t^T \left\{ \Phi_t(s, V_s^u) + \Phi_v(s, V_s^u) V_s^u [r + \pi_s^S \lambda_s \sigma + (\pi_s^P + \beta \pi_s^S) \lambda^P \sigma^P + \delta - k_s] \right. \\ \left. + 1/2 \Phi_{vv}(s, V_s^u) (V_s^u)^2 [(\pi_s^P + \beta \pi_s^S)^2 (\sigma^P)^2 + (\pi_s^S \sigma)^2] \right. \\ \left. - \kappa(V_s^u)^{1-\gamma} \frac{\lambda_s^\alpha}{\alpha} ds + \frac{(V_s^u k)^{1-\gamma}}{1-\gamma} \right\} \\ + \int_t^T \Phi_v(s, V_s^u) V_s^u (\pi_s^P + \beta \pi_s^S) \sigma^P dW_s^P + \int_t^T \Phi_v(s, V_s^u) V_s^u \pi_s^S \sigma dW_s. \quad (55) \end{aligned}$$

For the optimality candidates given in (19), the local martingale component in (55) disappears. A sufficient condition to verify this is the square integrability condition

$$\mathbb{E} \left[ \int_t^T (\Phi_v(s, V_s^{u^*}) V_s^{u^*})^2 ([\pi_s^{P^*} + \beta \pi_s^{S^*}]^2 (\sigma^P)^2 + [\pi_s^{S^*} \sigma]^2) ds \right] < \infty. \quad (*)$$

Now substituting the candidates from (40) - (44) yields

$$\begin{aligned} (\Phi_v(s, V_s^{u^*}) V_s^{u^*})^2 ([\pi_s^{P^*} + \beta \pi_s^{S^*}]^2 (\sigma^P)^2 + [\pi_s^{S^*} \sigma]^2) \\ = \frac{(V_s^{u^*})^{2(1-\gamma)} f(s)^2}{\gamma^2} \left[ (\lambda^P)^2 + \left( \frac{1}{\kappa \gamma} f(s) \right)^{\frac{1}{\gamma}} \right]. \quad (**) \end{aligned}$$

The RHS of (\*\*) is  $(V_s^{u^*})^{2(1-\gamma)}$  times a deterministic and continuous function on the compact set  $[0, T]$ . The deterministic part is uniformly bounded. Therefore it is sufficient to focus on the stochastic component:  $V_s^{u^*}$  satisfies the wealth equation

$$\begin{aligned} dV_t^{u^*} = V_t^{u^*} \left[ r dt + \frac{\lambda_P^2}{\gamma} dt + \frac{(\lambda^*(t, V_t^{u^*}))^2}{\gamma} dt - (f(t))^{-\frac{1}{\gamma}} dt + \delta dt \right. \\ \left. + \frac{\lambda_P}{\gamma} dW_t^P + \frac{\lambda^*(t, V_t^{u^*})}{\gamma} dW_t \right], \end{aligned}$$

for which we have substituted the optimality candidates (40) in the original wealth equation. Recalling that  $\lambda^*(t, v)$  is a deterministic function in  $t$  and

further does not depend on  $v$  and that  $f(t)$  is a deterministic function as well, we see that  $V_t^{u^*}$  follows a log-normal distribution for all  $t \geq 0$  with parameters being uniformly bounded for all  $t \in [0, T]$ . Since all moments of a log-normally distributed random variable exist, we have proven (\*). Furthermore  $\Phi$  satisfies the HJB equation (8), i.e. for  $u = u^* = (\pi^{P^*}, \pi^{S^*}, \lambda^*, k^*)$ , the choice (16) of the disutility function and the choice (15) of the consumption rate we have:

$$\begin{aligned} & \Phi_t(s, V_s^{u^*}) + \Phi_v(s, V_s^{u^*})V_s^{u^*} [r + \pi_s^{S^*} \lambda_s^* \sigma + (\pi_s^{P^*} + \beta \pi_s^{S^*}) \lambda^P \sigma^P + \delta - k_s^*] \\ & + 1/2 \Phi_{vv}(s, V_s^{u^*}) (V_s^{u^*})^2 [(\pi_u^{P^*} + \beta \pi_u^{S^*})^2 (\sigma^P)^2 + (\pi_s^{S^*} \sigma)^2] \\ & - \kappa (V_s^{u^*})^{1-\gamma} \frac{(\lambda_s^*)^\alpha}{\alpha} + \frac{(V_s^{u^*} k^*)^{1-\gamma}}{1-\gamma} = 0. \end{aligned}$$

Then, for  $u = u^*$ , the expectation of equation (55) using  $\Phi(T, v) = v^{1-\gamma}/(1-\gamma)$  is:

$$\begin{aligned} & \mathbb{E}^{t,v} \left[ \frac{(V_T^{u^*})^{1-\gamma}}{1-\gamma} \right] - \mathbb{E}^{t,v} \left[ \int_t^T \kappa (V_s^{u^*})^{1-\gamma} \frac{(\lambda_s^*)^\alpha}{\alpha} ds \right] + \mathbb{E}^{t,v} \left[ \int_t^T \frac{(V_s^{u^*} k^*)^{1-\gamma}}{1-\gamma} ds \right] \\ & = J'(t, v; \pi^*, \lambda^*, k^*) = \Phi(t, v). \end{aligned}$$

The optimality of our candidates is finally shown if we have for all  $(\pi, \lambda, k) \in A'_\gamma(t, v)$ :

$$\begin{aligned} & \mathbb{E}^{t,v} \left[ \frac{(V_T^u)^{1-\gamma}}{1-\gamma} \right] - \mathbb{E}^{t,v} \left[ \int_t^T \kappa (V_s^u)^{1-\gamma} \frac{(\lambda_s)^\alpha}{\alpha} ds \right] + \mathbb{E}^{t,v} \left[ \int_t^T \frac{(V_s^u k)^{1-\gamma}}{1-\gamma} ds \right] \\ & = J'(t, v; \pi, \lambda, k) \leq \Phi(t, v). \end{aligned} \tag{56}$$

Also, since  $\Phi$  satisfies the HJB equation (8), we get for all  $(\pi, \lambda, k) \in A'_\gamma(t, v)$ :

$$\begin{aligned} & \Phi_t(s, V_s^u) + \Phi_v(s, V_s^u)V_s^u [r + \pi_s^S \lambda_s \sigma + (\pi_s^P + \beta \pi_s^S) \lambda^P \sigma^P + \delta - k_s] \\ & + 1/2 \Phi_{vv}(s, V_s^u) (V_s^u)^2 [(\pi_u^P + \beta \pi_u^S)^2 (\sigma^P)^2 + (\pi_s^{S^*} \sigma)^2] \\ & - \kappa (V_s^u)^{1-\gamma} \frac{(\lambda_s)^\alpha}{\alpha} + \frac{(V_s^u k)^{1-\gamma}}{1-\gamma} \leq 0. \end{aligned}$$

Substituting this in (55), recalling that  $\Phi_v(t, v) = f(t) v^{-\gamma}$ , we get:

$$\begin{aligned} \Phi(T, V_T^u) - \int_t^T \kappa (V_s^u)^{1-\gamma} \frac{\lambda_s^\alpha}{\alpha} ds + \int_t^T \frac{(V_s^u k)^{1-\gamma}}{1-\gamma} ds \leq \Phi(t, v) \\ + \underbrace{\int_t^T (V_s^u)^{1-\gamma} f(s) (\pi_s^P + \beta \pi_s^S) \sigma^P dW_s^P + \int_t^T (V_s^u)^{1-\gamma} f(s) \pi_s^S \sigma dW_s^S}_{=: M_T^t}. \end{aligned} \quad (57)$$

To verify equation (56), we impose conditions under which the local martingale  $M^t$  is a martingale. Recall  $\Phi_v(t, v) = f(t) v^{-\gamma}$  and calculate the quadratic variation of  $M^t$

$$\begin{aligned} \langle M^t \rangle_T &= \int_t^T (V_s^u)^{2(1-\gamma)} f^2(s) ([\pi_s^P + \beta \pi_s^S]^2 (\sigma^P)^2 + [\sigma \pi_s^S]^2) ds \\ &\leq \frac{1}{2} \sup_{0 \leq s \leq T} f(s)^2 \left( \int_t^T (V_s^u)^{4(1-\gamma)} ds + \int_t^T ([\pi_s^P + \beta \pi_s^S]^2 (\sigma^P)^2 + [\sigma \pi_s^S]^2)^2 ds \right), \end{aligned} \quad (58)$$

where the second line is a straightforward upper bound. We show that  $M^t$  is a martingale by deriving the integrability of the quadratic variation  $\langle M^t \rangle_T$ . First we use that  $f$  is a continuous function on the compact set  $[0, T]$  and is uniformly bounded, and thus  $\sup_{0 \leq s \leq T} f(s)^2$  is finite. We are left to deal with the two expressions in the brackets of (58). The second expression is bounded in expectation by assumption, see (37) in Def. 3.2. In what follows we establish that the first expression is finite by showing that  $\mathbb{E}^{t,v}[(V_s^u)^\xi] < \infty$  uniformly, where  $\xi = 4(1-\gamma) < 0$  for  $\gamma > 1$ .

