Model-based fault diagnosis and fault-tolerant control for a nonlinear electro-hydraulic system

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Abstract

The work presented in this thesis discusses the model-based fault diagnosis and fault-tolerant control with application to a nonlinear electro-hydraulic system. High performance control with guaranteed safety and reliability for electro-hydraulic systems is a challenging task due to the high nonlinearity and system uncertainties. This thesis developed a diagnosis integrated fault-tolerant control (FTC) strategy for the electro-hydraulic system. In fault free case the nominal controller is in operation for achieving the best performance. If the fault occurs, the controller will be automatically reconfigured based on the fault information provided by the diagnosis system. Fault diagnosis and reconfigurable controller are the key parts for the proposed methodology. The system and sensor faults both are studied in the thesis.

Fault diagnosis consists of fault detection and isolation (FDI). A model-base residual generating is realized by calculating the redundant information from the system model and available signal. In this thesis differential-geometric approach is employed, which gives a general formulation of FDI problem and is more compact and transparent among various model-based approaches. The principle of residual construction with differential-geometric method is to find an unobservable distribution. It indicates the existence of a system transformation, with which the unknown system disturbance can be decoupled. With the observability codistribution algorithm the local weak observability of transformed system is ensured. A Fault detection observer for the transformed system can be constructed to generate the residual. This method can not isolated sensor faults. In the thesis the special decision making logic (DML) is designed based on the individual signal analysis of the residuals to isolate the fault.

The reconfigurable controller is designed with the backstepping technique. Backstepping method is a recursive Lyapunov-based approach and can deal with nonlinear systems. Some system variables are considered as “virtual controls” during the design procedure. Then the feedback control laws and the associate Lyapunov function can be constructed by following step-by-step routine. For the electro-hydraulic system adaptive backstepping controller is employed for compensate the impact of the unknown external load in the fault free case. As soon as the fault is identified, the controller can be reconfigured according to the new modeling of faulty system. The system fault is modeled as the uncertainty of system and can be tolerated by parameter adaption. The sensor fault acts to the system via controller. It can be modeled as parameter uncertainty of controller. All parameters coupled with the faulty measurement are replaced by its approximation. After the reconfiguration the pre-specified control performance can be recovered.
FDI integrated FTC based on backstepping technique is implemented successfully on the electro-hydraulic testbed. The on-line robust FDI and controller reconfiguration can be achieved. The tracking performance of the controlled system is guaranteed and the considered faults can be tolerated. But the problem of theoretical robustness analysis for the time delay caused by the fault diagnosis is still open.
Abstract (German)


Der rekonfigurierbare Regler wird mit dem Backstepping-Verfahren entworfen. Das Backstepping-Verfahren ist ein rekursiver Lyapunov-basierter Ansatz, der gut für nicht-lineare Systeme geeignet ist. Das Regelgesetz und die zugehörige Lyapunov-Funktion können durch folgendes schrittweise Verfahren konstruiert werden: Für das elektrohydraulische System wird ein adaptiver Backstepping-Regler angewendet, um den Einfluss der unbekannten äußeren Last im fehlerfreien Fall zu kompensieren. Sobald ein Fehler erkannt wird, wird der Regler rekonfiguriert basierend auf dem Modell des fehlerbe-

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- $v_c$: velocity of cylinder ................................................... 11
- $V$: Lyapunov function ...................................................... 56
- $V_H$: volume ................................................................. 10
- $V_{10}$: initial volume in chamber A ........................................ 11
- $V_{20}$: initial volume in chamber B ........................................ 11
- $V_{g_{\max}}$: maximal displacement of pump .............................. 8
- $x$: state vector ............................................................... 25
- $x_c$: stroke of cylinder .................................................... 11
- $x_r$: reference trajectory .................................................. 56
- $z$: state variable of residual generator .................................. 25
- $y_v$: valve spool stroke .................................................... 8
- $\alpha$: swshplate angle of pump .......................................... 8
- $\beta_2, \beta_3, \ldots, \beta_n$: stabilizing function of backstepping controller .......................... 55
- $\eta_v$: volumetric efficiency of pump .................................. 8
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<td>$\nu$</td>
<td>fault signal</td>
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<td>27</td>
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<td>$\omega_v$</td>
<td>natural frequency of valve</td>
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<tr>
<td>$\Omega_j$</td>
<td>coding set j</td>
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</table>
Terminology

The definitions of terminology listed here stem from [IB97] and [BFK+00]. They are suggested by the Safeprocess Technical Committee of IFAC.

**Fault** is an unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable/usual/standard condition.

**Failure** Permanent interruption of a system’s ability to perform a required function under specified operating conditions.

**Analytical redundancy** Use of more than one not necessarily identical ways to determine a variable, where one way uses a mathematical process model in analytical form.

**Hardware redundancy** Use of more than one independent instrument to accomplish a given function.

**Fault diagnosis** Determination of kind, size, location, and time of occurrence of a fault. Includes fault detection, isolation

**Fault detection** Determination of faults present in a system and time of detection.

**Fault isolation** Determination of kind, location, and time of detection of a fault. Follows fault detection.

**Residual** Fault information carrying signals, based on deviation between measurements and model based computations.

**Threshold** Limit value of a residual’s deviation from zero, so if exceeded, a fault is declared as detected.
1 Introduction

1.1 Background and motivation

Hydraulic systems are widely used in today’s industries. Compared to electrical drives they can generate large forces or torques very fast with simple structures. Hydraulic systems can broadly classified into two categories. The first is industrial hydraulics, which is typically applied in mechanical manufacture, like extrusion molding machines, pressers, rolling mills and other [Fin06]. The second is mobile hydraulics, which are the core components of steering, braking, suspension and power-train system of the vehicles [KZH04] and also popular in the robot manipulators, ship rudders, airplane landing gears and so on [Mün06].

Modern hydraulic systems are installed with electronic, for instance, sensors, servo-valves. Such systems are known as electro-hydraulic systems. The inputs are usually electrical signals and the relevant system variables are measured and transformed into electrical signals too. Thus, the hydraulic components can be precisely controlled to achieve better performance. But the control of electro-hydraulic systems is a challenging task due to inherent nonlinearities from complicated flow properties, friction in actuator, varying external load etc [BCR02]. Therefore, advanced control strategies are necessary for handling above problems [JK04].

Meanwhile, reliability and safety of the systems are most important issues of automated systems. Usually, a small fault can have a serious effect to control system, such as actuator malfunction or sensor offset, sometimes even crashes the system. If faults can be detected and identified earlier, the collapse of whole system can be avoided. This requirement motivates the research in fault diagnosis, which includes fault detection and isolation (FDI). If the redundant information is from the system model and available signal, it is labeled as model-based FDI, which is more attractive than conventional hardware redundancy based diagnosis, because the former gives more information about monitored system and improves the systems safety and reliability without additional hardware cost [CP99]. Furthermore, if a controller is able to tolerate possible faults automatically, the control is known as fault-tolerant control (FTC). Usually the FTC needs the fault information from FDI system. The controller can react to the fault and can be automatically reconfigured.

High performance control [AL00], [TSL98], [CLCL08] and model-based diagnosis [AS03], [Mün06] [GY05] of electro-hydraulic systems both have attracted a lot of researchers.
But there are few results, which combines the two aspect together. It is very meaningful to apply the FDI integrated FTC to electro-hydraulic systems, which work usually in critical places, like in automobiles, aircrafts. Besides the improvement of safety and reliability, if the fault in electro-hydraulic system can be detected early, the maintenance cost can be cut down. The whole system will be more intelligent and efficient.

1.2 Objectives and structure of the dissertation

Motivated by above considerations the objective of this dissertation is to develop an applicable model-based FDI integrated FTC scheme for the nonlinear electro-hydraulic system. The FDI system should be robust to the all uncertainties of the system but the considered faults. The high performance tracking control should be guaranteed even in the presence of the fault. The main tasks of this thesis are:

- modeling of system
- construction of faults
- design of robust model-based FDI systems
- design of FTC system
- keeping stability in the critical period during which the fault is present but not identified.

The rest of the dissertation is structured as follows.

In chapter 2, the model of the electro-hydraulic system is derived based on the physical relations among the system variables and experimental modeling technique. The derived model is validated with the measurements. The system is subject to a varying external load, which is model as an unknown disturbance to the system. The feature of system model gives the instructions for choice of FDI and FTC methods. In this chapter, the construction of the fault is introduced too. Among the most common faults in electro-hydraulic systems a system fault and two sensor faults are designed and constructed physically on the testbed. This chapter is the foundation of the rest of thesis.

In chapter 3, the modern model-based fault diagnosis methods are reviewed preliminarily. The differential-geometric approach is chosen for construction of FDI system because it gives a general formulation of FDI problem and is more compact and transparent than other model-based approaches. This approach gives a systematic design procedure to find a distribution, with which the system can be transformed so that it is only affected by one fault and unaffected by other system faults or disturbance. The local weak observability of the transformed system is also guaranteed with the proposed construction method. Thus, a fault detection observer based on the subsystem can be designed for residual generation. The fault isolation is achieved by a decision making logic (DML), which is
In chapter 4, the design of reconfigurable controller for FTC scheme is conducted. The State-of-the-Art of FTC is introduced at first. The backstepping control strategy is employed to deal with the high nonlinearity. The feedback control laws and the associate Lyapunov function can be constructed by following step-by-step routine. To compensate the effect of varying external load of the system parameter an adaption law is integrated. Then the tracking error can asymptotically converge to zero in the fault free case. The reconfigurations of controller in considered faulty cases are derived explicitly. Overall, the adaptive backstepping based reconfigurable controller and the FDI system proposed in chapter 3 construct the FTC scheme.

In chapter 5, the proposed FDI integrated FTC strategy is successfully implemented on the electro-hydraulic system. All considered fault scenarios are tested. Detailed results about FDI and FTC are presented.

In chapter 6, the summary and the advice to the work in future are given as the end of this dissertation.
2 Description of the electro-hydraulic system

2.1 Introduction

The researched testbed is built by company Mannesmann Rexroth and simulates the extruding machine, as shown in Fig. 2.1. It consists of two identical cylinders, which are named as working cylinder and load cylinder respectively. The working cylinder represents the press stem of an extrusion machine. Its fluid power is supplied by a variable displacement axial piston servo-pump. The load cylinder simulates the resistance force, which is generated when the material is pressed. It is connected with a fixed displacement pump so that load pressure can be built up quickly. With the equipment of a proportional pressure relief valve an arbitrary load can be achieved. The direction

\footnote{Now the company is named as Bosch Rexroth after the merge in 2001.}
change of fluid flow is realized through several 2-position directional valves. The operation pressure of whole system is limited under 250 $\text{Bar}$ [Wit97]. For simplicity it is assumed that the temperature of hydraulic circuit is constant and the pressure drop in pipeline is zero. The direct measured system variables are: swashplate angle, pressure in both chambers of working cylinder, pressure in chamber A of load cylinder and cylinder stroke. Cylinder velocity and angular velocity of swashplate can be calculated based on the respective position signals.

There is a Modular-4 card, in which signal measurements and control strategies are implemented. The card is actually a independent computer system with a 486-CPU. It can be installed in PC’s ISA slot and works parallel to the PC. The real-time application can be executed with a sampling rate up to $1k$ Hz. PC is used only for visualization, data saving and parameter setting.

Generally the hydraulic circuit of testbed can be divided in two main parts: servo-pump and cylinders. In the following sections of this chapter the modeling of the cylinders and pump, the verification of the models and construction of faults will be introduced in sequence.

2.2 Servo-pump

2.2.1 Functional description

![Cross section view of servo-pump](image.png)
2.2 Servo-pump

The pump, which is usually driven by electrical machine, transforms the mechanical energy into hydraulic energy. The variable displacement pump means the displacement, which is amount of fluid pumped per revolution of the pump’s input shaft, can be varied while the pump is running. The pump used here is actually a servo-pump and the structure is shown in Fig. 2.2. It consists of an axial piston pump with built-on proportional valve including inductive position transducer for sensing the swivel angle and valve position, a pressure transducer, and an external amplifier card for control.

The pump’s pistons slide axially in a rotating cylinder block to give a pumping action to the fluid. The piston shoes bear on a non-rotating swashplate. Variation of the swashplate angle changes the travel distance of the pistons, which then changes the hydraulic displacement of the pump. The swashplate angle is determined by offset cylinder and control cylinder. The stroke of control cylinder is controlled by the proportional valve.

The block diagram of servo-pump system is shown in Fig.2.3. If the pump is off, the swashplate angle is held on 100% because of the preload spring in offset cylinder side. With a rotating pump and de-energized proportional valve the big control cylinder is regulated to zero stroke as the valve spool is pushed to the right by the spring and therefore the pump pressure is applied to the control piston via valve port A. The balance
between the pump pressure and the spring force is achieved between 8 to 12 bar. Increase of the command value of swashplate angle makes the valve spool move from center position to left side until the swashplate angle (stroke of the control cylinder) reaches the setting value. The stroke of control cylinder reduces because of the connection of port A to tank. Thus the swashplate angle will increase. A reduction of command value of swashplate angle make the valve spool move from center to right. More fluid flows into the control cylinder leading to the increases of the stroke. Therefore the swashplate angle is reduced.

Theoretically the volume flow rate can be calculated through the formula

\[ Q_p = \frac{V_{g\text{max}} \cdot n \cdot \tan(\alpha) \cdot \eta_v}{1000 \cdot \tan(\alpha_{\text{max}})} \]  

(2.1)

where \( V_{g\text{max}} \) is the maximal displacement, \( \alpha \) stands for swashplate angle and \( \alpha_{\text{max}} \) is the maximum of the angle, \( n \) is the rotation speed of the pump and \( \eta_v \) stands for the volumetric efficiency [Fin06]. The swashplate angle varies usually in a small range, then \( \alpha \approx \tan(\alpha) \). The \( Q_p \) can be approximated as

\[ Q_p \approx K_Q \cdot \alpha, \]  

(2.2)

where \( K_Q \) is defined as pump flow coefficient, which is determined only by the mechanical structure of the pump. Hence the output flow of pump can be viewed as linear to the swashplate.