Applying variation of constants, the solution of the wealth equation (4) expressed with respect to the parameter  $\lambda$  is

$$V_t^u = V_0^u e^{(r+\delta)t + \int_0^t ((\pi_s^P + \beta \pi_s^S) \lambda^P \sigma^P + \pi_s^S \lambda_s \sigma - k_s) ds} e^{L_t - \frac{1}{2} \langle L \rangle_t},$$

where  $L_t = \int_0^t (\pi_s^P + \beta \pi_s^S) \sigma^P dW_u^P + \int_0^t \pi_s^S \sigma dW_u^S$  and  $\langle L \rangle_t = \int_0^t (\pi_s^P + \beta \pi_s^S)^2 (\sigma^P)^2 + (\pi_s^S \sigma)^2 ds$ .

Using this we have

$$(V_t^u)^\xi = (V_0^u)^\xi e^{\xi L_t - \frac{1}{2} \xi^2 \langle L \rangle_t} \times e^{\xi [\frac{1}{2} (\xi-1) \langle L \rangle_t + (r+\delta)t + \int_0^t ((\pi_s^P + \beta \pi_s^S) \lambda^P \sigma^P + \pi_s^S \lambda_s \sigma - k_s) ds]}.$$

The second factor is uniformly bounded by a constant, compare the conditions (37), (38) and (39) of Definition 3.2, recalling that  $\xi < 0$  for  $\gamma > 1$ , and

keeping in mind that  $k_t \geq 0$ ,  $t \leq s \leq T$ , by assumption. It remains to prove that the first factor,  $Z_t := e^{\xi L_t - \frac{1}{2} \xi^2 \langle L \rangle_t} \in L^2(P)$ ,  $t \leq u \leq T$ , is integrable. However,  $Z^t$  is strictly positive local martingale since it is the stochastic exponential of the local martingale  $\xi L^t$ . The Novikov condition holds by (37), i.e.:  $\mathbb{E}^{t,v}(e^{\frac{1}{2} \xi^2 \langle L^t \rangle_T}) < \infty$ , and hence  $Z^t$  is a true martingale and  $\mathbb{E}^{t,v}(Z_s^t) = 1$ ,  $t \leq s \leq T$ . The local martingale  $M^t$  is therefore a martingale vanishing in expectation in (57), implying (56) for  $u = (\pi, \lambda, k) \in \mathcal{A}'_\gamma(t, v)$ .  $\square$

## References

- Cadenillas A, Cvitanić J, Zapatero F (2004) Leverage decision and manager compensation with choice of effort and volatility. *Journal of Financial Economics* 73(1):71–92
- Core J, Guay W, Larcker D (2003) Executive equity compensation and incentives: A survey. *Economic Policy Review* 9:27–50
- Desmettre S, Gould J, Szimayer A (2010) Own-company stockholding and work effort preferences of an unconstrained executive. Working Paper available at SSRN: <http://ssrn.com/abstract=1330814>
- Holmstrom B (1979) Moral hazard and observability. *Bell Journal of Economics* 10:74–91
- Jensen M, Meckling W (1976) Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics* 3(4):305–360
- Korn R, Korn E (2001) Option pricing and portfolio optimization. *Graduate Studies in Mathematics, Volume 31*, American Mathematical Society
- Korn R, Kraft H (2008) Continuous-time delegated portfolio management with homogeneous expectations: Can an agency conflict be avoided? *Financial Markets and Portfolio Management* 22(1):67–90
- Murphy K (1999) Executive compensation. Ashenfelter and D. Card, eds., *Handbook of labor economics*, Vol. 3, Amsterdam, North-Holland.
- Ou-Yang H (2003) Optimal contracts in a continuous-time delegated portfolio management problem. *Review of Financial Studies* 16(1):173–208
- Ross S (1973) The economic theory of agency: The principal’s problem. *American Economic Review* 63(2):134–139

# Figures

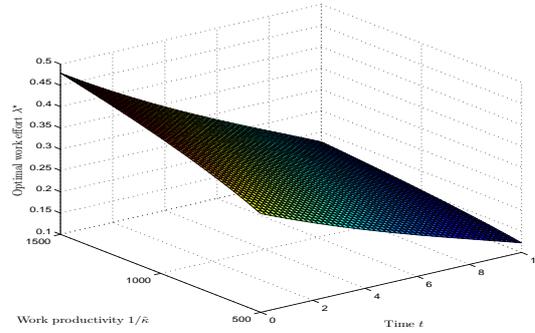


Figure 1: Optimal work effort  $\lambda^*$  w.r.t. work productivity  $1/\tilde{\kappa}$  and time  $t$  for fixed disutility stress  $\alpha = 5$ , time preferences  $\rho = 0.10$  and  $\tilde{\rho} = -0.10$ ,  $K = 1$ , base-level work effort  $\lambda_0 = 0.10$  and time horizon  $T = 10$  years.

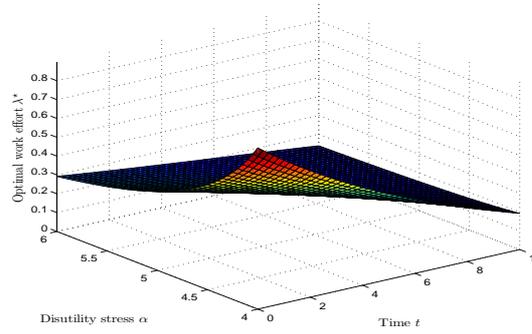


Figure 2: Optimal work effort  $\lambda^*$  w.r.t. disutility stress  $\alpha$  and time  $t$  for fixed work productivity  $1/\tilde{\kappa} = 1000$ , time preferences  $\rho = 0.10$  and  $\tilde{\rho} = -0.10$ ,  $K = 1$ , base-level work effort  $\lambda_0 = 0.10$  and time horizon  $T = 10$  years.

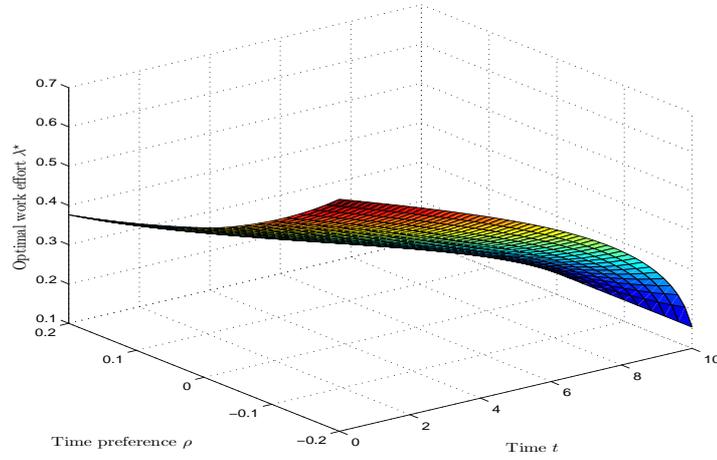


Figure 3: Optimal work effort  $\lambda^*$  w.r.t. the time preference of consumption  $\rho$  and time  $t$  for fixed work productivity  $1/\tilde{\kappa} = 1000$ ,  $\alpha = 5$ , time preference  $\tilde{\rho} = -0.10$ ,  $K = 1$ , base-level work effort  $\lambda_0 = 0.10$  and time horizon  $T = 10$  years.