2.2.2 Modeling of servo-pump

The servo-pump in fact is a closed-loop control system. The swashplate angle \( \alpha \) is chosen as the state variable of the pump model [MJ96] [Pra01]. Fig.2.4 shows the cascad structure of the control loop. \( u_{\text{in}} \) stands for the command value of the swashplate angle

![Figure 2.4: Closed control loop of servo pump](image)

and the \( y_v \) represents the stroke of valve spool. The angle control is the outer-loop. The angle error is amplified by the P controller and then passed on to the inner control-loop as a command value. The valve spool stroke is regulated by the inner PD controller. Thus the stroke of the control cylinder can be adjusted.

For precise modeling it is necessary to introduce the model of the proportional valve and control cylinder. The fluid flow of proportional valve \( Q_v \) usually is assumed to be
linear to the spool stroke $y_v$. The controlled valve can be approximated as a second order system \cite{SS00} \cite{Wat05}. In Laplace domain the relation of valve spool stroke $y_v$ to input voltage $u_v$ can be written as

$$
\frac{y_v}{u_v} = \frac{k_{vu}\omega_v^2}{s^2 + 2d_v\omega_v s + \omega_v^2}, \tag{2.3}
$$

where $d_v$ stands for damping coefficient, $\omega_v$ is the natural frequency and $k_{vu}$ represents the spool position-voltage gain.

By integrating the cylinder model the block scheme of the servo-pump can be expanded as Fig.2.5. It should be pointed out that the structure is partly simplified but it is still nonlinear because of the saturation of valve spool stroke, the friction in control cylinder, non-constant parameter $C_H$ and so on. This is a fifth-order system. Some of the parameters and state variables are still unknown. Consequently the model in Fig.2.5 is not appropriate for real-time application.

More simplifications should be made without losing much accuracy of modeling.

- The valve is simplified as a first-order system.
- The cylinder is approximated as an integrator by ignoring the friction, external load and leakage\(^2\).

Thus the servo-pump can be described by a second-order system as following:

$$
\frac{\alpha}{u_{in}} = \frac{k_{pu}k_{sp}}{A_sT_v s^2 + A_s s + k_{sp}}, \tag{2.4}
$$

where $k_{pu}$ is the angle-voltage gain, $u_{in}$ represents the command value of the swashplate angle, $k_{sp}$ stands for the pump gain, which depends on the controller parameters and valve flow coefficient and $T_v$ is the valve time constant.

\(^{2}\text{A detailed cylinder model can be found in chapter 2.3.}\)
2.3 Hydraulic cylinder

2.3.1 One-cylinder model

Cylinders are very common for linear actuation. The modeling of hydraulic cylinders can be easily found in textbooks on hydraulics [Mer67] [MR03] [AGS06]. Thus only a brief introduction will be made and the background of hydrodynamics will be omitted.

As shown in Fig.2.6 is a hydraulic differential cylinder. $P_A$ and $P_B$ represent the pressure in the chamber A and B respectively. $A_1$ and $A_2$ denote the piston area and ring-side area. $Q_{Lin}$ and $Q_{Lex}$ stand for the internal leakage flow and external leakage flow. $x_c$ is the cylinder displacement, $F_f$ is the friction force and $F_L$ is external load. The pressure dynamics is derived according the rule of mass conservation

$$\sum Q_{in} - \sum Q_{out} = \dot{V}_H + \frac{V_H}{E_H} \dot{P}, \quad (2.5)$$

where $Q$ means liquid flow, $V_H$ is the volume and $E_H$ is the bulk modulus. The bulk modulus indicates the compressibility of the fluid and can be formed as

$$E_H = -V_H \frac{\partial P_H}{\partial V_H}. \quad (2.6)$$

For mineral oil the value is around $1.4 \sim 1.6 \times 10^4$ Bar. The bulk modulus is affected by the system pressure, entrained air, mechanical compliance and temperature. For engineering applications only the effective bulk modulus, which is denoted as $E_e$, is utilized, which is usually expressed empirically. Considering the significant influence of entrained-air the following approximation is employed [Wat05].

$$E_e = \frac{P}{P_o} + k_{air} E_A + \frac{P}{P_o}, \quad (2.7)$$

where $P_o$ is the atmospheric pressure and constant parameter $k_{air}$ is the air/oil volume ratio.
Now the pressure dynamics in both chambers can be derived as

\[
\dot{P}_A = \frac{E_A(P_A)}{V_{10} + A_1x_c} (-A_1v_c + Q_A - Q_{Lin}) \tag{2.8}
\]

\[
\dot{P}_B = \frac{E_B(P_B)}{V_{20} - A_2x_c} (A_2v_c - Q_B + Q_{Lin} - Q_{Lex}). \tag{2.9}
\]

where \(V_{10}\) and \(V_{20}\) are the initial volume of chamber A and B respectively, \(v_c\) stands for the piston velocity, \(E_A\) and \(E_B\) can be calculated according to Eq.(2.7). \(Q_A\) is the flow rate of chamber A, which is equal to the pump output flow \(Q_A = KQ\alpha\) because of the pump controlled structure of the system. \(Q_B\) is the flow rate of chamber B. The pressure in chamber B is low, therefore the effect of the directional valves cannot be ignored here. According to the characteristic curve of the directional valve \(Q_B\) can be approximated by

\[
Q_B = \begin{cases} 
\sqrt{k_{q1}(P_B - P_o) + k_{q2}} & \text{if } k_{q1}(P_B - P_o) + k_{q2} > 0; \\
0 & \text{if } k_{q1}(P_B - P_o) + k_{q2} \leq 0.
\end{cases} \tag{2.10}
\]

The parameters \(k_{q1}\) and \(k_{q2}\) can be identified through experiments. By ignoring the leakages and combining Newton’s second law the dynamic model of working cylinder can be described as:

\[
\dot{x}_c = v_c \tag{2.11}
\]

\[
\dot{v}_c = \frac{1}{m_c}(P_A A_1 - P_B A_2 - F_f - F_L) \tag{2.12}
\]

\[
\dot{P}_A = \frac{E_A(P_A)}{V_{10} + A_1x_c} (-A_1v_c + KQ\alpha) \tag{2.13}
\]

\[
\dot{P}_B = \frac{E_B(P_B)}{V_{20} - A_2x_c} (A_2v_c - Q_B), \tag{2.14}
\]

where \(x_c\) is the stroke of the cylinder, \(v_c\) is the velocity of the cylinder piston, \(m_c\) means the mass of the piston, \(F_f\) and \(F_L\) represent the friction and external load respectively.

### 2.3.2 Two-cylinder model

According to the cylinder model derived in last section the external load \(F_L\) is generated by the extrusion of two cylinder rods. This force cannot be measured with the existing hardware. Now a two-cylinder model will be derived, in which the external load is same as the pressure in load cylinder chamber A, which can be measured directly.

The load cylinder shown in Fig.2.7 is exactly same as the working cylinder. If the working cylinder moves forwards, it moves backwards with the same velocity. Assuming the friction in load cylinder is same as in working cylinder piston motion can be written
as:

\[ \dot{x}_c = v_c \] (2.15)
\[ \dot{v}_c = \frac{1}{2m_c}(P_{BL}A_2 - P_{AL}A_1 - F_f + F_L), \] (2.16)

here \( P_{AL} \) and \( P_{BL} \) represent the pressure in chamber A and B respectively. The chamber B is in subpressure state and the pressure \( P_{BL} \) is close to tank pressure, so \( P_{BL} \) can be ignored. Now add (2.16) to (2.12). The following equation can be derived.

\[ \dot{v}_c = \frac{1}{2m_c}(P_AA_1 - P_BA_2 - 2F_f - P_{AL}A_1) \] (2.17)

Define \( 2m_c = m, \) \( 2F_f = F_{fric} \) and \( P_{AL}A_1 = F_{load} \), so the velocity dynamics is governed by

\[ \dot{v}_c = \frac{1}{m}(P_AA_1 - P_BA_2 - F_{fric} - F_{load}). \] (2.18)

In this equation the external load is equal to the pressure in chamber A of load cylinder, which can be directly adjusted by the proportional pressure relief valve. The two-cylinder model can be written as

\[ \dot{x}_c = v_c \] (2.19)
\[ \dot{v}_c = \frac{1}{m}(P_AA_1 - P_BA_2 - F_{fric} - F_{load}) \] (2.20)
\[ \dot{P}_A = \frac{E_A(P_A)}{V_{10} + A_1x_c}(-A_1v_c + KQ\alpha) \] (2.21)
\[ \dot{P}_B = \frac{E_B(P_B)}{V_{20} - A_2x_c}(A_2v_c - Q_B), \] (2.22)
2.4 System identification and verification

2.4.1 Parameter identification for the pump

Some parameters of the pump model should be identified by input-output data. The pump model with the unknown parameters $p_1$ and $p_2$ is written in state-space format.

\[
\dot{\alpha} = v_\alpha \tag{2.23}
\]

\[
\dot{v}_\alpha = -p_1 \alpha - p_2 v_\alpha + p_1 k_{pu} \alpha_r, \tag{2.24}
\]

where $v_\alpha$ stands for the angular velocity of pump swashplate. It is important for system identification to design appropriate inputs [Lju99]. Here there are three kinds of inputs:

- Pseudo random multi-level (PRMS) signals
- Step signal
- Sinusoidal signal.

PRMS signal is recommended for system identification for exciting all amplitudes and frequencies of the system. Step signal is used to test the time domain response to the system. Sinusoidal signal is additionally applied here because later the reference values for control is sinusoidal-like curves.

The identification procedure is carried out with System Identification Toolbox of Matlab. The sampling time is chosen as $T_s = 5ms$, which is a compromise with respect to the sampling theory and hardware characteristic [AW96]. By analyzing the test results responding to the three inputs the final values of estimated parameters are $p_1 = 44414.0$ and $p_2 = 333.4$. The system verifying results are shown in Fig. 2.8–2.11. If the frequency of input signal is under 100 rad/s and amplitude change is under 4$\alpha$ the model estimations match well with measurements.

![Figure 2.8: Swashplate time response to PMRS input](image-url)
2.4.2 Parameter identification for the cylinder

In the cylinder model the friction force $F_{fric}$, flow coefficients $k_{qb1}$ and $k_{qb2}$ should be identified.
The friction in hydraulic cylinder consists of three parts [JK04] [Nis02]: viscous friction $f_v$, static friction $f_s$ and Coulomb friction $f_c$. The velocity-friction curve usually has the form like Fig. 2.12. It is obvious that in the low velocity area the friction is strongly nonlinear and hardly representable mathematically. Here only the viscous and Coulomb friction are considered. The rest can be treated as the uncertainty in external load. Then the friction model is written as

$$F_{fric} = D_v v + f_c. \quad (2.25)$$

Estimation of $D_v$ and $f_c$ requires the value of friction. By making cylinder move with constant velocity the friction is calculated through

$$F_{fric} = P_A A_1 - P_B A_2 - F_{load}$$

The optimal values of $D_v = 2.718e3$ $N/mm$ and $f_c = 3.596e3$ $N$ are obtained by
using polynomial fitting Technique. Fig. 2.13 shows the good linearity of friction. The maximal error occurs at the smallest velocity because of the influence of static friction. Identification of $Q_B$ is similar to friction. During the constant velocity motion of cylinder the pressure dynamics in chamber B is almost zero, then the $Q_B$ can be calculated by

$$Q_B = A_2 v_c.$$  

(2.26)

With optimal values of $k_{q_b1} = 61.275$ and $k_{q_b2} = -63.478$ the verifying result is shown in Fig. 2.14. With bigger pressure difference the approximation error is smaller.

### 2.4.3 Verification of whole system

Applying the identified parameters the system model can be represented as following

$$\dot{x}_c = v_c$$

$$\dot{v}_c = \frac{1}{m} (P_A A_1 - P_B A_2 - D v_c f_c - F_{load})$$

$$\dot{P}_B = \frac{E_B (P_B)}{V_{20} - A_2 x_c} \left( A_2 v_c - \sqrt{k_{q_b1} (P_B - P_o) + k_{q_b2}} \right)$$

$$\dot{P}_A = \frac{E_A (P_A)}{V_{10} + A_1 x_c} (-A_1 v_c + K_Q \alpha)$$

$$\dot{\alpha} = v_\alpha$$

$$\dot{v}_\alpha = -p_1 \alpha - p_2 v_\alpha + p_1 k_{pu} u_{in}.$$ 

(2.27)

The external load $F_{load}$ has significant influence to the system dynamics. Unfortunately it can not be precisely simulated. The proportional pressure relief valve can only determine a rough area of the pressure in load cylinder, because the pressure varies with velocity change. For accurate verification of the whole system a look-up table based on measured load pressure is applied for simulation.
The setting of system verification:

- The input voltage is sinusoidal-like;
- Pressure relief valve is set as 50 Bar;
- Parameter $k_{air}$ is assumed as 0.005.

![Figure 2.15: Pressure in chamber A of working cylinder](image1)

![Figure 2.16: Pressure in chamber B of working cylinder](image2)

![Figure 2.17: Velocity of working cylinder](image3)
Fig. 2.15 and 2.16 show the good match between the modeling and measurements. The modeling error is mainly caused by the approximation of the effective bulk modulus, especially in low pressure situation. It is remarkable that the cylinder velocity of the simulation model is also very noisy. This possibly results from the varied load.

### 2.5 Fault construction

Like all the technical devices the components of electro-hydraulic system can be faulty due to aging, wrong operation or sudden change of work environment, etc. The common faults in electro-hydraulic systems are briefly listed in table 2.1.

<table>
<thead>
<tr>
<th>Components</th>
<th>Faults</th>
<th>Consequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump</td>
<td>failure in electrical machine,</td>
<td>breakdown of whole system,</td>
</tr>
<tr>
<td></td>
<td>leakage</td>
<td>insufficient output</td>
</tr>
<tr>
<td>Valve</td>
<td>destroyed valve seat</td>
<td>breakdown of whole system</td>
</tr>
<tr>
<td>Cylinder, Motor</td>
<td>leakage</td>
<td>degraded control performance</td>
</tr>
<tr>
<td>Sensor</td>
<td>aging, cable break</td>
<td>degraded control performance, even system breakdown</td>
</tr>
<tr>
<td>Others</td>
<td>polluted oil</td>
<td>accelerated aging</td>
</tr>
</tbody>
</table>

Table 2.1: Common faults in electro-hydraulic systems

Not all listed faults can be studied within this research, because some faults are difficult to be simulated, like oil pollution or impossible to be tolerated, such as a sudden breakdown of electrical power. On the other hand it is not economical to simulate a
2.5 Fault construction

failure with high cost. As mentioned in literature [Mün06] leakage and senor faults occur frequently and can be simulated by simple reconstruction in hardware. Therefore these two kinds of faults are constructed on the testbed. In the range of this thesis the assumption is always held that only one fault is present for faulty system.

<table>
<thead>
<tr>
<th>Designed faults</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>internal leakage in cylinder</td>
<td>$Q_{Lin}$</td>
</tr>
<tr>
<td>sensor fault of $P_A$</td>
<td>$\Delta P_A$</td>
</tr>
<tr>
<td>sensor fault of $P_B$</td>
<td>$\Delta P_B$</td>
</tr>
</tbody>
</table>

Table 2.2: The designed faults of testbed

2.5.1 Internal leakage in cylinder

The main cause of cylinder internal leakage is the wear of moving component, namely the piston. The man-made internal leakage is constructed through the bypass pipeline between the chamber A and B of working cylinder, like Fig.2.19. The volume flow of leakage is controlled by a proportional directional valve. The leakage volume flow will not be measured directly by the flow meter for economy. The value can be obtained according to the characteristic curves of the valve. The pictures of the constructed internal leakage on real testbed are shown in Fig. 2.20.

2.5.2 Sensor faults

Sensor failures are very common in the electro-hydraulic systems. The electrical signals from sensors are influenced by noise, offsets, etc. The noise usually come from the
electromagnetic compatibility (EMC) problem and consists of high frequency components. The noise can be easily filtered by anti-aliasing filters. Therefore sensor noises are excluded in this dissertation.

Offsets of the sensors are caused by sensor aging, potential difference or similar reasons. The offsets in pressure sensors of chamber A and B of working cylinder are seen as sensor faults and realized by addition of an amount after A/D conversion.

2.6 Summary

This chapter introduces the mathematical model of the electro-hydraulic system. The model is built based on the physical characteristic with some reasonable simplifications. System parameters are identified through experiments. Then the system model is verified with the measurements. They have good consistency. The design of the faults is also introduced in this chapter. This chapter is the base of whole thesis.
3 Model-based fault detection and isolation

3.1 Introduction

3.1.1 Background

Engineering systems are always subject to unexpected changes, such as components malfunction, change of working condition, etc. The guarantee of safety and reliability is extremely important for automated systems. For safety-critical systems, like aircraft or nuclear power plant, hardware redundancy is usually applied to make the systems normal running even if fault occurs. This solution can not be widely used because of high cost of hardware. Furthermore the extra installed hardware can also introduce faults into the system. On the other hand the progression of a fault is usually gradual as shown in Fig. 3.1. If the fault can be detected and identified early, not only the total breakdown of the system can be avoided but also the maintenance cost can be reduced significantly. Hardware redundancy methods can not fulfill these requirements.

![Figure 3.1: Fault progression over time [Zog02]](image)
Thus, since 1970s analytical redundancy methods have been intensive researched and investigated [Bea71]. Analytical redundancy means to calculate the required redundant information by using the system model and the available signal information. This approach is also referred to as model-based FDI [Loo01]. The major advantage of model-based FDI is that no additional hardware is needed, except a powerful computer system. Although a great deal of real-time computations are necessary for on-line diagnosis, it is not so arduous with today’s PCs and measurement cards.

Model-based FDI basically consists of two main procedures: residual generation and decision making, as shown in Fig. 3.2. Residual generation is the key part of the diagnosis system. The residual is zero or almost zero if the system is fault free. The signal should distinguished deviate from zero if the fault occurs. A “good” residual signal should be sensitive to one special fault but robust to the all system uncertainties (modeling error, disturbance, etc) and other faults. Then the fault can be detected and isolated. Decision making is based on the residual signals. It can be carried out by simply setting thresholds or using statistical methods, like hypothesis test or generalized likelihood ratio (GLR) test, etc. Usually a compromise between false alarm occurrence and fault detection sensitivity should be made to obtain satisfactory results.

![Figure 3.2: Structure of model-based diagnosis [CP99]](image)

### 3.1.2 Residual generation

It is necessary to give a brief review of model-based residual generation approaches. Three major methods are introduced here.

1. **Parity space approach** is first presented by Chow and Willsky in 1984 [CW84]. The parity equations are usually derived by system dynamical model, as state space or transfer functions. Residual signal can be generated by checking the consistency
of the mathematical relations between the outputs and inputs in a time window (i.e. the parity check), during which the information of occurred faults are present in measurement. Robustness to additive model uncertainty can also be realized by orthogonal parity equations. The same principle can be applied for fault isolation by treating the uninterested faults as unknown inputs [GS90], [MV88], [PC91]. Further researches involved parity space based FDI can be found in [GCF+95] and [Ger97]. The main advantage of parity relation approach is its simplicity. The algorithm can be easily implemented with computers. If the system model is available, the parity relation always can be established. But it is difficult for this method to deal with nonlinear systems or noisy measurements. Using linearized model can introduce so much modeling uncertainties that the fault diagnosis is not possible.

2. **Parameter estimation and identification** approach works based on the idea that faults affect the system physical parameters such as resistance, friction, viscosity, etc. The parameters of actual process are continuously estimated. Residual generation can be achieved by comparison estimated parameters to their nominal values. An substantial difference indicates the occurrence of one or more faults. Several researches in this area can be found in [Ise84], [IF91] and [Ise93]. The basic procedure for carrying out parameter estimation based FDI can be found in [CP99]. This approach can be applied for both linear and nonlinear systems and is very effective to parameter faults. An accurate parameter identification requires persistently exciting input signal, i.e. all modes of the system should be excited during the identification procedure. For closed-loop control systems such a condition can not always be satisfied. Fault isolation could be difficult if the physical parameters do not uniquely correspond to model parameters. If the physical parameters varies with the system dynamics, it will be difficult for the decision making logic to distinguish them from faults. These restrictions limit this approach in application.

3. **Observer based approach** is the most active research area of model-based FDI. The idea is to reconstruct the states or outputs of the system from the measurements by using either observers for deterministic systems or Kalman-Filters for stochastic systems [Yan97]. The difference between the estimation and the measurement is usually viewed as residual. Various design schemes have been proposed: in [BZH04], [Bes03] and [JV00] high-gain observers are applied for nonlinear systems; in [TE02] and [CS07] sliding-mode observers are employed for robust residual generation; in [CH98] Kalman Filters are utilized for detecting faults. Furthermore many advanced methods can be integrated for improving the robustness of diagnosis system, such as $H\infty$ technique [PH97] [PMZ06], unknown input observers method [CPZ96] [SO05]; differential-geometric algorithm [HKK00] [DI01]; , [KS03]; adaptive techniques [WD96] [JSC04], etc. Fault isolation for observer based approaches is more complicated. Different methods are employed, such as Multi-Model approach, unknown input decoupling approach and so on. The com-
mon of these methods is that a band of observers or filters are running parallel for isolating a fault. It should be pointed out that adaptive observers differ from other observer approaches. Adaptive observers combine observer and parameter identification techniques [Din08]. The fault can be directly estimated by parameter adaption law. Residual generating can be archived according to the fault amplitude. It is very convenient to accomplish fault detection, isolation and estimation simultaneously. But this approach also has more restrict constraints to the system structure [JSC04].

More details and advanced techniques can be found in recent published books [SFP02], [Ise06], [Wit07] and [Din08]. No general method can deal with all kinds of systems. Which approach should be employed depends on the system model.

3.2 Differential-geometric approach

Among the various model-based fault diagnosis approaches introduced in last section the differential geometric method is of high interest. This approach provides a lot of advantages as it gives a general formulation of FDI problem and is more compact and transparent than other methods [Loo01]. Massoumnia first proposed this method in [Mas86a] and [Mas86b] for linear systems. It has been proven that a necessary and sufficient condition for the linear fundamental problem of residual generation (FPRG) to be solvable is the existence of an unobservability subspace leading to a quotient (observable) subsystem unaffected by all fault signals but one\(^1\). DePersis and Isidori extended this concept to nonlinear system by means of construction of the unobservability distribution, which can be viewed as the nonlinear analogue of the unobservability subspace [DI01].

In this section the differential-geometric approaches for linear and nonlinear systems will be reviewed.

3.2.1 FDI for linear time-invariant systems

Consider a linear time-invariant system (LTI) as following:

\[
\dot{x} = Ax + Bu + \sum_{i=1}^{k} L_i \nu_i \\
y = Cx
\]

where \(x \in \mathbb{R}^n\) is the state vector, \(u \in \mathbb{R}^w\) represents the control input, \(\nu_i \in \mathbb{R}\) is the fault signal and \(y \in \mathbb{R}^q\) is the output vector. \(A, B\) and \(C\) are the system matrices with

---

\(^1\)Definition of affected/unaffected can be found in Appendix A
appropriate dimensions. Sensor faults model is omitted here. By augmenting system
dynamics (3.1) the sensor faults can be modeled as pseudo-actuator faults. The details
can be found in [MVW89].

The general FDI filter problem (FDIFP) of LTI systems is formulated by [MVW89] as
following:

Considering the system (3.1) and (3.2), the FDIFP is to design an LTI dynamic residual
generator that takes the known signals $u$ and $y$ (observables of the system) as inputs and
generates a set of residual vectors $r_i$, $i \in q = \{1, 2, \ldots, q\}$, with the following properties:

1. When no fault is present, all the residuals $r_i$ decay asymptotically to zero. Hence,
   the net transmission from $u$ to the residuals is zero, and the modes observable from
   the residuals are asymptotically stable.

2. In the $j_{th}$ fault mode (i.e., when $\nu_j \neq 0$), the residuals $r_i$ for $i \in \Omega_j$ are nonzero,
   and the other residuals $r_a$ for $a \in q - \Omega_j$ decay asymptotically to zero. Here the
   pre-specified family of coding sets $\Omega_j \subseteq q$, $j \in k = \{1, 2, \ldots, k\}$ is chosen such
   that, by knowing which of the $r_i$ are (or decay to) zero and which are not, the fault
   $\nu_j$ can be uniquely identified.

The concept of coding set used in the above formulation contains a set of numbers that
represents a specific subset of the different residuals $r_i$; $i \in q$, i.e. $\Omega_j \subseteq q$, $j \in k$ where
$k$ denotes the number of faults and $q$ the number of residuals. If the complete set of
residuals defined by the coding set $\Omega_j$ is affected by an occurring fault it can be said that
the occurring fault is the $j_{th}$ fault. If $q = k$ and $\Omega_j = \{j\}$, then the simplest coding set
can be obtained and the $j_{th}$ residual is only affected by the $j_{th}$ fault. More details can
be found in [Loo01] and [MVW89]. Massoumnia et al. have first proposed the definition
of the fundamental problem of residual generation (FPRG) for LTI system [MVW89].
FPRG is in fact a restricted version of FDIFP. It considers only two-faults case. The
residual signal is only sensitive to one fault and unaffected by another one.

Assume a LTI system with two fault signals and the vector $L_1$ is monic, i.e. it has full
column rank.

$$
\dot{x} = Ax + Bu + L_1\nu_1 + L_2\nu_2 \\
y = Cx 
$$  \hspace{1cm} (3.3)

The residual generator has the following form:

$$
\dot{z} = Fz - Ey + Gu \\
r = Mz - Hy + Ku 
$$  \hspace{1cm} (3.4)

where $z \in \mathbb{R}^q$, $r \in \mathbb{R}^p$ and $E$, $F$, $G$, $H$, $K$, $M$ are the matrices with appropriate
dimensions. By combining the system and residual generator equations the extended
system and residual generator can be obtained.

\[
\begin{bmatrix}
\dot{x} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
A & 0 \\
-EC & F
\end{bmatrix}
\begin{bmatrix}
x \\
z
\end{bmatrix} +
\begin{bmatrix}
B & L_2 \\
G & 0
\end{bmatrix}
\begin{bmatrix}
u \\
\nu_2
\end{bmatrix} +
\begin{bmatrix}
L_1 \\
0
\end{bmatrix}
u_1
\]

(3.5)

\[r = \begin{bmatrix}
-HC & M
\end{bmatrix}
\begin{bmatrix}
x \\
z
\end{bmatrix} +
\begin{bmatrix}
K & 0
\end{bmatrix}
\begin{bmatrix}
u \\
\nu_2
\end{bmatrix}
\]

(3.6)

The system (3.5) and (3.6) can be formulated as:

\[
\begin{align*}
\dot{x}^e & = A^e x^e + B^e u^e + L^e \nu_1 \\
r & = H^e x^e + K^e u^e
\end{align*}
\]

(3.7) (3.8)

where \(x^e \in \mathbb{R}^{n+q}\), \(u^e \in \mathbb{R}^{w+1}\) and \(A^e, B^e, L^e, H^e,\) and \(K^e\) are the matrices with appropriate dimensions. Based on the equations (3.7) and (3.8) the definition of FPRG can be proposed, which is same as the definition in [Loo01].