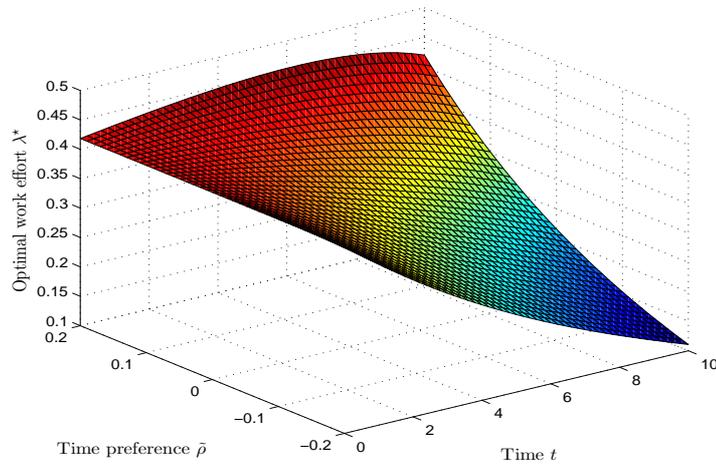


Figure 4: Optimal work effort  $\lambda^*$  w.r.t. the time preference of disutility  $\tilde{\rho}$  and time  $t$  for fixed work productivity  $1/\tilde{\kappa} = 1000$ , disutility stress  $\alpha = 5$ , time preference  $\rho = 0.10$ ,  $K = 1$ , base-level work effort  $\lambda_0 = 0.10$  and time horizon  $T = 10$  years.

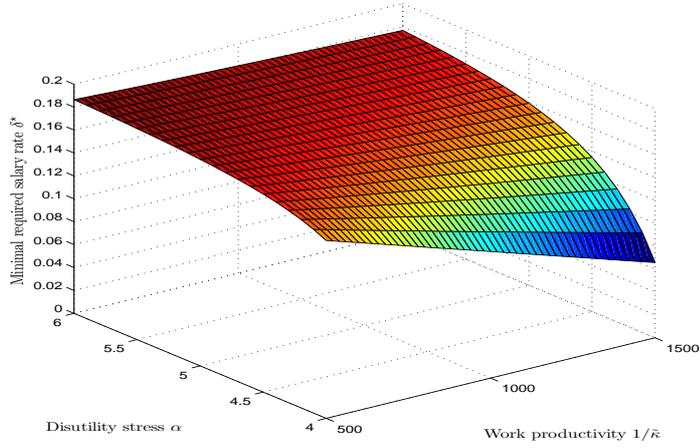


Figure 5: Minimal required salary rate  $\delta^*$  w.r.t. disutility stress  $\alpha$  and work productivity  $1/\tilde{\kappa}$  for fixed time preferences  $\rho = 0.10$  and  $\tilde{\rho} = -0.10$ ,  $K = 1$ , base-level work effort  $\lambda_0 = 0.10$ , outside salary rate  $\hat{\delta} = 0.2$ , and time horizon  $T = 10$  years.

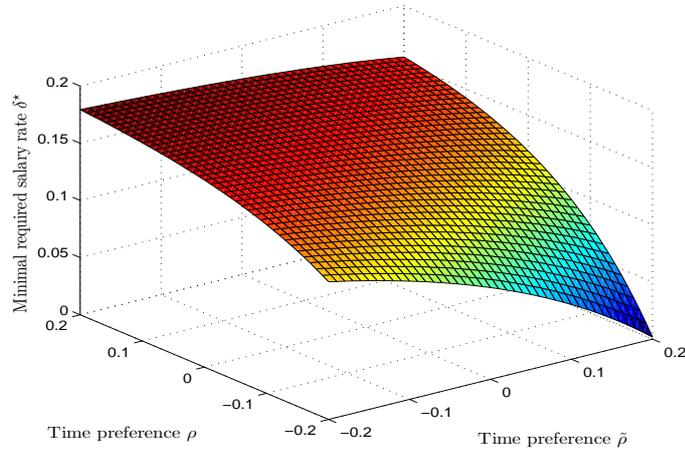


Figure 6: Minimal required salary rate  $\delta^*$  w.r.t. the time preferences  $\rho$  and  $\tilde{\rho}$  for fixed work productivity  $1/\tilde{\kappa} = 1000$ , disutility stress  $\alpha = 5$ ,  $K = 1$ , base-level work effort  $\lambda_0 = 0.10$  and time horizon  $T = 10$  years.

# Published reports of the Fraunhofer ITWM

The PDF-files of the following reports are available under:

[www.itwm.fraunhofer.de/de/zentral\\_\\_berichte/berichte](http://www.itwm.fraunhofer.de/de/zentral__berichte/berichte)

1. D. Hietel, K. Steiner, J. Struckmeier  
**A Finite - Volume Particle Method for Compressible Flows**  
(19 pages, 1998)
2. M. Feldmann, S. Seibold  
**Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing**  
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics  
(23 pages, 1998)
3. Y. Ben-Haim, S. Seibold  
**Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery**  
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis  
(24 pages, 1998)
4. F.-Th. Lentens, N. Siedow  
**Three-dimensional Radiative Heat Transfer in Glass Cooling Processes**  
(23 pages, 1998)
5. A. Klar, R. Wegener  
**A hierarchy of models for multilane vehicular traffic**  
**Part I: Modeling**  
(23 pages, 1998)  
**Part II: Numerical and stochastic investigations**  
(17 pages, 1998)
6. A. Klar, N. Siedow  
**Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes**  
(24 pages, 1998)
7. I. Choquet  
**Heterogeneous catalysis modelling and numerical simulation in rarified gas flows**  
**Part I: Coverage locally at equilibrium**  
(24 pages, 1998)
8. J. Ohser, B. Steinbach, C. Lang  
**Efficient Texture Analysis of Binary Images**  
(17 pages, 1998)
9. J. Orlik  
**Homogenization for viscoelasticity of the integral type with aging and shrinkage**  
(20 pages, 1998)
10. J. Mohring  
**Helmholtz Resonators with Large Aperture**  
(21 pages, 1998)
11. H. W. Hamacher, A. Schöbel  
**On Center Cycles in Grid Graphs**  
(15 pages, 1998)
12. H. W. Hamacher, K.-H. Küfer  
**Inverse radiation therapy planning - a multiple objective optimisation approach**  
(14 pages, 1999)
13. C. Lang, J. Ohser, R. Hilfer  
**On the Analysis of Spatial Binary Images**  
(20 pages, 1999)
14. M. Junk  
**On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes**  
(24 pages, 1999)
15. M. Junk, S. V. Raghurame Rao  
**A new discrete velocity method for Navier-Stokes equations**  
(20 pages, 1999)
16. H. Neunzert  
**Mathematics as a Key to Key Technologies**  
(39 pages (4 PDF-Files), 1999)
17. J. Ohser, K. Sandau  
**Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem**  
(18 pages, 1999)
18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel  
**Solving nonconvex planar location problems by finite dominating sets**  
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm  
(19 pages, 2000)
19. A. Becker  
**A Review on Image Distortion Measures**  
Keywords: Distortion measure, human visual system  
(26 pages, 2000)
20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn  
**Polyhedral Properties of the Uncapacitated Multiple Allocation Hub Location Problem**  
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut  
(21 pages, 2000)
21. H. W. Hamacher, A. Schöbel  
**Design of Zone Tariff Systems in Public Transportation**  
(30 pages, 2001)
22. D. Hietel, M. Junk, R. Keck, D. Teleaga  
**The Finite-Volume-Particle Method for Conservation Laws**  
(16 pages, 2001)
23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel  
**Location Software and Interface with GIS and Supply Chain Management**  
Keywords: facility location, software development, geographical information systems, supply chain management  
(48 pages, 2001)
24. H. W. Hamacher, S. A. Tjandra  
**Mathematical Modelling of Evacuation Problems: A State of Art**  
(44 pages, 2001)
25. J. Kuhnert, S. Tiwari  
**Grid free method for solving the Poisson equation**  
Keywords: Poisson equation, Least squares method, Grid free method  
(19 pages, 2001)
26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier  
**Simulation of the fiber spinning process**  
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD  
(19 pages, 2001)
27. A. Zemitis  
**On interaction of a liquid film with an obstacle**  
Keywords: impinging jets, liquid film, models, numerical solution, shape  
(22 pages, 2001)
28. I. Ginzburg, K. Steiner  
**Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids**  
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models  
(22 pages, 2001)
29. H. Neunzert  
**»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«**  
**Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001**  
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrrskalalanalyse, Strömungsmechanik  
(18 pages, 2001)
30. J. Kuhnert, S. Tiwari  
**Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations**  
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation  
AMS subject classification: 76D05, 76M28  
(25 pages, 2001)
31. R. Korn, M. Krekel  
**Optimal Portfolios with Fixed Consumption or Income Streams**  
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems  
(23 pages, 2002)
32. M. Krekel  
**Optimal portfolios with a loan dependent credit spread**  
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics  
(25 pages, 2002)
33. J. Ohser, W. Nagel, K. Schladitz  
**The Euler number of discretized sets – on the choice of adjacency in homogeneous lattices**  
Keywords: image analysis, Euler number, neighborhood relationships, cuboidal lattice  
(32 pages, 2002)