**Definition 3.2.1.** *Fundamental problem of residual generation (FPRG) of the system (3.3) is to design a LTI dynamic residual generator by finding the appropriate matrices in (3.4) such that the following conditions are satisfied for system (3.7) and (3.8):*

1. \(r\) is unaffected by \(u^e\).
2. The map from \(\nu_1\) to \(r\) is input observable.
3. The observable modes of the pair \((H^e, A^e)\) should be asymptotically stable.

With the condition 1 the residual is unaffected by uninteresting inputs (control input and another fault signal), which prevents the false alarm of the diagnosis system and enables the isolation of fault \(\nu_1\) from \(\nu_2\). It also enhances the robustness of the residual generator if signal \(\nu_2\) is regarded as disturbance. The condition 2 assures the detectability of fault \(\nu_1\) with the designed residuals, which guarantees that the residual is affected by the fault signal. The condition 3 makes the residual \(r\) asymptotically approach to zero if no fault is present.

The nature way to solve the problem is to view the \(\nu_1\) as input to system and check whether the system for \(\nu_1\) is input observable. It means that for a system with triple \((C, A, B)\), \(B\) is monic and the image of \(B\) does not intersect the unobservable subspace of \((C, A)\). This approach can only ensure that the condition 2 and 3 are satisfied. The fault isolation problem is not considered by this approach.

The solution of definition 3.2.1 can be derived in terms of differential-geometric method as represented in [MVW89].
Theorem 3.2.1. The linear fundamental problem of residual generation (FPRG) in definition 3.2.1, has a solution if and only if:

\[ S^* \cap L_1 = 0 \quad \text{with} \quad S^* = \text{Inf} S(L_2) \quad L_1 = \text{Im} L_1 \quad L_2 = \text{Im} L_2 \quad (3.9) \]

\( S(L_2) \) stands for set of all \((C, A)\) unobservability subspaces\(^2\) containing the subspace \(L_2\). \(\text{Inf} S(L_2)\) can be understood as the minimal element of set \(S(L_2)\). \(\text{Im}\) represents the image of matrix. If condition (3.9) is satisfied, the dynamics of residual generator can be arbitrarily designed. Theorem 3.2.1 is the base of the differential-geometric methodology. Therefore, the proof is repeated here.

Proof. \(^3\) (Sufficiency)

The proof of sufficiency part is in fact a design procedure.

Assume \(D_0 \in D(S^*)\), i.e. \((A + D_0 C)S^* \subseteq S^*\). There is a canonical projection \(P : \mathcal{X} \to \mathcal{X}/S^*\). \(P\) is a system transformation matrix that the transformed system states are unaffected by the another fault \(\nu_2\). Calculate the matrix \(A_0 = (A + D_0 C : \mathcal{X}/S^*)\) and make \(A_0 P = P(A + D_0 C)\). Let the matrix \(H\) from system (3.4) be the solution of \(\text{Ker} HC = S^* + \text{Ker} C\) and matrix \(M\) is the solution of \(MP = HC\), where \(\text{Ker}\) represents the null space. The pair \((M, A_0)\) is constructed as observable. Then there exist a matrix \(D_1\) such that \(\sigma(F) = \Lambda\), where \(F = A_0 + D_1 M\) and \(\Lambda\) is an arbitrary self-conjugate set and \(\sigma(F)\) is the spectrum of \(F\).

Let \(P\) denote the right inverse of \(P\). Then let \(D = D_0 + P_r D_1 H\), \(E = PD\), \(G = PB\) and \(K = 0\).

Define \(e = z - Px\). So the following can be conducted

\[ \dot{e} = Fe - PL_1 \nu_1 \quad (3.10) \]
\[ r = Mz - Hy = Me \quad (3.11) \]

It is obvious that residual vector \(r\) in Eq. (3.11) is only affected by the fault \(\nu_1\). If the condition (3.9) holds, the FPRG for LTI system has a solution.

(Necessity) If the FPRG for LTI system has a solution, the \(r\) in system (3.8) can not be affected by \(u^e\) in any case. Hence, the \(K^e = 0\) and

\[ \langle A|B^e \rangle \subseteq S^e \quad (3.12) \]

where \(B^e = \text{Im} B^e\) and \(S^e = \langle \text{Ker} H^e | A \rangle\). \(S^e\) is the unobservability subspace of \((H^e, A^e)\). According to the proposition 1 in [MVW89] there exists a matrix \(Q\) that \(Q^{-1}S^e\) is a \((C, A)\)-unobservability subspace.

\(^2\)see Appendix A.7  
\(^3\)see Appendix A about the basic of differential geometry
Let $S = Q^{-1}S^e$, then $Q^{-1}B^e \subseteq S$. As $L_2 \subseteq B^e$ and $S(L_2)$ denotes the set of all $(C, A)$ unobservability subspaces containing the subspace $L_2$, then
\[ S \in S(L_2). \] (3.13)

$L_1$ is assumed to be monic so $L^e$ should be monic too. For condition 2 of definition 3.2.1 to hold $L^e \cap S^e = 0$. Thus
\[
Q^{-1}(L^e \cap S^e) = Q^{-1}L^e \cap Q^{-1}S^e = L_1 \cap S = 0.
\] (3.14)

Now it is obvious that (3.13) and (3.14) hold only if (3.9) is true. \qed

It is worth pointing out that there could be other subspaces $S^* \subset S$, which means the dimension of residual generator can be further reduced. But it is more difficult to find a lower dimension residual generator because there is no good design routine to find the unobservability subspace except $S^*$. The above procedure gives a systemic and straightforward way to find the unobservability subspace. With this procedure the detectability and isolatability of faults can be easily tested. If the fault is detectable and isolatable, then the residual generator with desired performance can be directly constructed based on this approach.

Although the solution (3.13) is derived from two faults case, it can be easily extended to multi-faults case. The multi-faults case is defined as extension of the fundamental problem in residual generation (EFPRG).

Consider a system with faults and disturbances
\[
\begin{align*}
\dot{x} &= Ax + Bu + L_1\nu_1 + P\xi \\
y &= Cx
\end{align*}
\] (3.15)

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^w$ represents the control input, $\nu_1 \in \mathbb{R}$ is the interested fault signal, $y \in \mathbb{R}^q$ is the output vector and $\xi \in \mathbb{R}^d$ denotes the disturbances (including uninteresting faults). Vector $L_1$ is monic. $A, B, C,$ and $P$ are the system matrices or vectors with appropriate dimensions.

**Theorem 3.2.2.** The extension of linear fundamental problem of residual generation (EFPRG), has a solution if and only if:
\[
S^* \cap L_1 = 0 \quad \text{with} \quad S^* = \text{Inf} S(\mathcal{P}), \quad L_1 = \text{Im}L_1 \quad \text{and} \quad \mathcal{P} = \text{Im}P
\] (3.16)

The proof of theorem 3.2.2 is similar to 3.2.1. The details can be found in [MVW89]. If the solution of EFPRG can be found, then it is possible to detect and isolate the fault even in simultaneous faults occurring situation.
3.2 Differential-geometric approach

3.2.2 FDI for nonlinear input-affine systems

In the preceding section the linear fundamental problem of residual generation (FPRG) for LTI system is formulated. The systematic design procedure of residual generator based on the ideal of [MVW89] is represented. But it is restricted to linear systems. Fault detection and isolation for nonlinear systems is more difficult. Linearization around one or more operation points can introduce a lot of modeling errors into the diagnosis system. After the decoupling of these uncertainties the residual generating is hardly possible. It is necessary to extend geometric approach into nonlinear systems. The modeling uncertainties can be reduced massively by precise nonlinear description. Thus the robustness of residual generator can be significantly improved. The limited measurements can be kept for generating residual signals.

Geometric approach based fault diagnosis for nonlinear systems is intensive researched in [DI99], [DI00a] and [DI01]. They introduced the concept of unobservable distribution. The existence and regularity of this distribution indicated the solution of fault detection and isolation for nonlinear systems. The principle of construction of residual generator is to carry out a system transformation according to the unobservable distribution for disturbance decoupling or fault isolation. The transformed system is only affected by one fault and fault detection observer is designed for the transformed system. In the following of this section this method will be introduced in detail.

Consider an input-affine nonlinear system as modeled by equations of the form

\[
\begin{align*}
\dot{x} &= f(x) + g(x)u + l(x)\nu + p(x)\xi \\
y &= h(x)
\end{align*}
\] (3.17)

with state vector \(x \in \mathbb{R}^n\), control input \(u \in \mathbb{R}^w\), specific fault signal \(\nu \in \mathbb{R}\), disturbance signals (including other fault signals) \(\xi \in \mathbb{R}^d\) and output \(y \in \mathbb{R}^q\). \(f(x), g(x)\) and \(p(x)\) are smooth vector fields and \(h(x)\) is a smooth mapping with \(f(0) = 0\) and \(h(0) = 0\). Here only the actuator faults are considered. Similar to linear systems case the following definition is formulated

**Definition 3.2.2. (Local nonlinear fundamental problem of residual generation (l-NLFPRG))**[Loo01]: Considering the system 3.17 the l-NLFPRG is to find, if possible, a filter

\[
\begin{align*}
\dot{z} &= \hat{f}(z, y) + \hat{g}(z, y)u \\
r &= \hat{h}(z, y).
\end{align*}
\] (3.18)

\(\hat{f}(z, y), \hat{h}(z, y)\) and \(\hat{g}(z, y)\) are smooth too with \(\hat{f}(0, 0) = 0\) and \(\hat{h}(z, y) = 0\). \(z \in \mathbb{R}^n\) and \(r \in \mathbb{R}^q\). An augmented system can be constructed based on the systems (3.17) and (3.18).

\[
\begin{pmatrix}
\dot{x} \\
\dot{z}
\end{pmatrix} =
\begin{pmatrix}
f(x) \\
\hat{f}(z, h(x))
\end{pmatrix} +
\begin{pmatrix}
g(x) \\
\hat{g}(z, y)
\end{pmatrix}u +
\begin{pmatrix}
l(x) \\
0
\end{pmatrix}\nu +
\begin{pmatrix}
p(x) \\
0
\end{pmatrix}\xi
\] (3.19)
which can be reformulated as

\[
\dot{x}^e = f^e(x^e) + g^e(x^e)u + l^e(x^e)\nu_1 + p^e(x^e)\xi
\]  

(3.21)

\[
r = h^e(x^e)
\]  

(3.22)

and is defined on neighborhood \(U^e\) of the origin \((x; z) = (0; 0)\). The following properties hold:

1. if \(\nu = 0\), then \(r\) is unaffected by \(u\) and \(\xi\);
2. \(r\) is affected by \(\nu\);
3. if \(\nu = 0\),

\[
\lim_{t \to \infty} \|r\| = 0, \quad \forall t > 0
\]

for any initial condition \((x^0; z^0)\) in a suitable set containing the origin \((x; z) = (0; 0)\) and any set of admissible input.

Condition 1 ensures the robustness of the residual generator if the uncertainty of the system can be modeled as an additive disturbance and isolatability to other faults. Condition 2 relates the fault \(\nu\) to residual vector so that the effect of the fault can be reflected by residual signals. The condition 3 guarantees the stability of the residual generator. Here \(l\) indicates the fact that fault detection and isolation features are required to hold locally [AAE01].

Solution of the l-NLFPRG is derived in [DI99], [DI01] and [AAE01]. Assume the observation distribution \(\mathcal{O}^e\) of the system (3.21) and (3.22). The codistribution of \(\mathcal{O}^e\) can be defined as

\[
\Omega_{\mathcal{O}^e} = \text{span}\{dR(x^e), R \in \mathcal{O}^e\},
\]  

(3.23)

where \(d\) stands for the differential.

Based on the proof in [Isi95] solution of the definition 3.2.2 can be described by

\[
\text{span}\{g^e(x^e), p^e(x^e)\} \subset (\Omega_{\mathcal{O}^e})^\perp \quad \text{and} \quad l^e(x^e) \notin (\Omega_{\mathcal{O}^e})^\perp.
\]  

(3.24)

It is not easy to find the subspace \((\Omega_{\mathcal{O}^e})^\perp\). An assumption to the system is inducted to simplify the construction of \((\Omega_{\mathcal{O}^e})^\perp\).

The key problem is to find the distribution \(\Omega_{\mathcal{O}^e}\). If \(x^e = (x; z) = (0; 0)\) is a regular point of \(\Omega_{\mathcal{O}^e}\), then in the neighborhood of this point \(\Omega_{\mathcal{O}^e}\) can be described by the smallest codistribution \(M^e\), which is invariant under \(\{f^e, g^e, p^e\}\) and contains \(\text{span}\{dh^e\}\). In this case the formulation in (3.24) can be represented as

\[
\text{span}\{g^e(x^e), p^e(x^e)\} \subset (M^e)^\perp \quad \text{and} \quad l^e(x^e) \notin (M^e)^\perp.
\]  

(3.25)
3.2 Differential-geometric approach

Now the problem is transformed to the construction of the codistribution \(M^{c}\), which is more convenient in calculation. Then the solution of l-NLFPRG \(^4\) is [AAE01]:

**Theorem 3.2.3.** Let \(M\) be an involutive conditioned invariant distribution (unobservability subspace) of system (3.17) such that

\[
\text{span}\{p\} \subset M \subset \text{Ker}(\varphi \circ h) \quad \text{and} \quad l \notin M
\]

for some surjection: \(\varphi : \mathbb{R}^q \rightarrow \mathbb{R}^\tilde{n}\), defined locally around \(y = 0\) and with \(\varphi(0) = 0\). Then, there exists a change of state coordinates \(z = \Phi(x)\) and a change of output coordinates \(\tilde{y} = \Psi(y)\), defined locally around \(x = 0\) and, respectively, \(y = 0\), such that, in the new coordinates, the system (3.17) admits the normal form:

\[
\dot{z}_1 = f_1(z_1, \tilde{y}_2) + \tilde{g}_1(z_1, \tilde{y}_2)u_1 + \tilde{l}_1(z_1, z_2)\nu \\
\dot{z}_2 = f_1(z_1, z_2) + \tilde{g}_2(z_1, z_2)u_2 + \tilde{l}_2(z_1, z_2)\nu + \tilde{p}(z_1, z_2)\xi \\
\tilde{y}_1 = \tilde{h}_1(z_1) \\
\tilde{y}_2 = \tilde{h}_2(z_1, z_2),
\]

with \(z_1 \in \mathbb{R}^\tilde{n}, \tilde{n} = n - \text{dim}(Q) = \text{codim}(Q), \tilde{l}_1(z_1, z_2) \neq 0\) locally around \(\Phi(0) = 0\).