34. I. Ginzburg, K. Steiner  
**Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting**  
Keywords: Lattice Boltzmann models; free-surface phenomena; interface boundary conditions; filling processes; injection molding; volume of fluid method; interface boundary conditions; advection-schemes; up-wind-schemes (54 pages, 2002)
35. M. Günther, A. Klar, T. Materne, R. Wegener  
**Multivalued fundamental diagrams and stop and go waves for continuum traffic equations**  
Keywords: traffic flow, macroscopic equations, kinetic derivation, multivalued fundamental diagram, stop and go waves, phase transitions (25 pages, 2002)
36. S. Feldmann, P. Lang, D. Prätzel-Wolters  
**Parameter influence on the zeros of network determinants**  
Keywords: Networks, Equicofactor matrix polynomials, Realization theory, Matrix perturbation theory (30 pages, 2002)
37. K. Koch, J. Ohser, K. Schladitz  
**Spectral theory for random closed sets and estimating the covariance via frequency space**  
Keywords: Random set, Bartlett spectrum, fast Fourier transform, power spectrum (28 pages, 2002)
38. D. d'Humières, I. Ginzburg  
**Multi-reflection boundary conditions for lattice Boltzmann models**  
Keywords: lattice Boltzmann equation, boundary conditions, bounce-back rule, Navier-Stokes equation (72 pages, 2002)
39. R. Korn  
**Elementare Finanzmathematik**  
Keywords: Finanzmathematik, Aktien, Optionen, Portfolio-Optimierung, Börse, Lehrerweiterbildung, Mathematikunterricht (98 pages, 2002)
40. J. Kallrath, M. C. Müller, S. Nickel  
**Batch Presorting Problems: Models and Complexity Results**  
Keywords: Complexity theory, Integer programming, Assignment, Logistics (19 pages, 2002)
41. J. Linn  
**On the frame-invariant description of the phase space of the Folgar-Tucker equation**  
Key words: fiber orientation, Folgar-Tucker equation, injection molding (5 pages, 2003)
42. T. Hanne, S. Nickel  
**A Multi-Objective Evolutionary Algorithm for Scheduling and Inspection Planning in Software Development Projects**  
Key words: multiple objective programming, project management and scheduling, software development, evolutionary algorithms, efficient set (29 pages, 2003)
43. T. Bortfeld, K.-H. Küfer, M. Monz, A. Scherrer, C. Thieke, H. Trinkaus  
**Intensity-Modulated Radiotherapy - A Large Scale Multi-Criteria Programming Problem**  
Keywords: multiple criteria optimization, representative systems of Pareto solutions, adaptive triangulation, clustering and disaggregation techniques, visualization of Pareto solutions, medical physics, external beam radiotherapy planning, intensity modulated radiotherapy (31 pages, 2003)
44. T. Halfmann, T. Wichmann  
**Overview of Symbolic Methods in Industrial Analog Circuit Design**  
Keywords: CAD, automated analog circuit design, symbolic analysis, computer algebra, behavioral modeling, system simulation, circuit sizing, macro modeling, differential-algebraic equations, index (17 pages, 2003)
45. S. E. Mikhailov, J. Orlik  
**Asymptotic Homogenisation in Strength and Fatigue Durability Analysis of Composites**  
Keywords: multiscale structures, asymptotic homogenization, strength, fatigue, singularity, non-local conditions (14 pages, 2003)
46. P. Domínguez-Marín, P. Hansen, N. Mladenović, S. Nickel  
**Heuristic Procedures for Solving the Discrete Ordered Median Problem**  
Keywords: genetic algorithms, variable neighborhood search, discrete facility location (31 pages, 2003)
47. N. Boland, P. Domínguez-Marín, S. Nickel, J. Puerto  
**Exact Procedures for Solving the Discrete Ordered Median Problem**  
Keywords: discrete location, Integer programming (41 pages, 2003)
48. S. Feldmann, P. Lang  
**Padé-like reduction of stable discrete linear systems preserving their stability**  
Keywords: Discrete linear systems, model reduction, stability, Hankel matrix, Stein equation (16 pages, 2003)
49. J. Kallrath, S. Nickel  
**A Polynomial Case of the Batch Presorting Problem**  
Keywords: batch presorting problem, online optimization, competitive analysis, polynomial algorithms, logistics (17 pages, 2003)
50. T. Hanne, H. L. Trinkaus  
**knowCube for MCDM – Visual and Interactive Support for Multicriteria Decision Making**  
Key words: Multicriteria decision making, knowledge management, decision support systems, visual interfaces, interactive navigation, real-life applications. (26 pages, 2003)
51. O. Iliev, V. Laptev  
**On Numerical Simulation of Flow Through Oil Filters**  
Keywords: oil filters, coupled flow in plain and porous media, Navier-Stokes, Brinkman, numerical simulation (8 pages, 2003)
52. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva  
**On a Multigrid Adaptive Refinement Solver for Saturated Non-Newtonian Flow in Porous Media**  
Keywords: Nonlinear multigrid, adaptive refinement, non-Newtonian flow in porous media (17 pages, 2003)
53. S. Kruse  
**On the Pricing of Forward Starting Options under Stochastic Volatility**  
Keywords: Option pricing, forward starting options, Heston model, stochastic volatility, cliquet options (11 pages, 2003)
54. O. Iliev, D. Stoyanov  
**Multigrid – adaptive local refinement solver for incompressible flows**  
Keywords: Navier-Stokes equations, incompressible flow, projection-type splitting, SIMPLE, multigrid methods, adaptive local refinement, lid-driven flow in a cavity (37 pages, 2003)
55. V. Starikovicus  
**The multiphase flow and heat transfer in porous media**  
Keywords: Two-phase flow in porous media, various formulations, global pressure, multiphase mixture model, numerical simulation (30 pages, 2003)
56. P. Lang, A. Sarishvili, A. Wirsén  
**Blocked neural networks for knowledge extraction in the software development process**  
Keywords: Blocked Neural Networks, Nonlinear Regression, Knowledge Extraction, Code Inspection (21 pages, 2003)
57. H. Knaf, P. Lang, S. Zeiser  
**Diagnosis aiding in Regulation Thermography using Fuzzy Logic**  
Keywords: fuzzy logic, knowledge representation, expert system (22 pages, 2003)
58. M. T. Melo, S. Nickel, F. Saldanha da Gama  
**Largescale models for dynamic multi-commodity capacitated facility location**  
Keywords: supply chain management, strategic planning, dynamic location, modeling (40 pages, 2003)
59. J. Orlik  
**Homogenization for contact problems with periodically rough surfaces**  
Keywords: asymptotic homogenization, contact problems (28 pages, 2004)
60. A. Scherrer, K.-H. Küfer, M. Monz, F. Alonso, T. Bortfeld  
**IMRT planning on adaptive volume structures – a significant advance of computational complexity**  
Keywords: Intensity-modulated radiation therapy (IMRT), inverse treatment planning, adaptive volume structures, hierarchical clustering, local refinement, adaptive clustering, convex programming, mesh generation, multi-grid methods (24 pages, 2004)
61. D. Kehrwald  
**Parallel lattice Boltzmann simulation of complex flows**  
Keywords: Lattice Boltzmann methods, parallel computing, microstructure simulation, virtual material design, pseudo-plastic fluids, liquid composite moulding (12 pages, 2004)
62. O. Iliev, J. Linn, M. Moog, D. Niedziela, V. Starikovicus  
**On the Performance of Certain Iterative Solvers for Coupled Systems Arising in Discretization of Non-Newtonian Flow Equations**

Keywords: Performance of iterative solvers, Preconditioners, Non-Newtonian flow (17 pages, 2004)

63. R. Ciegis, O. Iliev, S. Rief, K. Steiner  
**On Modelling and Simulation of Different Regimes for Liquid Polymer Moulding**  
Keywords: Liquid Polymer Moulding, Modelling, Simulation, Infiltration, Front Propagation, non-Newtonian flow in porous media (43 pages, 2004)

64. T. Hanne, H. Neu  
**Simulating Human Resources in Software Development Processes**  
Keywords: Human resource modeling, software process, productivity, human factors, learning curve (14 pages, 2004)

65. O. Iliev, A. Mikelic, P. Popov  
**Fluid structure interaction problems in deformable porous media: Toward permeability of deformable porous media**  
Keywords: fluid-structure interaction, deformable porous media, upscaling, linear elasticity, stokes, finite elements (28 pages, 2004)

66. F. Gaspar, O. Iliev, F. Lisbona, A. Naumovich, P. Vabishchevich  
**On numerical solution of 1-D poroelasticity equations in a multilayered domain**  
Keywords: poroelasticity, multilayered material, finite volume discretization, MAC type grid (41 pages, 2004)

67. J. Ohser, K. Schladitz, K. Koch, M. Nöthe  
**Diffraction by image processing and its application in materials science**  
Keywords: porous microstructure, image analysis, random set, fast Fourier transform, power spectrum, Bartlett spectrum (13 pages, 2004)

68. H. Neunzert  
**Mathematics as a Technology: Challenges for the next 10 Years**  
Keywords: applied mathematics, technology, modelling, simulation, visualization, optimization, glass processing, spinning processes, fiber-fluid interaction, turbulence effects, topological optimization, multicriteria optimization, Uncertainty and Risk, financial mathematics, Malliavin calculus, Monte-Carlo methods, virtual material design, filtration, bio-informatics, system biology (29 pages, 2004)

69. R. Ewing, O. Iliev, R. Lazarov, A. Naumovich  
**On convergence of certain finite difference discretizations for 1D poroelasticity interface problems**  
Keywords: poroelasticity, multilayered material, finite volume discretizations, MAC type grid, error estimates (26 pages, 2004)

70. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva  
**On Efficient Simulation of Non-Newtonian Flow in Saturated Porous Media with a Multigrid Adaptive Refinement Solver**  
Keywords: Nonlinear multigrid, adaptive renement, non-Newtonian in porous media (25 pages, 2004)

71. J. Kalcsics, S. Nickel, M. Schröder  
**Towards a Unified Territory Design Approach – Applications, Algorithms and GIS Integration**  
Keywords: territory design, political districting, sales territory alignment, optimization algorithms, Geographical Information Systems (40 pages, 2005)

72. K. Schladitz, S. Peters, D. Reinel-Bitzer, A. Wiegmann, J. Ohser  
**Design of acoustic trim based on geometric modeling and flow simulation for non-woven**  
Keywords: random system of fibers, Poisson line process, flow resistivity, acoustic absorption, Lattice-Boltzmann method, non-woven (21 pages, 2005)

73. V. Rutka, A. Wiegmann  
**Explicit Jump Immersed Interface Method for virtual material design of the effective elastic moduli of composite materials**  
Keywords: virtual material design, explicit jump immersed interface method, effective elastic moduli, composite materials (22 pages, 2005)

74. T. Hanne  
**Eine Übersicht zum Scheduling von Baustellen**  
Keywords: Projektplanung, Scheduling, Bauplanung, Bauindustrie (32 pages, 2005)

75. J. Linn  
**The Folgar-Tucker Model as a Differential Algebraic System for Fiber Orientation Calculation**  
Keywords: fiber orientation, Folgar-Tucker model, invariants, algebraic constraints, phase space, trace stability (15 pages, 2005)

76. M. Speckert, K. Dreßler, H. Mauch, A. Lion, G. J. Wierda  
**Simulation eines neuartigen Prüfsystems für Achserprobungen durch MKS-Modellierung einschließlich Regelung**  
Keywords: virtual test rig, suspension testing, multibody simulation, modeling hexapod test rig, optimization of test rig configuration (20 pages, 2005)

77. K.-H. Küfer, M. Monz, A. Scherrer, P. Süß, F. Alonso, A. S. A. Sultan, Th. Bortfeld, D. Craft, Chr. Thieke  
**Multicriteria optimization in intensity modulated radiotherapy planning**  
Keywords: multicriteria optimization, extreme solutions, real-time decision making, adaptive approximation schemes, clustering methods, IMRT planning, reverse engineering (51 pages, 2005)

78. S. Amstutz, H. Andrä  
**A new algorithm for topology optimization using a level-set method**  
Keywords: shape optimization, topology optimization, topological sensitivity, level-set (22 pages, 2005)

79. N. Ettrich  
**Generation of surface elevation models for urban drainage simulation**  
Keywords: Flooding, simulation, urban elevation models, laser scanning (22 pages, 2005)

80. H. Andrä, J. Linn, I. Matei, I. Shklyar, K. Steiner, E. Teichmann  
**OPTCAST – Entwicklung adäquater Strukturoptimierungsverfahren für Gießereien Technischer Bericht (KURZFASSUNG)**  
Keywords: Topologieoptimierung, Level-Set-Methode, Gießprozesssimulation, Gießtechnische Restriktionen, CAE-Kette zur Strukturoptimierung (77 pages, 2005)

81. N. Marheineke, R. Wegener  
**Fiber Dynamics in Turbulent Flows Part I: General Modeling Framework**  
Keywords: fiber-fluid interaction; Cosserat rod; turbulence modeling; Kolmogorov's energy spectrum; double-velocity correlations; differentiable Gaussian fields (20 pages, 2005)

**Part II: Specific Taylor Drag**  
Keywords: flexible fibers;  $k-\epsilon$  turbulence model; fiber-turbulence interaction scales; air drag; random Gaussian aerodynamic force; white noise; stochastic differential equations; ARMA process (18 pages, 2005)

82. C. H. Lampert, O. Wirjadi  
**An Optimal Non-Orthogonal Separation of the Anisotropic Gaussian Convolution Filter**  
Keywords: Anisotropic Gaussian filter, linear filtering, orientation space, nD image processing, separable filters (25 pages, 2005)

83. H. Andrä, D. Stoyanov  
**Error indicators in the parallel finite element solver for linear elasticity DDFEM**  
Keywords: linear elasticity, finite element method, hierarchical shape functions, domain decomposition, parallel implementation, a posteriori error estimates (21 pages, 2006)

84. M. Schröder, I. Solchenbach  
**Optimization of Transfer Quality in Regional Public Transit**  
Keywords: public transit, transfer quality, quadratic assignment problem (16 pages, 2006)

85. A. Naumovich, F. J. Gaspar  
**On a multigrid solver for the three-dimensional Biot poroelasticity system in multilayered domains**  
Keywords: poroelasticity, interface problem, multigrid, operator-dependent prolongation (11 pages, 2006)

86. S. Panda, R. Wegener, N. Marheineke  
**Slender Body Theory for the Dynamics of Curved Viscous Fibers**  
Keywords: curved viscous fibers; fluid dynamics; Navier-Stokes equations; free boundary value problem; asymptotic expansions; slender body theory (14 pages, 2006)

87. E. Ivanov, H. Andrä, A. Kudryavtsev  
**Domain Decomposition Approach for Automatic Parallel Generation of Tetrahedral Grids**  
Key words: Grid Generation, Unstructured Grid, Delaunay Triangulation, Parallel Programming, Domain Decomposition, Load Balancing (18 pages, 2006)

88. S. Tiwari, S. Antonov, D. Hietel, J. Kuhnert, R. Wegener  
**A Meshfree Method for Simulations of Interactions between Fluids and Flexible Structures**  
Key words: Meshfree Method, FPM, Fluid Structure Interaction, Sheet of Paper, Dynamical Coupling (16 pages, 2006)

89. R. Ciegis, O. Iliev, V. Starikovicius, K. Steiner  
**Numerical Algorithms for Solving Problems of Multiphase Flows in Porous Media**  
Keywords: nonlinear algorithms, finite-volume method, software tools, porous media, flows (16 pages, 2006)