The proof of the theorem 3.2.3 is omitted here. The details can be found in [DI01]. What is important is how to find the distribution \(M\). If \(M\) is a generic conditioned invariant distribution, it is not ensured that the subsystem (3.27) and (3.29) is observable. The observability of system (3.27) and (3.29) depends on the construction method of \(M\). DePersis and Isidori [DI00b] have developed the observability codistribution algorithm, which guarantees the locally weak observability of subsystem (3.27) and (3.29) after system transformation. This method will be introduced in the following.

### 3.2.2.1 Calculation of the involutive conditioned invariant distribution \(M\) (unobservability distribution)

The construction of distribution \(M\) is carried out in two steps. First, the distribution, which is involutive, conditioned invariant, containing the \(\text{span}\{p\}\), is constructed. Next, based on the algorithm of first step the observability codistribution \(M^\perp\) can be obtained. Then the unobservability distribution is simply \(M = (M^\perp)^\perp\).

**Step 1:** Consider a nonlinear input-affine system

\[
\dot{x} = g_0(x) + \sum_{i=1}^{w} g_i(x)u_i \\
y = h(x)
\]

---

\(^4\)Because it is the “regular” (stronger) version of the fundamental problem of residual generation, it is also defined as r\(l\)-NLFPRG in some literatures.
3 Model-based fault detection and isolation

with \( x \in \mathbb{R}^n, \ u_i \in \mathbb{R}, \ y \in \mathbb{R}^q, \) in which \( g_0(x), g_1(x), \ldots, g_s(x) \) are smooth vector fields and \( h(x) \) is a smooth map. Recall the definition in [Isi95] that a distribution \( \Delta \) is said to be conditioned invariant for system (3.31) if it satisfies

\[
[g_i, \Delta \cap Ker \{dh\}] \subset \Delta, \quad \text{for all} \quad i = 0, \ldots, m
\]

(3.32)

where \([ \ ]\) is the Lie Bracket. Now let \( p_1(x), \ldots, p_d(x) \) be the set of additional smooth vector fields and consider the distribution

\[
P = \text{span} \{ p_1, \ldots, p_d \}.
\]

(3.33)

Define that \( \bar{P} \) denotes the smallest involutive distribution that contains \( P \) (involutive closure of \( P \)). Then consider the non-decreasing sequence of distributions

\[
S_0 = \bar{P}
\]

(3.34)

\[
S_{k+1} = \bar{S}_k + \sum_{i=0}^{w} [g_i, \bar{S}_k \cap Ker \{dh\}]
\]

(3.35)

According to the proof in [DI00b] there exists an integer \( k^* \) such that

\[
S_{k^*+1} = \bar{S}_{k^*}
\]

(3.36)

and \( \bar{S}_{k^*} \) is also denoted as \( \Gamma^*_P \). Then \( \Gamma^*_P \) is involutive, contains \( P \) and conditioned invariant. If there exists other distribution \( \Delta \) that is involutive, contains \( P \) and conditioned invariant, \( \Gamma^*_P \) satisfies \( \Gamma^*_P \subset \Delta \).

Now suppose that the distribution \( \Gamma^*_P \) is well-defined and nonsingular, so that its annihilator \( (\Gamma^*_P)\perp \) is locally spanned by exact differentials. Suppose also \( \Gamma^*_P \cap Ker \{dh\} \) is also a smooth distribution. Then it can be said that \( (\Gamma^*_P)\perp \) is the maximal conditioned codistribution which is locally spanned by exact differentials and contained in \( P\perp \). More details can be found in [DI00b].

**Step 2:** There exists a fixed codistribution \( \Theta \) of system (3.31) and the following non-decreasing sequence of codistribution:

\[
M_0 = \Theta \cap \text{span} \{dh\}
\]

(3.37)

\[
M_{k+1} = \Theta \cap \left( \sum_{i=0}^{s} L_{g_i}M_k + \text{span} \{dh\} \right),
\]

(3.38)

where \( L_{g_i}M_k \) is the Lie Derivative of \( M_k \). Suppose that all codistribution of this sequence are nonsingular, so that there is an integer \( k^* \leq n - 1 \) such that \( M_k = M_{k^*} \) for all \( k > k^* \), and set \( \xi^* = M_{k^*} \). Then it is usually denoted as

\[
\xi^* = o.c.a.(\Theta),
\]
where o.c.a is short for observability codistribution algorithm. The algorithm has the property that \( o.c.a.(\Theta) = o.c.a.(o.c.a.(\Theta)) \) and if \( \Theta \) is conditioned invariant, so is the codistribution \( \xi^* \). A codistribution \( \xi \) is called observability codistribution if

\[
L_{g_i} \xi \subset \xi + \text{span}\{dh\} \quad \forall i = 0, 1, \ldots, s
\]

(3.39)

\[
\Omega = o.c.a.(\Omega).
\]

(3.40)

Furthermore if the codistribution \( \Omega = \Delta^\perp \), then \( \Delta \) is the unobservability distribution.

After substituting the \((\Gamma^P)^\perp\), \( o.c.a.((\Gamma^P)^\perp) \) is the maximal observability codistribution which is locally spanned by exact differentials and contained in \( P^\perp \). The corresponding unobservability distribution \( M \) can be construed by

\[
M = (o.c.a.((\Gamma^P)^\perp))^\perp
\]

(3.41)

As a result of the maximality of \( o.c.a.((\Gamma^P)^\perp) \), \( M \) is the smallest involutive conditioned invariant unobservability distribution that contains \( P \). The locally weak observability of the transformed system in fault free situation can be guaranteed, which is proved in [DI00b]. Therefor the \( M \) is most possible distribution to fulfill the condition in theorem 3.2.3.

### 3.3 FDI for the electro-hydraulic system

Fault diagnosis for electro-hydraulic systems is a challenging task because of high nonlinear dynamics and unknown disturbance of the system. The nonlinear model is employed for a precise modeling. The difficulty is to archive the robustness to the unknown disturbance and keep the sensitivity to the considered faults simultaneously. Several researches in this area have been presented: Gayaka and Yao [GY08] applied the adaptive robust approaches to construct the fault detection observer for residual generation; An and Sepehri [AS03] used extended Kalman Filter for detecting the pump pressure drop; Shi and his colleagues [SGLB05] employed linear observer and adaptive threshold for robust FDI of a nonlinear electro-hydraulic system.

What is different from the systems discussed in references is that our electro-hydraulic testbed has a varied external load, which has significant effect to system dynamics. If the change of the external load is ignored, the estimations of the system states can not converge to the actual values. So far there is no precise modeling of the varying external load. Therefore, the only way to make the diagnosis system robust to the external load is to decouple it from the mathematical model.

#### 3.3.1 Disturbance decoupling

For the electro-hydraulic testbed the external load is viewed as unknown disturbance. The considered faults are internal leakage and sensor offset on pressure transducers of
chamber A and B. Firstly, only the internal leakage, which belongs to actuator faults, is studied. The sensor faults are researched later.

Based on the differential-geometric approach an unobservability distribution of the nonlinear system should be constructed. Then system transformation in states and output space can be carried out. Finally an observable subsystem, which is affected only by the specified fault, can be extracted for fault diagnosis. Design a fault detection observer for the subsystem. Then the residual generation is obtained.

Recall the electro-hydraulic system with internal leakage:

\[ \dot{x}_c = v_c \]
\[ \dot{v}_c = \frac{1}{m} (PA_1 - PB A_2 - Dv_c v - f_c - F_{load}) \]
\[ \dot{P}_B = \frac{E_B(P_B)}{V_{20} - A_2 x_c} \left( A_2 v_c - \sqrt{k_{q1}(PB - Po) + k_{q2}} + Q_{Lin} \right) \]
\[ \dot{P}_A = \frac{E_A(P_A)}{V_{10} + A_1 x_c} (-A_1 v_c - Q_{Lin} + K_Q \alpha) \]
\[ \dot{\alpha} = v_\alpha \]
\[ \dot{v}_\alpha = -p_1 \alpha - p_2 v_\alpha + p_1 k_{pu} u_{in} \] \hspace{1cm} (3.42)

The state variables are cylinder position \( x_c \), cylinder velocity \( v_c \), the pressure \( P_B \), the pressure chamber \( P_A \), the swashplate angle \( \alpha \) and the angular velocity \( v_\alpha \). The bulk modulus \( E_A(P_A) \) and \( E_B(P_B) \) are nonlinear functions of respective pressures as described in Eq.(2.7). The physical meanings of other parameters can be found in chapter 2. It is obvious that the pump dynamics has no intersection to cylinder dynamics except the pressure change in chamber A. Neither the internal leakage nor the external load can affect the pump dynamics. It is nature to consider only the cylinder dynamics for residual generator and treat the swashplate angle as control input of cylinder system.

Then the new system for fault diagnosis is represented as:

\[ \dot{x}_c = v_c \]
\[ \dot{v}_c = \frac{1}{m} (PA_1 - PB A_2 - Dv_c v - f_c) - \frac{1}{m} F_{load} \]
\[ \dot{P}_B = \frac{E_B(P_B)}{V_{20} - A_2 x_c} \left( A_2 v_c - \sqrt{k_{q1}(PB - Po) + k_{q2}} + Q_{Lin} \right) \]
\[ \dot{P}_A = -\frac{E_A(P_A)}{V_{10} + A_1 x_c} A_1 v_c - \frac{E_A(P_A)}{V_{10} + A_1 x_c} Q_{Lin} + \frac{E_A(P_A)}{V_{10} + A_1 x_c} K_Q \alpha; \]

with the state vector \( x = [x_c \ v_c \ P_B \ P_A]^T \). All state variables are measurable, then

\[ y = [x_c \ v_c \ P_B \ P_A]^T. \] \hspace{1cm} (3.44)

Now it should be tested whether the desired unobservability distribution does exist. The corresponding vectors are (For avoiding confusion the state variable \( x \) is omitted in
3.3 FDI for the electro-hydraulic system

denotation.):

\[
g_0 = \begin{pmatrix}
\frac{1}{m} (P_A A_1 - P_B A_2 - D_c v_c - f_c) \\
\frac{E_B(P_B)}{V_{20} - A_2 x_c} \left( A_2 v_c - \sqrt{k_q b_1 (P_B - P_o) + k_q b_2} \right) \\
-\frac{E_A(P_A)}{V_{10} + A_1 x_c} A_1 v_c
\end{pmatrix}
\]

\[
g_1 = \begin{pmatrix}
0 \\
0 \\
0 \\
K_Q \frac{E_A(P_A)}{V_{10} + A_1 x_c}
\end{pmatrix}
\]

\[
l = \begin{pmatrix}
0 \\
0 \\
0 \\
-\frac{E_B(P_B)}{V_{20} - A_2 x_c} & \frac{E_A(P_A)}{V_{10} + A_1 x_c}
\end{pmatrix}
\]

\[
p = \begin{pmatrix}
1/m \\
0 \\
0 \\
0
\end{pmatrix}
\]

The terms \(\frac{E_B(P_B)}{V_{20} - A_2 x_c}, \frac{E_A(P_A)}{V_{10} + A_1 x_c}, K_Q\) and \(m\) will never be zero because of the physical constraints. Therefore, \(g_1, l, p\) are all regular distributions.

Following the procedure (3.34) and (3.35) to compute \(\Gamma_P^*\) begins with:

\[S_0 = \bar{P} = P, \quad \text{with} \quad P = \text{span} \{p\} \quad (3.45)\]

As \(P\) is one dimension subspace and constant, the involutive closure is \(\bar{P}\) equal to \(P\). Obviously, \(\bar{S}_0 = S_0\). All states are measurable, then

\[dh = I_4 \Rightarrow Ker(dh) = 0 \Rightarrow \bar{S}_0 \cap Ker(dh) = 0\]

where \(I_4\) is the \(4 \times 4\) identity matrix. Furthermore:

\[S_1 = \bar{S}_0 + \sum_{i=0}^w [g_i, 0] \Rightarrow S_1 = \bar{S}_0 + 0 = \bar{S}_0 \]

\[\Rightarrow k^* = 0 \Rightarrow \Gamma_P^* = P\]

This leads to

\[(\Gamma_P^*)^\perp = P^\perp = \text{span} \{(1 0 0 0), (0 0 1 0), (0 0 0 1)\}\]

Meanwhile

\[\text{span} \{dh\} = \text{span} \{(1 0 0 0), (0 1 0 0), (0 0 1 0), (0 0 0 1)\}\]

Now calculate the involutive conditioned invariant distribution \(M\) according to the observability codistribution algorithm (o.c.a.).

\[M_0 = P^\perp \cap \text{span} \{dh\} = P^\perp\]
Furthermore

\[ M_1 = M_0 \cap \left( \sum_{i=0}^{w} L_0 M_0 + \operatorname{span}\{dh\} \right) \]

\[ = M_0 \cap (0 + \operatorname{span}\{dh\}) \]

\[ = M_0. \]

This leads to

\[ M = (P_0^\perp)^\perp = P \quad (3.46) \]

Obviously, \( l \notin P \). Thus according to the theorem 3.2.3 there exists a subsystem, which is unaffected by external load and affected by fault internal leakage.

Next step, a system transformation based on the observability distribution can be implemented. The procedure is derived in [DI00b]. It will be repeated here.

If \( n_1 \) is the dimension of the observability distribution \( \Delta \) and \( q - n_2 \) is the dimension of the subspace \( \Delta \cap \operatorname{span}\{dh\} \), then there exists a smooth surjection \( \Psi_1 : \mathbb{R}^q \to \mathbb{R}^{q - n_2} \) such that

\[ \Delta \cap \operatorname{span}\{dh\} = \operatorname{span}\{d(\Psi_1 \circ h)\}, \]

where \( q \) is the dimension of the output space.

Furthermore two local diffeomorphisms \( \Psi(y) \) and \( \Phi(x) \) can be found with the format

\[ \Psi(y) = \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix} = \begin{pmatrix} \Psi_1(y) \\ H_2 y \end{pmatrix}, \quad \Phi(x) = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} \Phi_1(x) \\ H_2 h(x) \\ \Phi_3(x) \end{pmatrix}, \]

where \( H_2 \) is a \( n_2 \times p \) selection matrix, function \( \Phi_1 \) satisfies \( \Delta = \operatorname{span}\{d\Phi_1\} \) and \( \Phi_3 : \mathbb{R}^n \to \mathbb{R}^{n - n_2 - n_1} \) is an arbitrary function.

For the cylinder system it can be obtained \( \dim(\Delta) = \dim(P_0^\perp) = 3 \). It leads to \( n_1 = 3 \) and \( n_2 = 1 \). Define \( x = \{x_1 \ x_2 \ x_3 \ x_4\}^T = \{x_c \ v_c \ P_B \ P_A\}^T \) and \( y = \{y_1 \ y_2 \ y_3 \ y_4\}^T = \{x_1 \ x_2 \ x_3 \ x_4\}^T \). Then it is nature to choose:

\[ \Psi_1 = \{y_1 \ y_3 \ y_4\}^T; \quad H_2 = \{0 \ 1 \ 0 \ 0\}; \quad \Phi_1(x) = \{x_1 \ x_3 \ x_4\}^T. \]

As \( n - n_1 - n_2 = 0 \), \( \Phi_3(x) \) does not exist. It leads to

\[ z_1 = \tilde{y}_1 = \{x_1 \ x_3 \ x_4\}^T \quad z_2 = \tilde{y}_2 = \{x_2\} \]

In the new coordinates the system is rewritten as

\[ \dot{x}_c = v_c \]

\[ \dot{P}_B = -\frac{E_B(P_B)}{V_{20} - A_2 x_c} \sqrt{k_{q1}(P_B - P_o) + k_{q2}} + \frac{E_B(P_B)}{V_{20} - A_2 x_c} A_2 v_c + \frac{E_B(P_B)}{V_{20} - A_2 x_c} Q_{Lin} \quad (3.47) \]

\[ \dot{P}_A = -\frac{E_A(P_A)}{V_{10} + A_1 x_c} A_1 v_c - \frac{E_A(P_A)}{V_{10} + A_1 x_c} Q_{Lin} + \frac{E_A(P_A)}{V_{10} + A_1 x_c} K_{Qc}. \]
The transformed system has three orders. It is locally weakly observable in fault free case. The cylinder velocity is regarded as input signal.

### 3.3.2 Fault detection observer

Nonlinear observer of system (3.47) and (3.48) should be designed for fault detection. The difficulty is that the new system dynamics is mainly described by the input signals. The nonlinear observer design methodology can not be applied due to the “abnormal” structure of system. Fortunately, all state variables are measurable. The system nonlinearity can be compensated by output feedback. Thus, a linear Luenberger observer is constructed for fault detection.

The structure of the observer is designed as:

\[
\dot{\hat{x}} = Ax + f(y) + g(y)u + L_o(y - \hat{y})
\]
\[
\hat{y} = C\hat{x}
\]

(3.49)

Define observer error \( e_o = x - \hat{x} \) and residual \( r = y - \hat{y} = ce_o \). Then it can be obtained

\[
\dot{e}_o = (A - L_oC)e_o
\]
\[
r = Ce_o.
\]

(3.50)

\( L_o \) is the observer gain, which ensures that \( (A - L_oC) \) is stable. Therefor, the residual \( r \) satisfies

\[
\lim_{t \to \infty} ||r|| = 0 \quad \forall t > 0,
\]

(3.51)

if no fault occurs.

#### 3.3.2.1 The internal leakage

The internal leakage can be viewed as actuator fault. If the fault occurs, the error dynamics of the observer is

\[
\dot{e}_o = (A - L_oC)e_o + E_{Lin}f_{Lin}
\]
\[
r = Ce_o.
\]

(3.52)

where

\[
E_{Lin} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad f_{Lin} = \begin{bmatrix} \frac{E_y(P_B)}{V_{20} - A_2x_c}Q_{Lin} \\ -\frac{E_A(P_A)}{V_{10} + A_1x_c}Q_{Lin} \end{bmatrix}.
\]

Then the residual vector is the response of linear filter with transfer function \( C(sI - (A - L_oC))^{-1}E_{Lin} \) to the input vector \( f_{Lin} \). The elements of the \( f_{Lin} \) are always bounded and the transfer function is stable. Therefore, the boundedness of the residual can be guaranteed if the internal leakage occurs.
3.3.2.2 The sensor faults

It is always difficult to detect sensor faults by applying the observers with closed structure. There is no systematic method to design an observer detecting sensor fault of nonlinear systems. The detectability depends on the structures of systems and observers. Usually, the observers work so “well” that they track the measurements even if the measurements drift from the real values. Thus, the sensor fault effect of the electro-hydraulic system should be studied in individual.

If there is an offset $\Delta P_B$ in sensor of pressure $P_B$, then the outputs of system are

$$y_1 = \{x_c \quad P_B + \Delta P_B \quad P_A\}^T$$

It can be obtained that the observer dynamics related to pressure estimation $\hat{P}_B$ is affected. Then,

$$\dot{e}_{P_B} = -L_{o2}e_{P_B} - L_{o2}\Delta P_B + \Pi(\Delta P_B)$$

$$r_{P_B} = e_{P_B} + \Delta P_B,$$  \hspace{1cm} (3.53)

where $e_{P_B} = P_B - \hat{P}_B$ and

$$\Pi(f_B) = \frac{E_B(P_B)}{V_{20} - A_2x_c} \left( A_2v_c - \sqrt{k_{q1}(P_B - P_o) + k_{q2}} \right)$$

$$- \frac{E_B(P_B + \Delta P_B)}{V_{20} - A_2x_c} \left( A_2v_c - \sqrt{k_{q1}(P_B + \Delta P_B - P_o) + k_{q2}} \right).$$

If the effect of $\Pi(f_B)$ is not considered, then the residual $r_{P_B}$ is the response of the linear filter $\frac{s}{s + L_{o2}}$ to input $\Delta P_B$. It is obvious that if $\Delta P_B$ is assumed to be a step signal

$$\lim_{t \to \infty} \|r_{P_B}\| = 0 \quad \forall t > 0.$$

It can be concluded that the sensor fault effect depends only on the term $\Pi(\Delta P_B)$. Theoretically, fault detection observer can sense the sensor fault, but the amplitude of the residual depends on not only the amount of offset but also the system structure.

The similar analysis can be carried out for the sensor of $P_A$ by defining $e_{P_A} = P_A - \hat{P}_A$. The same conclusion can be obtained.

3.3.3 Decision making

Residual signals are generated by the fault detection observer. But it can not be determined yet if a fault is happened or which fault is happened. Usually a decision making logic (DML) is designed for residual evaluation and fault isolation. An intelligent DML sometimes can also improve the robustness of the fault diagnosis system. In this thesis the procedure is divided into two steps.
Step 1: Signal processing
The residual signals cannot be sent to the DML directly due to the measurement noise, which can trigger false alarms. Simply increasing the range of thresholds can prevent from the impact of noise but bring about the risk of missed detection of the fault. An alternative is filtering of the residuals. The original residuals are filtered by a low-pass filter before they are used for decision making. The part of fault information probably is also filtered out, which can delay the FDI. In this thesis the later method is employed.

Based on the practical test it can be observed that the pressure signals are coupled with noise. The position signal delivered by the absolute-encode is so precise and smooth that it can be viewed as noise free. Therefore, the estimation errors of $P_A$ and $P_B$ are filtered firstly. Then the residuals of the cylinder systems are defined as

$$r_{xc}(k) = x_c(k) - \hat{x}_c(k), r_{P_B}(k) = G_{f_1}(z^{-1})e_{P_B}(k) \quad r_{P_A}(k) = G_{f_2}(z^{-1})e_{P_A}(k),$$

where $G_{f_1}(z^{-1})$ and $G_{f_2}(z^{-1})$ are the discrete transfer functions of the filters. Here the $k$ denotes the residual signal of $k$th sampling time.

Step 1: Decision making logic
According to the structure of fault detection observer the connection between faults and residuals can be derived as shown in table (3.1), where 1 means affected and 0 is unaffected. It is obvious that only the residuals $r_{P_A}$ and $r_{P_B}$ are sensitive to the faults. The residual $r_{xc}$ is not affected by the faults because the velocity is a input signal for transformed system. As the internal leakage affects two residuals $r_{P_A}$ and $r_{P_B}$, the product of them $r_{Lin}$ is defined as the special residual to the leakage. The advantage of introducing the $r_{Lin}$ is that all useful information is concentrated on one signal, which can simplify the decision making procedure. Tab. 3.1 shows how the fault can be identified, where O means the alarm is triggered and × means not.

<table>
<thead>
<tr>
<th>Fault</th>
<th>Residual</th>
<th>$r_{xc}$</th>
<th>$r_{P_B}$</th>
<th>$r_{P_A}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{Lin}$</td>
<td></td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\Delta P_B$</td>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta P_A$</td>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.1: The relation between the faults and the residuals
Furthermore, timer is set for recording how long is the residual out of the range of the threshold. If the time interval $t_{re}$ is bigger than the predefined value $t_{pf}$. The fault alarm will be activated. The principle is illustrated in the Fig. 3.3.

Let $t_{r1}$, $t_{r2}$ and $t_{r3}$ be the recorded time intervals and $t_{pf1}$, $t_{pf2}$ and $t_{pf3}$ be the predefined values of $r_{PB}$, $r_{PA}$ and $r_{Lin}$ respectively. Three boolean signals are defined for decision making:

$$DML_{PB} = \begin{cases} 
1 & t_{r1} \geq t_{pf1} \quad \text{and} \quad DML_{QLin} = DML_{PA} = 0 \\
0 & \text{otherwise}
\end{cases}$$

(3.54)

$$DML_{PA} = \begin{cases} 
1 & t_{r2} \geq t_{pf2} \quad \text{and} \quad DML_{QLin} = DML_{PB} = 0 \\
0 & \text{otherwise}
\end{cases}$$

(3.55)

$$DML_{QLin} = \begin{cases} 
1 & t_{r3} \geq t_{pf3} \quad \text{and} \quad DML_{QA} = DML_{QPB} = 0 \\
0 & \text{otherwise}
\end{cases}$$

(3.56)

where 1 means alarm triggered and 0 means not. This DML is based on the assumption that the effect of the fault can be persistently reflected by the residual. It enable to
isolate three faults with only two real residuals. The predefined values can be obtained by simulations but should be adapted through experimental tests. Because the leakage affects the \( r_{PA} \) and \( r_{PB} \) both, \( t_{r3} \) should be smaller than the others. Otherwise the DML will regard it as a sensor fault. It is assumed only one fault is present. Therefore, as soon as one fault is identified, then the alarm for other faults will never be triggered.

### 3.3.4 Sensor total failure checking

It could happen for the sensors that the total information is lost during the operation. Such failure should be identified as soon as possible. So it is necessary to make a total failure checking for some important measurements by every sampling time. The rule is very simple: if the residual is extremely large, for example, ten times as the threshold, and the actual measured value is almost zero. Then it can be concluded that a total failure is happened. Such a failure has the highest priority and the alarm should be triggered immediately. In this thesis the total failure of \( P_A \) is considered. Because the cylinder works under high pressure, a sudden information lost could cause serious consequence. How should the controller react to sensor total failure \( P_A \) is discussed in chapter 4.

### 3.4 Simulation

#### 3.4.1 Simulation settings

![Figure 3.4: Characteristic curves of proportional directional valve](image)
Simulation model of the electro-hydraulic system for Matlab/Simulink is built based on the system description (3.42). External load is also imitated for testing the robustness of the fault generator. It is not exact same as the actual load pressure due to the modeling error of the proportional pressure relief valve. The internal leakage flow is simulated according to the data of characteristic curves of the proportional directional valve as shown in Fig. 3.4. The command value is set as 40%, whose curve possesses good linearity. Measurement noise of $P_A$ and $P_B$ signal is added for matching the real situation. The operation pressure is about 50 Bar. But for testing the robustness of the observer the external load changes until the value is around 100 Bar as shown in Fig. 3.5.

![Figure 3.5: External load](image)

### 3.4.2 Simulation results

#### 3.4.2.1 Case 1: fault free

The first simulation is executed for fault free case. The results are shown in Fig. 3.6 and Fig. 3.7.
The estimation tracks the measurement very well even the external load varies a lot. All the residuals are near zero. The value of $r_{PA}$ is much larger than other residuals due to the big value of $PA$ itself. It indicates that making selection of threshold should consider the range of the variables.

### 3.4.2.2 Case 2: with the internal leakage $Q_{Lin}$

Now the internal leakage is added in the simulation. The leakage flow is set as a ramp-like signal and saturates on 5 ($L/min$) as shown in Fig. 3.8.
Residuals $r_{PA}$, $r_{PB}$ and $r_{Lin}$ are affected by the leakage. These residuals deviates significantly deviated from the value range in fault free case. It can be said the residuals can indicate the fault occurrence quickly and clearly. The fault isolation is based on the designed DML.
3.4.2.3 Case 3: with the fault $\Delta P_B$

The sensor offset is a step-like signal and the value is 20% of the measurement.

![Figure 3.11: Estimation with fault $\Delta P_B$](image)

The residual $r_{PB}$ diverged obviously from zero after the sensor fault occurrence. The other residuals kept in the near of zero.