90. D. Niedziela, O. Iliev, A. Latz  
**On 3D Numerical Simulations of Viscoelastic Fluids**  
Keywords: non-Newtonian fluids, anisotropic viscosity, integral constitutive equation  
(18 pages, 2006)
91. A. Winterfeld  
**Application of general semi-infinite Programming to Lapidary Cutting Problems**  
Keywords: large scale optimization, nonlinear programming, general semi-infinite optimization, design centering, clustering  
(26 pages, 2006)
92. J. Orlik, A. Ostrovska  
**Space-Time Finite Element Approximation and Numerical Solution of Hereditary Linear Viscoelasticity Problems**  
Keywords: hereditary viscoelasticity; kern approximation by interpolation; space-time finite element approximation, stability and a priori estimate  
(24 pages, 2006)
93. V. Rutka, A. Wiegmann, H. Andrä  
**EJIM for Calculation of effective Elastic Moduli in 3D Linear Elasticity**  
Keywords: Elliptic PDE, linear elasticity, irregular domain, finite differences, fast solvers, effective elastic moduli  
(24 pages, 2006)
94. A. Wiegmann, A. Zemitis  
**EJ-HEAT: A Fast Explicit Jump Harmonic Averaging Solver for the Effective Heat Conductivity of Composite Materials**  
Keywords: Stationary heat equation, effective thermal conductivity, explicit jump, discontinuous coefficients, virtual material design, microstructure simulation, EJ-HEAT  
(21 pages, 2006)
95. A. Naumovich  
**On a finite volume discretization of the three-dimensional Biot poroelasticity system in multilayered domains**  
Keywords: Biot poroelasticity system, interface problems, finite volume discretization, finite difference method  
(21 pages, 2006)
96. M. Krekel, J. Wenzel  
**A unified approach to Credit Default Swap-tion and Constant Maturity Credit Default Swap valuation**  
Keywords: LIBOR market model, credit risk, Credit Default Swap-tion, Constant Maturity Credit Default Swap-method  
(43 pages, 2006)
97. A. Dreyer  
**Interval Methods for Analog Circuits**  
Keywords: interval arithmetic, analog circuits, tolerance analysis, parametric linear systems, frequency response, symbolic analysis, CAD, computer algebra  
(36 pages, 2006)
98. N. Weigel, S. Weihe, G. Bitsch, K. Dreßler  
**Usage of Simulation for Design and Optimization of Testing**  
Keywords: Vehicle test rigs, MBS, control, hydraulics, testing philosophy  
(14 pages, 2006)
99. H. Lang, G. Bitsch, K. Dreßler, M. Speckert  
**Comparison of the solutions of the elastic and elastoplastic boundary value problems**  
Keywords: Elastic BVP, elastoplastic BVP, variational inequalities, rate-independency, hysteresis, linear kinematic hardening, stop- and play-operator  
(21 pages, 2006)
100. M. Speckert, K. Dreßler, H. Mauch  
**MBS Simulation of a hexapod based suspension test rig**  
Keywords: Test rig, MBS simulation, suspension, hydraulics, controlling, design optimization  
(12 pages, 2006)
101. S. Azizi Sultan, K.-H. Küfer  
**A dynamic algorithm for beam orientations in multicriteria IMRT planning**  
Keywords: radiotherapy planning, beam orientation optimization, dynamic approach, evolutionary algorithm, global optimization  
(14 pages, 2006)
102. T. Götz, A. Klar, N. Marheineke, R. Wegener  
**A Stochastic Model for the Fiber Lay-down Process in the Nonwoven Production**  
Keywords: fiber dynamics, stochastic Hamiltonian system, stochastic averaging  
(17 pages, 2006)
103. Ph. Süß, K.-H. Küfer  
**Balancing control and simplicity: a variable aggregation method in intensity modulated radiation therapy planning**  
Keywords: IMRT planning, variable aggregation, clustering methods  
(22 pages, 2006)
104. A. Beaudry, G. Laporte, T. Melo, S. Nickel  
**Dynamic transportation of patients in hospitals**  
Keywords: in-house hospital transportation, dial-a-ride, dynamic mode, tabu search  
(37 pages, 2006)
105. Th. Hanne  
**Applying multiobjective evolutionary algorithms in industrial projects**  
Keywords: multiobjective evolutionary algorithms, discrete optimization, continuous optimization, electronic circuit design, semi-infinite programming, scheduling  
(18 pages, 2006)
106. J. Franke, S. Halim  
**Wild bootstrap tests for comparing signals and images**  
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression  
(13 pages, 2007)
107. Z. Drezner, S. Nickel  
**Solving the ordered one-median problem in the plane**  
Keywords: planar location, global optimization, ordered median, big triangle small triangle method, bounds, numerical experiments  
(21 pages, 2007)
108. Th. Götz, A. Klar, A. Unterreiter, R. Wegener  
**Numerical evidence for the non-existing of solutions of the equations describing rotational fiber spinning**  
Keywords: rotational fiber spinning, viscous fibers, boundary value problem, existence of solutions  
(11 pages, 2007)
109. Ph. Süß, K.-H. Küfer  
**Smooth intensity maps and the Bortfeld-Boyer sequencer**  
Keywords: probabilistic analysis, intensity modulated radiotherapy treatment (IMRT), IMRT plan application, step-and-shoot sequencing  
(8 pages, 2007)
110. E. Ivanov, O. Gluchshenko, H. Andrä, A. Kudryavtsev  
**Parallel software tool for decomposing and meshing of 3d structures**  
Keywords: a-priori domain decomposition, unstructured grid, Delaunay mesh generation  
(14 pages, 2007)
111. O. Iliev, R. Lazarov, J. Willems  
**Numerical study of two-grid preconditioners for 1d elliptic problems with highly oscillating discontinuous coefficients**  
Keywords: two-grid algorithm, oscillating coefficients, preconditioner  
(20 pages, 2007)
112. L. Bonilla, T. Götz, A. Klar, N. Marheineke, R. Wegener  
**Hydrodynamic limit of the Fokker-Planck equation describing fiber lay-down processes**  
Keywords: stochastic differential equations, Fokker-Planck equation, asymptotic expansion, Ornstein-Uhlenbeck process  
(17 pages, 2007)
113. S. Rief  
**Modeling and simulation of the pressing section of a paper machine**  
Keywords: paper machine, computational fluid dynamics, porous media  
(41 pages, 2007)
114. R. Ciegis, O. Iliev, Z. Lakdawala  
**On parallel numerical algorithms for simulating industrial filtration problems**  
Keywords: Navier-Stokes-Brinkmann equations, finite volume discretization method, SIMPLE, parallel computing, data decomposition method  
(24 pages, 2007)
115. N. Marheineke, R. Wegener  
**Dynamics of curved viscous fibers with surface tension**  
Keywords: Slender body theory, curved viscous fibers with surface tension, free boundary value problem  
(25 pages, 2007)
116. S. Feth, J. Franke, M. Speckert  
**Resampling-Methoden zur mse-Korrektur und Anwendungen in der Betriebsfestigkeit**  
Keywords: Weibull, Bootstrap, Maximum-Likelihood, Betriebsfestigkeit  
(16 pages, 2007)
117. H. Knaf  
**Kernel Fisher discriminant functions – a concise and rigorous introduction**  
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression  
(30 pages, 2007)
118. O. Iliev, I. Rybak  
**On numerical upscaling for flows in heterogeneous porous media**

- Keywords: numerical upscaling, heterogeneous porous media, single phase flow, Darcy's law, multiscale problem, effective permeability, multipoint flux approximation, anisotropy (17 pages, 2007)
119. O. Iliev, I. Rybak  
**On approximation property of multipoint flux approximation method**  
Keywords: Multipoint flux approximation, finite volume method, elliptic equation, discontinuous tensor coefficients, anisotropy (15 pages, 2007)
120. O. Iliev, I. Rybak, J. Willems  
**On upscaling heat conductivity for a class of industrial problems**  
Keywords: Multiscale problems, effective heat conductivity, numerical upscaling, domain decomposition (21 pages, 2007)
121. R. Ewing, O. Iliev, R. Lazarov, I. Rybak  
**On two-level preconditioners for flow in porous media**  
Keywords: Multiscale problem, Darcy's law, single phase flow, anisotropic heterogeneous porous media, numerical upscaling, multigrid, domain decomposition, efficient preconditioner (18 pages, 2007)
122. M. Brickenstein, A. Dreyer  
**POLYBORI: A Gröbner basis framework for Boolean polynomials**  
Keywords: Gröbner basis, formal verification, Boolean polynomials, algebraic cryptanalysis, satisfiability (23 pages, 2007)
123. O. Wirjadi  
**Survey of 3d image segmentation methods**  
Keywords: image processing, 3d, image segmentation, binarization (20 pages, 2007)
124. S. Zeytun, A. Gupta  
**A Comparative Study of the Vasicek and the CIR Model of the Short Rate**  
Keywords: interest rates, Vasicek model, CIR-model, calibration, parameter estimation (17 pages, 2007)
125. G. Hanselmann, A. Sarishvili  
**Heterogeneous redundancy in software quality prediction using a hybrid Bayesian approach**  
Keywords: reliability prediction, fault prediction, non-homogeneous poisson process, Bayesian model averaging (17 pages, 2007)
126. V. Maag, M. Berger, A. Winterfeld, K.-H. Küfer  
**A novel non-linear approach to minimal area rectangular packing**  
Keywords: rectangular packing, non-overlapping constraints, non-linear optimization, regularization, relaxation (18 pages, 2007)
127. M. Monz, K.-H. Küfer, T. Bortfeld, C. Thieke  
**Pareto navigation – systematic multi-criteria-based IMRT treatment plan determination**  
Keywords: convex, interactive multi-objective optimization, intensity modulated radiotherapy planning (15 pages, 2007)
128. M. Krause, A. Scherrer  
**On the role of modeling parameters in IMRT plan optimization**  
Keywords: intensity-modulated radiotherapy (IMRT), inverse IMRT planning, convex optimization, sensitivity analysis, elasticity, modeling parameters, equivalent uniform dose (EUD) (18 pages, 2007)
129. A. Wiegmann  
**Computation of the permeability of porous materials from their microstructure by FFF-Stokes**  
Keywords: permeability, numerical homogenization, fast Stokes solver (24 pages, 2007)
130. T. Melo, S. Nickel, F. Saldanha da Gama  
**Facility Location and Supply Chain Management – A comprehensive review**  
Keywords: facility location, supply chain management, network design (54 pages, 2007)
131. T. Hanne, T. Melo, S. Nickel  
**Bringing robustness to patient flow management through optimized patient transports in hospitals**  
Keywords: Dial-a-Ride problem, online problem, case study, tabu search, hospital logistics (23 pages, 2007)
132. R. Ewing, O. Iliev, R. Lazarov, I. Rybak, J. Willems  
**An efficient approach for upscaling properties of composite materials with high contrast of coefficients**  
Keywords: effective heat conductivity, permeability of fractured porous media, numerical upscaling, fibrous insulation materials, metal foams (16 pages, 2008)
133. S. Gelareh, S. Nickel  
**New approaches to hub location problems in public transport planning**  
Keywords: integer programming, hub location, transportation, decomposition, heuristic (25 pages, 2008)
134. G. Thömmes, J. Becker, M. Junk, A. K. Vainkuntam, D. Kehrwald, A. Klar, K. Steiner, A. Wiegmann  
**A Lattice Boltzmann Method for immiscible multiphase flow simulations using the Level Set Method**  
Keywords: Lattice Boltzmann method, Level Set method, free surface, multiphase flow (28 pages, 2008)
135. J. Orlik  
**Homogenization in elasto-plasticity**  
Keywords: multiscale structures, asymptotic homogenization, nonlinear energy (40 pages, 2008)
136. J. Almqvist, H. Schmidt, P. Lang, J. Deitmer, M. Jirstrand, D. Prätzel-Wolters, H. Becker  
**Determination of interaction between MCT1 and CAII via a mathematical and physiological approach**  
Keywords: mathematical modeling; model reduction; electrophysiology; pH-sensitive microelectrodes; proton antenna (20 pages, 2008)
137. E. Savenkov, H. Andrä, O. Iliev  
**An analysis of one regularization approach for solution of pure Neumann problem**  
Keywords: pure Neumann problem, elasticity, regularization, finite element method, condition number (27 pages, 2008)
138. O. Berman, J. Kalcsics, D. Krass, S. Nickel  
**The ordered gradual covering location problem on a network**  
Keywords: gradual covering, ordered median function, network location (32 pages, 2008)
139. S. Gelareh, S. Nickel  
**Multi-period public transport design: A novel model and solution approaches**  
Keywords: Integer programming, hub location, public transport, multi-period planning, heuristics (31 pages, 2008)
140. T. Melo, S. Nickel, F. Saldanha-da-Gama  
**Network design decisions in supply chain planning**  
Keywords: supply chain design, integer programming models, location models, heuristics (20 pages, 2008)
141. C. Lautensack, A. Särkkä, J. Freitag, K. Schladitz  
**Anisotropy analysis of pressed point processes**  
Keywords: estimation of compression, isotropy test, nearest neighbour distance, orientation analysis, polar ice, Ripley's K function (35 pages, 2008)
142. O. Iliev, R. Lazarov, J. Willems  
**A Graph-Laplacian approach for calculating the effective thermal conductivity of complicated fiber geometries**  
Keywords: graph laplacian, effective heat conductivity, numerical upscaling, fibrous materials (14 pages, 2008)
143. J. Linn, T. Stephan, J. Carlsson, R. Bohlin  
**Fast simulation of quasistatic rod deformations for VR applications**  
Keywords: quasistatic deformations, geometrically exact rod models, variational formulation, energy minimization, finite differences, nonlinear conjugate gradients (7 pages, 2008)
144. J. Linn, T. Stephan  
**Simulation of quasistatic deformations using discrete rod models**  
Keywords: quasistatic deformations, geometrically exact rod models, variational formulation, energy minimization, finite differences, nonlinear conjugate gradients (9 pages, 2008)
145. J. Marburger, N. Marheineke, R. Pinnau  
**Adjoint based optimal control using mesh-less discretizations**  
Keywords: Mesh-less methods, particle methods, Eulerian-Lagrangian formulation, optimization strategies, adjoint method, hyperbolic equations (14 pages, 2008)
146. S. Desmettre, J. Gould, A. Szimayer  
**Own-company stockholding and work effort preferences of an unconstrained executive**  
Keywords: optimal portfolio choice, executive compensation (33 pages, 2008)