3.4.2.4 Case 4: with the fault $\Delta P_A$

The sensor offset is also step-like signal and the value is 20% of the measurement.
The fault $\Delta P_A$ can affect residual $r_{P_A}$. But the shape of residual is like a impulse, which is far away from zero only at the beginning of fault occurrence. After the transient phase, the residual $r_{P_A}$ returned back to zero, although the fault is still present. Following the definition in [CP99] the sensor fault $\Delta P_A$ is only detectable and not strong detectable by the $r_{P_A}$.

3.4.3 Conclusion

The simulation results shows that the designed fault detection observer is robust to the time-variant disturbance. The residuals are almost zero if the system is fault free. If
the fault occurs, the fault detection observer responds quickly and meanwhile keeps the boundedness. But the disturbance decoupling takes the cost of one measurement (cylinder velocity), which is a input signal for transformed system. This leads to that the residual $r_{x}$ is unaffected by the faults. The transformed system has a “bad” structure. Here “bad” means that the linear part of the system dynamics is zero, which results in the non-strong detectability of the fault $\Delta P_A$. Therefore, the DML should be modified according to the residual response to $\Delta P_A$. Then, the $DML_{P_A}$ in (3.55) is modified as

$$DML_{P_A} = \begin{cases} 1 & \tilde{t}_{pf2} \leq t_{r2} \leq \tilde{t}_{pf2} \quad (\tilde{t}_{pf2} > \tilde{t}_{pf2}) \\ 0 & \text{otherwise} \end{cases} \quad (3.57)$$

where $t_{r2}$ is the time interval, during which the $r_{P_A}$ always exceeds the threshold. The lower limit prevents from the false alarm caused by noise or other stochastic disturbances. The upper limit isolates the sensor offset from the leakage. The residuals now can be distinguished from each other, if different faults occur. Therefore, the discussed three faults can be detected and isolated by the proposed approach.

In the simulations the values of thresholds and parameters for DML are not presented. Based on the simulation results an optimal setting of these parameters is always possible. More details can be found in chapter 5, which shows the results of experimental tests.

### 3.5 Summary

This chapter gives the detailed design procedure of fault diagnosis system for the electro-hydraulic system. Firstly, the state of the art of model-based detection and isolation is introduced. The differential-geometric approach is employed for robust fault detection. With differential-geometric method a system transformation can be carried out. The transformed system is robust to system disturbance (the varying external load) but affected by the considered faults. Then the fault detection observer is designed for the transformed system. Furthermore, the faults can be isolated by the designed decision-making logic (DML). The simulation shows good results of the proposed fault diagnosis strategy. One problem is that after system transformation the measurement of position can not be handled as residual, because the velocity is an input of the transformed system.
4 Fault-tolerant control for the electro-hydraulic system

4.1 Introduction

4.1.1 Background

Fault-tolerant control (FTC) namely is a kind of reconfigurable control, which can maintain the control performance or at least, guarantee the stability of whole system in the event of faults. FTC is different from the conventional control. It contains not only a control law but also the fault diagnosis subsystem, which helps the reconfiguration of controller. The structure of FTC can be generally described as shown in Fig. 4.1. FTC system contains two layers: supervision and execution [BKLS03]. The supervision layer monitors the system behavior through the inputs, measurements and system model. The fault diagnosis block will give all the information about faults. The results are used by controller redesign mechanism to make an appropriate decision. The execution layer is similar to usual control loop. The feature is that the controller should be reconfigurable, which means the controller can react to the fault. With this architecture the reliability and safety of the system can be improved.

Figure 4.1: The structure of fault-tolerant control [BKLS03]
4.1.2 State-of-the-Art

FTC system has attracted more and more attention in industry and academia. Generally speaking FTC system can be divided into two classes: passive FTC and active FTC. The both are briefly reviewed below.

4.1.2.1 passive FTC

Passive FTC system is designed to achieve insensitivity to certain faults by means of making the system robust to them. The faults are usually modeled as the system uncertainties so that robust controller can be designed to guarantee the desired performance. The control system take into account all possible faults in design stage. If the controller exists, it is fixed. Basically the robust control approaches can be applied for passive FTC, for instance, robust $H_{\infty}$ approach in [ZR01], [NS03] and [CP01], quantitative feedback theory in [NS02], reliable control technique in [ZJ98], [Fer02].

The advantage of the passive FTC is that as long as the controller exists, the system can maintain the desired performance in all considered fault scenarios. The design procedure is carried out off-line. It is not constrained to real-time computation problem in implementation. Fault diagnosis system is not necessary. Therefore, the passive method is very applicable for the systems, which have only limited computational resource. However the design of passive FTC system is subject to all considered faults so that the designed controller is usually conservative. Usually the best control performance will be lost due to the conservatism, although the system runs mostly in nominal state.

4.1.2.2 active FTC

In contrast, active FTC systems react to faults actively by reconfiguring control actions so that the pre-specified performance of the entire system can be ensured. The reconfiguration mechanism is based on the latest fault information from a real-time fault diagnosis system and executed on-line. Various active FTC methods have been developed. Some of them are introduced here as overview.

**Pseudo inverse**

The pseudo inverse is one of the most used approaches due to its computational simplicity and its ability to handle a very large class of system faults. The basic version of pseudo inverse is derived as follows. Consider a nominal system

\[
\begin{align*}
x_{k+1} &= A_k x_k + B_k u_k \\
y_k &= c_k x_k.
\end{align*}
\]

(4.1)

with state-feedback control law $u_k = F_n x_k$. Furthermore, the description of the faulty
system (without sensor faults) is
\[ x_{k+1}^f = A_k^f x_k + B_k^f u_k^f, \]
\[ y_k^f = c_k^f x_k^f. \] (4.2)

The reconfigured controller is designed with the same structure, i.e. \( u_k^f = F^f x_k^f \). Now the problem is transformed to find the gain matrix \( F^f \), which should minimize the distance between the matrix \( (A_k + B_k F_n) \) and \( (A_k^f + B_k^f F_f) \). Namely,
\[
F_f = \arg \min_{F_f} \| (A_k + B_k F_n) - (A_k^f + B_k^f F_f) \|
\]
\[ = B^\dagger (A_k + B_k F_n - A_k^f), \] (4.3)

where \( B^\dagger \) is the pseudo-inverse of the matrix \( B_k^f \). The computation cost of this approach is low. It can be easily implemented for real-time applications. One of the serious disadvantages is that the optimal control law computed by (4.3) does not always stabilize the closed-loop system. Simple examples are demonstrated in [GA91]. Therefore, the modified passive method is developed to overcome this problem by introducing system constraints, which complicates the optimal solution. Other disadvantages are that all system states should be measurable and sensor fault and system uncertainties are not considered. The recent research in this area can be found in [YB00] and [KV00].

**Multiple Model**

The multiple model (MM) is based on a set of linear models \( M_i \) \( i = 1, 2, \ldots, N \). Each model represents a different faulty system. The key part of the multiple model approach is to design an on-line procedure, which determines the global control action through a (probabilistically) weighted combination of the different control actions that can be taken as described in [MS91] and [ZJ01]. The control inputs of each model are sometimes mixed, which is known as blending [MS97]. The mixing procedure is usually based on a bank of Kalman-Filters, in which each Kalman-Filter is designed for one of the local models \( M_i \). Through the residuals of the Kalman-Filters the probabilities \( \varsigma_i \) of each model in effect are computed. By weighting the control with probability factor the real input is constructed as
\[ u = \sum_{i=1}^{N} \varsigma_i u_i, \quad \sum_{i=1}^{N} \varsigma_i = 1 \]

The multiple model can be applied for nonlinear system or combined with different control strategies for achieving better performance, such as model predictive control, adaptive control, etc. Fault diagnosis system can also be easily integrated. The main disadvantage is the high computation burden for real-time implementation. Also the modeling uncertainties may be neglected.

**Model predictive control (MPC)**

The MPC is a kind of optimal control based on matching a prediction of the system
output to some desired reference trajectory. The optimization is executed at every time instant, which is the main difference from normal optimal control strategies. The MPC architecture allows fault-tolerance to be simply integrated by redefining the constraints to represent certain faults such as actuator faults or changing the internal model or changing the control objectives to reflect limitations due to the faulty components [AAE01]. Therefore, as soon as the fault is diagnosed the controller reconfiguration is accomplished automatically. More details about MPC based FTC can be found in [KM99], [MJ03].

Generally speaking, the MPC approach is very suitable for FTC. But it can not handle the modeling uncertainties and hardly can be applied for nonlinear systems.

**Adaptive control**

Adaptive methods based FTC has attracted many researchers due to its inherent self-reconfiguration capacity. The effect of the faults can be compensated by parameter adaption. Adaptive algorithms usually can not handle the sensor fault and the parameter estimation can not asymptotically converge to the actual value. Then an additional FDI system is necessary to assist the reconfiguration of the controller. This structure is also utilized by this thesis. Adaptive control can be applied to nonlinear system by combining nonlinear control techniques such as feedback linearization, backstepping and etc [TaXTJ04]. More references on this topic can be found in [AZIGR91], [IS98] [KTT03].

More approaches for active FTC can be found in references [ZJ08], [Jia05], [BKLS03], [JMZ03]. The active FTC system is more intelligent than the passive FTC system. The reconfiguration of the controller is carried out on-line according to the fault information. Hence, in fault free case the system possesses best performance. The serious problem of the active FTC is time delay caused by FDI. The time delay can not be avoided. Can system still be stable during the period that the fault occurs before it is identified? Therefore, the controller should be not only reconfigurable but also robust, at least for a short time. Thus, the performance of the active FTC also depends on the efficiency of FDI system. Another problem is high computation burden, which limits the active FTC strategies in practical applications.

**4.2 Backstepping based FTC**

The backstepping methodology is a recursive Lyapunov-based approach. By following a step-by-step algorithm feedback control laws and associated Lyapunov functions can be constructed systematically. A major advantage of backstepping is the flexibility to avoid cancellations of useful nonlinearities and achieve regulation and tracking properties [KKK95]. The backstepping controller can be reconfigured by integrating parameter adaption or changing the internal model. The system or actuator faults can be modeled as unknown parameters of the system. Hence, these faults can be tolerated by parameter adaption. The sensor faults are brought into the system by the controller and can be modeled as uncertainties of the controller parameters. Hence, the system model for
controller design should be changed so that the effect of the sensor fault can be compensated or eliminated. An alternative for dealing with the sensor fault is to replace the faulty measurement by the estimation of an observer, in which the faulty measurement is abandoned. In this case the observer-based control structure is turned out. For the electro-hydraulic system it is impossible to construct such an observer due to the decoupling of the external load. Therefore, in this thesis only the first method is under consideration. The details of construction the backstpping controller and reconfiguration for the fault accommodation are shown in following.

4.2.1 Preliminaries

Firstly, some basic knowledge about nonlinear systems is reviewed for completeness and convenience [KKK95].

4.2.1.1 $\mathcal{K}$ and $\mathcal{KL}$ function

Definition 4.2.1. A continuous function $\gamma : [0, a) \to \mathbb{R}_+$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\gamma(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\gamma(r) \to \infty$ as $r \to \infty$.

Definition 4.2.2. A continuous function $\beta : [0, a) \times \mathbb{R}_+ \to \mathbb{R}_+$ is said to belong class $\mathcal{KL}$ if for each fixed $s$ the mapping $\beta(r, s)$ belong to class $\mathcal{K}$ with respect to $r$, and for each fixed $r$ the mapping $\beta(r, s)$ is decreasing with respect to $s$ and $\beta(r, s) \to 0$ as $s \to \infty$. It is said to belong to class $\mathcal{KL}_\infty$.

4.2.1.2 Stability

Consider the nonautonomous system

$$\dot{x} = f(x, t), \quad x(t) \in \mathbb{R}^n$$

where $x(t_0) = x_0$ and $f(0, t) = 0, \forall t \geq t_0$. Then the stability properties of the equilibrium point $x_e = 0$ are defined as follows.

Definition 4.2.3. The $x_e = 0$ of system (4.4) is

uniformly stable , if there exists a class $\mathcal{K}$ function $\gamma(\cdot)$ and a positive constant $c$, independent of $t_0$, such that

$$|x(t)| \leq \gamma(|x(t)|), \quad \text{with} \quad t \geq t_0 \geq 0 \text{ and } |x(t)| < c; \quad (4.5)$$

uniformly asymptotically stable, if there exists a class $\mathcal{KL}$ function $\beta(\cdot, \cdot)$ and a positive constant $c$, independent of $t_0$, such that

$$|x(t)| \leq \beta(|x(t_0)|, t - t_0), \quad \text{with} \quad t \geq t_0 \geq 0 \text{ and } |x(t)| < c; \quad (4.6)$$
exponentially stable, if (4.6) is satisfied with
\[ \beta(r, s) = kre^{-\rho s}, \quad \text{with} \quad k > 0 \text{ and } \rho > 0. \] (4.7)

Then the LaSalle-Yoshizawa theorem can be introduced here.

**Theorem 4.2.1. (LaSalle-Yoshizawa Theorem)** Let \( x = 0 \) be an equilibrium point of (4.4) and suppose \( f \) is locally Lipschitz in \( x \) uniformly in \( t \). Let \( V : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}_+ \) be a continuously differentiable function such that
\[
\gamma_1(|x(t)|) \leq V(x, t) \leq \gamma_2(|x(t)|)
\]
\[
\dot{V} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x, t) \leq -W(x) \leq 0
\]
where \( \gamma_1 \) and \( \gamma_2 \) are class \( K_\infty \) functions and \( W \) is a continuous function. Then, all solutions of (4.4) are globally uniformly bounded and satisfy
\[
\lim_{t \to \infty} W(x(t)) = 0.
\] (4.10)

If \( W(x) \) is positive definite, then the equilibrium is globally uniformly asymptotically stable.

The proof [KKK95] is omitted here.