147. M. Berger, M. Schröder, K.-H. Küfer  
**A constraint programming approach for the two-dimensional rectangular packing problem with orthogonal orientations**  
Keywords: rectangular packing, orthogonal orientations non-overlapping constraints, constraint propagation (13 pages, 2008)
148. K. Schladitz, C. Redenbach, T. Sych, M. Godehardt  
**Microstructural characterisation of open foams using 3d images**  
Keywords: virtual material design, image analysis, open foams (30 pages, 2008)
149. E. Fernández, J. Kalcsics, S. Nickel, R. Ríos-Mercado  
**A novel territory design model arising in the implementation of the WEEE-Directive**  
Keywords: heuristics, optimization, logistics, recycling (28 pages, 2008)
150. H. Lang, J. Linn  
**Lagrangian field theory in space-time for geometrically exact Cosserat rods**  
Keywords: Cosserat rods, geometrically exact rods, small strain, large deformation, deformable bodies, Lagrangian field theory, variational calculus (19 pages, 2009)
151. K. Dreßler, M. Speckert, R. Müller, Ch. Weber  
**Customer loads correlation in truck engineering**  
Keywords: Customer distribution, safety critical components, quantile estimation, Monte-Carlo methods (11 pages, 2009)
152. H. Lang, K. Dreßler  
**An improved multiaxial stress-strain correction model for elastic FE postprocessing**  
Keywords: Jiang's model of elastoplasticity, stress-strain correction, parameter identification, automatic differentiation, least-squares optimization, Coleman-Li algorithm (6 pages, 2009)
153. J. Kalcsics, S. Nickel, M. Schröder  
**A generic geometric approach to territory design and districting**  
Keywords: Territory design, districting, combinatorial optimization, heuristics, computational geometry (32 pages, 2009)
154. Th. Fütterer, A. Klar, R. Wegener  
**An energy conserving numerical scheme for the dynamics of hyperelastic rods**  
Keywords: Cosserat rod, hyperelastic, energy conservation, finite differences (16 pages, 2009)
155. A. Wiegmann, L. Cheng, E. Glatt, O. Iliev, S. Rief  
**Design of pleated filters by computer simulations**  
Keywords: Solid-gas separation, solid-liquid separation, pleated filter, design, simulation (21 pages, 2009)
156. A. Klar, N. Marheineke, R. Wegener  
**Hierarchy of mathematical models for production processes of technical textiles**  
Keywords: Fiber-fluid interaction, slender-body theory, turbulence modeling, model reduction, stochastic differential equations, Fokker-Planck equation, asymptotic expansions, parameter identification (21 pages, 2009)
157. E. Glatt, S. Rief, A. Wiegmann, M. Knefel, E. Wegenke  
**Structure and pressure drop of real and virtual metal wire meshes**  
Keywords: metal wire mesh, structure simulation, model calibration, CFD simulation, pressure loss (7 pages, 2009)
158. S. Kruse, M. Müller  
**Pricing American call options under the assumption of stochastic dividends – An application of the Korn-Rogers model**  
Keywords: option pricing, American options, dividends, dividend discount model, Black-Scholes model (22 pages, 2009)
159. H. Lang, J. Linn, M. Arnold  
**Multibody dynamics simulation of geometrically exact Cosserat rods**  
Keywords: flexible multibody dynamics, large deformations, finite rotations, constrained mechanical systems, structural dynamics (20 pages, 2009)
160. P. Jung, S. Leyendecker, J. Linn, M. Ortiz  
**Discrete Lagrangian mechanics and geometrically exact Cosserat rods**  
Keywords: special Cosserat rods, Lagrangian mechanics, Noether's theorem, discrete mechanics, frame-indifference, holonomic constraints (14 pages, 2009)
161. M. Burger, K. Dreßler, A. Marquardt, M. Speckert  
**Calculating invariant loads for system simulation in vehicle engineering**  
Keywords: iterative learning control, optimal control theory, differential algebraic equations(DAEs) (18 pages, 2009)
162. M. Speckert, N. Ruf, K. Dreßler  
**Undesired drift of multibody models excited by measured accelerations or forces**  
Keywords: multibody simulation, full vehicle model, force-based simulation, drift due to noise (19 pages, 2009)
163. A. Streit, K. Dreßler, M. Speckert, J. Lichter, T. Zenner, P. Bach  
**Anwendung statistischer Methoden zur Erstellung von Nutzungsprofilen für die Auslegung von Mobilbaggern**  
Keywords: Nutzungsvielfalt, Kundenbeanspruchung, Bemessungsgrundlagen (13 pages, 2009)
164. I. Correia, S. Nickel, F. Saldanha-da-Gama  
**Anwendung statistischer Methoden zur Erstellung von Nutzungsprofilen für die Auslegung von Mobilbaggern**  
Keywords: Capacitated Hub Location, MIP formulations (10 pages, 2009)
165. F. Yaneva, T. Grebe, A. Scherrer  
**An alternative view on global radiotherapy optimization problems**  
Keywords: radiotherapy planning, path-connected sub-levelsets, modified gradient projection method, improving and feasible directions (14 pages, 2009)
166. J. I. Serna, M. Monz, K.-H. Küfer, C. Thieke  
**Trade-off bounds and their effect in multi-criteria IMRT planning**  
Keywords: trade-off bounds, multi-criteria optimization, IMRT, Pareto surface (15 pages, 2009)
167. W. Arne, N. Marheineke, A. Meister, R. Wegener  
**Numerical analysis of Cosserat rod and string models for viscous jets in rotational spinning processes**  
Keywords: Rotational spinning process, curved viscous fibers, asymptotic Cosserat models, boundary value problem, existence of numerical solutions (18 pages, 2009)
168. T. Melo, S. Nickel, F. Saldanha-da-Gama  
**An LP-rounding heuristic to solve a multi-period facility relocation problem**  
Keywords: supply chain design, heuristic, linear programming, rounding (37 pages, 2009)
169. I. Correia, S. Nickel, F. Saldanha-da-Gama  
**Single-allocation hub location problems with capacity choices**  
Keywords: hub location, capacity decisions, MILP formulations (27 pages, 2009)
170. S. Acar, K. Natcheva-Acar  
**A guide on the implementation of the Heath-Jarrow-Morton Two-Factor Gaussian Short Rate Model (HJM-G2++)**  
Keywords: short rate model, two factor Gaussian, G2++, option pricing, calibration (30 pages, 2009)
171. A. Szimayer, G. Dimitroff, S. Lorenz  
**A parsimonious multi-asset Heston model: calibration and derivative pricing**  
Keywords: Heston model, multi-asset, option pricing, calibration, correlation (28 pages, 2009)
172. N. Marheineke, R. Wegener  
**Modeling and validation of a stochastic drag for fibers in turbulent flows**  
Keywords: fiber-fluid interactions, long slender fibers, turbulence modelling, aerodynamic drag, dimensional analysis, data interpolation, stochastic partial differential algebraic equation, numerical simulations, experimental validations (19 pages, 2009)
173. S. Nickel, M. Schröder, J. Steeg  
**Planning for home health care services**  
Keywords: home health care, route planning, meta-heuristics, constraint programming (23 pages, 2009)
174. G. Dimitroff, A. Szimayer, A. Wagner  
**Quanto option pricing in the parsimonious Heston model**  
Keywords: Heston model, multi asset, quanto options, option pricing (14 pages, 2009)
174. G. Dimitroff, A. Szimayer, A. Wagner  
**Model reduction of nonlinear problems in structural mechanics**  
Keywords: flexible bodies, FEM, nonlinear model reduction, POD (13 pages, 2009)

176. M. K. Ahmad, S. Didas, J. Iqbal  
**Using the Sharp Operator for edge detection and nonlinear diffusion**  
 Keywords: maximal function, sharp function, image processing, edge detection, nonlinear diffusion  
 (17 pages, 2009)
177. M. Speckert, N. Ruf, K. Dreßler, R. Müller, C. Weber, S. Weihe  
**Ein neuer Ansatz zur Ermittlung von Erprobungslasten für sicherheitsrelevante Bauteile**  
 Keywords: sicherheitsrelevante Bauteile, Kundenbeanspruchung, Festigkeitsverteilung, Ausfallwahrscheinlichkeit, Konfidenz, statistische Unsicherheit, Sicherheitsfaktoren  
 (16 pages, 2009)
178. J. Jegorovs  
**Wave based method: new applicability areas**  
 Keywords: Elliptic boundary value problems, inhomogeneous Helmholtz type differential equations in bounded domains, numerical methods, wave based method, uniform B-splines  
 (10 pages, 2009)
179. H. Lang, M. Arnold  
**Numerical aspects in the dynamic simulation of geometrically exact rods**  
 Keywords: Kirchhoff and Cosserat rods, geometrically exact rods, deformable bodies, multibody dynamics, partial differential algebraic equations, method of lines, time integration  
 (21 pages, 2009)
180. H. Lang  
**Comparison of quaternionic and rotation-free null space formalisms for multibody dynamics**  
 Keywords: Parametrisation of rotations, differential-algebraic equations, multibody dynamics, constrained mechanical systems, Lagrangian mechanics  
 (40 pages, 2010)
181. S. Nickel, F. Saldanha-da-Gama, H.-P. Ziegler  
**Stochastic programming approaches for risk aware supply chain network design problems**  
 Keywords: Supply Chain Management, multi-stage stochastic programming, financial decisions, risk  
 (37 pages, 2010)
182. P. Ruckdeschel, N. Horbenko  
**Robustness properties of estimators in generalized Pareto Models**  
 Keywords: global robustness, local robustness, finite sample breakdown point, generalized Pareto distribution  
 (58 pages, 2010)
183. P. Jung, S. Leyendecker, J. Linn, M. Ortiz  
**A discrete mechanics approach to Cosserat rod theory – Part 1: static equilibria**  
 Keywords: Special Cosserat rods; Lagrangian mechanics; Noether's theorem; discrete mechanics; frame-indifference; holonomic constraints; variational formulation  
 (35 pages, 2010)
184. R. Eymard, G. Printsypar  
**A proof of convergence of a finite volume scheme for modified steady Richards' equation describing transport processes in the pressing section of a paper machine**  
 Keywords: flow in porous media, steady Richards' equation, finite volume methods, convergence of approximate solution  
 (14 pages, 2010)
185. P. Ruckdeschel  
**Optimally Robust Kalman Filtering**  
 Keywords: robustness, Kalman Filter, innovation outlier, additive outlier  
 (42 pages, 2010)
186. S. Repke, N. Marheineke, R. Pinnau  
**On adjoint-based optimization of a free surface Stokes flow**  
 Keywords: film casting process, thin films, free surface Stokes flow, optimal control, Lagrange formalism  
 (13 pages, 2010)
187. O. Iliev, R. Lazarov, J. Willems  
**Variational multiscale Finite Element Method for flows in highly porous media**  
 Keywords: numerical upscaling, flow in heterogeneous porous media, Brinkman equations, Darcy's law, subgrid approximation, discontinuous Galerkin mixed FEM  
 (21 pages, 2010)
188. S. Desmettre, A. Szimayer  
**Work effort, consumption, and portfolio selection: When the occupational choice matters**  
 Keywords: portfolio choice, work effort, consumption, occupational choice  
 (34 pages, 2010)

Status quo: July 2010