**4.2.2 Standard backstepping**

The idea of backstepping is to design a controller recursively by considering some of the state variables as “virtual controls” and designing for them intermediate control laws. The derivation of standard backstepping can be illustrated by the following example.

Assume a second-order system
\[
x_1 = f_1(x_1) + g_1(x_1)x_2
\]
\[
x_2 = f_2(x_1, x_2) + g_2(x_1, x_2)u,
\] (4.11)
where \( g_i \neq 0 \) with \( i = 1, 2 \). The reference trajectory of \( x_1 \) is denoted by \( x_r \) and its second order derivatives are piecewise continuous and bounded. The control input \( u \) should guarantee that
\[
\lim_{t \to \infty} (x_1 - x_r) = 0.
\] (4.12)

The design procedure is illustrated as follows.
**Step 1:** Introduce $e_1 = x_1 - x_r$ and $e_2 = x_2 - \beta_2$, where $\beta_2$ is a stabilizing function to be designed. Let the Lyapunov function $V_1 = \frac{1}{2}\rho_1 e_1^2$ ($\rho_1 > 0$), then the derivative is

$$
\dot{V}_1 = \rho_1 e_1 \dot{e}_1 = \rho_1 e_1 \left[ f_1(x_1) + g_1(x_1)x_2 - \dot{x}_r \right].
$$

(4.13)

Substitute $e_2 = x_2 - \beta_2$ into (4.13)

$$
\dot{V}_1 = \rho_1 e_1 (f_1(x_1) + g_1(x_1)e_2 + g_1(x_1)\beta_2 - \dot{x}_r).
$$

(4.14)

Choose the stabilizing function as $\beta_2 = \frac{1}{g_1(x_1)} \left[ -f_1(x_1) + \dot{x}_r - k_1 e_1 \right]$ with $k_1 > 0$.

(4.15)

Then the derivative of $V_1$ is

$$
\dot{V}_1 = -\rho_1 k_1 e_1^2 + \rho_1 g_1(x_1)e_1 e_2.
$$

(4.16)

So far the $\dot{V}_1 \leq 0$ is not ensured due to the term $\rho_1 g_1(x_1)e_1 e_2$.

**Step 2:** Now augment the Lyapunov function as $V_2 = V_1 + \frac{1}{2}\rho_2 e_2^2$ ($\rho_2 > 0$). The derivative of $V_2$ is

$$
\dot{V}_2 = \dot{V}_1 + \rho_2 e_2 \dot{e}_2 = -\rho_1 k_1 e_1^2 + \rho_1 g_1(x_1)e_1 e_2 + \rho_2 e_2 \left[ f_2(x_1,x_2) + g_2(x_1,x_2)u - \dot{\beta}_2 \right],
$$

(4.17)

where $u$ is chosen to ensure $\dot{V}_2 \leq 0$ that leads to

$$
u = \frac{1}{g_2(x_1,x_2)} \left[ -f_2(x_1,x_2) - \frac{\rho_1}{\rho_2} g_1(x_1)e_1 + \dot{\beta}_2 - k_2 e_2 \right] \text{ with } k_2 > 0,
$$

(4.18)

where

$$
\dot{\beta}_2 = \frac{1}{g_1} \left[ g_1 \left( -f_1 - k_1 \dot{e}_1 + \dot{x}_r \right) - \dot{g}_1 \left( -f_1 - k_1 e_1 + \dot{x}_r \right) \right].
$$

Then the derivative of $V_2$ is

$$
\dot{V}_2 = -\rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2.
$$

(4.19)

Now it can be obtained

$$
\dot{V}_2 \leq 0 \quad \text{and} \quad V_2 = 0 \quad \text{only by} \quad e_1 = e_2 = 0.
$$

(4.20)

Because $g_i \neq 0$ and $x_r$ are second order differentiable, the designed $u$ can be obtained and is bounded. By applying the LaSalle-Yoshizawa theorem, the global uniform asymptotic stability of $e_1$ and $e_2$ is guaranteed. It follows that

$$
\lim_{t \to \infty} e_i = 0.
$$

(4.21)
Then the desired tracking performance in (4.12) can be achieved.

This recursive design procedure can be easily extended to $n$-th order system. Assume the nonlinear system has the structure

\[ \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 \\
\vdots \\
\dot{x}_{n-1} = f_{n-1}(x_1, \cdots, x_{n-1}) + g_{n-1}(x_1, \cdots, x_{n-1})x_n \\
\dot{x}_n = f_n(x_1, \cdots, x_n) + g_n(x_1, \cdots, x_n)u \]

(4.22)

with $g_i \neq 0$ ($i = 1, 2, \cdots, n$). In addition, system with such a form is known as strict-feedback system. The reference trajectory $x_r$ is $n$th order differentiable. Define

\[ e_i = x_i - \beta_i, \]

(4.23)

where $\beta_1 = x_r$ and $\beta_i (i > 1)$ are the virtual control signals, which are determined in the recursive procedure.

The Lyapunov function is

\[ V = \frac{1}{2} \sum_{i=1}^{n} \rho_i e_i^2 \quad \text{with} \quad \rho_i > 0, \ i = 1, 2, \cdots, n. \]

(4.24)

The control input based on backstepping approach can be recursively derived as

\[ u = \frac{1}{g_n} \left( -f_n - \frac{\rho_{n-1}}{\rho_n} g_{n-1}e_{n-1} - k_n e_n + \dot{\beta}_n \right) \quad (k_i > 0) \]

(4.25)

Then the derivative of the Lyapunov function (4.24) is

\[ \dot{V} = -\sum_{i=1}^{n} \rho_i k_i e_i^2. \]

(4.26)

Thus, the tracking error is asymptotic stable at the origin according to LaSalle-Yoshizawa theorem.

### 4.2.3 Adaptive backstepping

Adaptive techniques can be easily inserted in backstepping design procedure. Assume a second order system with an unknown, constant parameter. The parameter appears in the system linearly. The system is described as

\[ \dot{x}_1 = f_1(x_1)\theta_1 + g_1(x_1)x_2 \\
\dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)u \]

(4.27)
with $g_i \neq 0 \ (i = 1, 2)$ and second order differentiable reference trajectory $x_r$. $\theta_1$ stands for the unknown parameter. The control input $u$ should ensure that the tracking error $x_1 - x_r$ converges to zero. Carry out the similar procedure as the illustration example (4.11).

**Step 1:** Introduce $e_1 = x_1 - x_r$, $e_2 = x_2 - \beta_2$ and $e_{\theta_1} = \theta_1 - \hat{\theta}_1$, where $\hat{\theta}_1$ is the estimation of $\theta_1$. Let the Lyapunov function $V_i = \frac{1}{2} \rho_1 e_1^2$, then the derivative is

$$
\begin{align*}
\dot{V}_1 &= \rho_1 e_1 \dot{e}_1 \\
&= \rho_1 e_1 [\theta_1 f_1(x_1) + g_1(x_1) x_2 - \dot{x}_r]
\end{align*}
$$

(4.28)

Choose the stabilizing function as

$$
\beta_2 = \frac{1}{g_1(x_1)} [-\theta_1 f_1(x_1) + \dot{x}_r - k_1 e_1] \quad \text{with} \quad k_1 > 0.
$$

(4.29)

But the parameter $\theta_1$ is unknown, it is substituted by its estimation $\hat{\theta}_1$. Then

$$
\beta_2 = \frac{1}{g_1(x_1)} [-\hat{\theta}_1 f_1(x_1) + \dot{x}_r - k_1 e_1].
$$

(4.30)

That leads to

$$
\dot{V}_1 = -\rho_1 k_1 e_1^2 + \rho_1 g_1(x_1) e_1 e_2 + \rho_1 f_1(x_1) e_1 e_{\theta_1}.
$$

(4.31)

**Step 2:** Now augment the Lyapunov function as $V_2 = V_1 + \frac{1}{2} \rho_2 e_2^2 + \frac{1}{2} \rho_\theta e_{\theta_1}^2$. The derivative of $V_2$ is

$$
\begin{align*}
\dot{V}_2 &= \dot{V}_1 + \rho_2 e_2 \dot{e}_2 + \rho_\theta e_{\theta_1} \dot{\theta}_1 \\
&= -\rho_1 k_1 e_1^2 + \rho_1 g_1(x_1) e_1 e_2 + \rho_1 f_1(x_1) e_1 e_{\theta_1} \\
&\quad + \rho_2 e_2 \left[ f_2(x_1, x_2) + g_2(x_1, x_2) u - \beta_2 \right] + \rho_\theta e_{\theta_1} \dot{\theta}_1.
\end{align*}
$$

(4.32)

Choose the control input as

$$
u = \frac{1}{g_2(x_1, x_2)} \left[ -f_2(x_1, x_2) - \frac{\rho_1}{\rho_2} g_1(x_1) e_1 + \beta_2 - k_2 e_2 \right] \quad \text{with} \quad k_2 > 0.
$$

(4.33)

Then the derivative of Lyapunov function $V_2$ is

$$
\dot{V}_2 = -\rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2 + \rho_1 f_1(x_1) e_1 e_{\theta_1} + \rho_\theta e_{\theta_1} \dot{\theta}_1.
$$

(4.34)

To guarantee a non-positive $\dot{V}_2$ for arbitrary $\theta_1$, the updating law of $\theta_1$ is designed to make the sum of last two terms in (4.34) is zero. Because $\theta_1$ is constant, then $\dot{e}_{\theta_1} = -\dot{\theta}_1$. 

Thus, the adaption law of $\theta_1$ is

$$\dot{\theta}_1 = \frac{\rho_1}{\rho_{\theta_1}} e_1 f_1(x_1)$$  \hspace{1cm} (4.35)$$

That leads to $\dot{V}_2 \leq 0$ and $\dot{V}_2 = 0$ only if $e_1 = e_2 = 0$. The tracking error is asymptotic stable on origin even with the unknown parameter in system. The estimation error $e_{\theta_1}$ is bounded based on the LaSalle-Yoshizawa.

This procedure can be generalized to the parametric strict-feedback system, which has the following form

$$\dot{x}_1 = f(x_1) + g_1(x_1)x_2 + \Phi_1^T(x_1)\theta$$
$$\dot{x}_2 = f(x_1, x_2) + g_2(x_1, x_2)x_3 + \Phi_2^T(x_1, x_2)\theta$$
$$\vdots$$
$$\dot{x}_{n-1} = f(x_1, \cdots, x_{n-1}) + g_{n-1}(x_1, \cdots, x_{n-1})x_n + \Phi_{n-1}^T(x_1, \cdots, x_{n-1})\theta$$
$$\dot{x}_n = f_n(x_1, \cdots, x_n) + \Phi_n^T(x_1, \cdots, x_n)\theta + g_n(x_1, \cdots, x_n)u$$

where $\theta$ is the unknown parameter vector, which is constant or changes slowly. $g_i \neq 0$ and is known with $i = 1, 2, \cdots, n$. The reference $x_r$ is $n$th order differentiable. Then the Lyapunov function can always be constructed as

$$V_a = \frac{1}{2} \sum_{i=1}^{n} \rho_i e_i^2 + \frac{1}{2} e_\theta^T \Gamma e_\theta$$  \hspace{1cm} (4.37)$$

where $e_i = x_i - \beta_i$, $\beta_1 = x_r$ and $\beta_i (i > 1)$ are the stabilizing functions, which are determined by backstepping procedure are coupled with the estimation of parameters. $\Gamma$ is a positive definite matrix. Then there exists a control input $u$ to ensure that

$$\dot{V}_a = - \sum_{i=1}^{n} \rho_i k_i e_i^2$$  \hspace{1cm} (k_i > 0),$$

which implies that the tracking error will asymptotically converge to zero. The estimation error $e_\theta$ is bounded. The proof is also based on the LaSalle-Yoshizawa theorem. It is noted that $e_\theta$ is only bounded and but not asymptotically approaches to zero.

### 4.2.4 Backstepping based FTC for electro-hydraulic system

The backstepping method is applied to realize the position tracking for the electro-hydraulic system, which is subject to the varied external load $F_{load}$ and the considered faults $Q_{Lin}$, $\Delta P_A$ and $\Delta P_B$. $F_{load}$ and $Q_{Lin}$ can be viewed as parameter uncertainties of the system. So the adaptive backstepping should be employed to tolerate these uncertainties. The sensor faults $\Delta P_A$ and $\Delta P_B$ affect the system through the backstepping
controller itself. Some parameters of system model are dependent on the measurements of $P_A$ and $P_B$, such as $E_A(P_A)$ and $E_B(P_B)$. Then the system model must be adapted for reconfiguration of the backstepping controller.

The structure of backstepping based FTC for the electro-hydraulic system is shown in Fig. 4.2. In fault free case the adaptive backstepping is utilized for compensation of the variation of $F_{load}$. If the fault is present, the controller is reconfigured based on a new model. Which model should be adapted is dependent on the up-to-date information of the fault from FDI system. The new model usually is not so precise as the model in nominal state. It may cause performance degradation.

![Figure 4.2: The structure of backstepping based FTC for electro-hydraulic system](image)

### 4.2.4.1 System transformation

It is necessary for the system to be formulated into strict-feedback or parametric-strict-feedback structure for the implementation of backstepping approach. Recall the model of the electro-hydraulic system

\[
\begin{align*}
\dot{x}_c &= v_c \\
\dot{v}_c &= \frac{1}{m}(P_AA_1 - P_BA_2 - Dvv_c - f_c - F_{load}) \\
\dot{P}_B &= \frac{E_B(P_B)}{V_{20} - A_2x_c} \left( A_2v_c - \sqrt{k_{q1}(P_B - P_o) + k_{q2} + Q_{Lin}} \right) \\
\dot{P}_A &= \frac{E_A(P_A)}{V_{10} + A_1x_c}(-A_1v_c - Q_{Lin} + K_Q\alpha) \\
\dot{\alpha} &= v_\alpha \\
\dot{v}_\alpha &= -p_1\alpha - p_2v_\alpha + p_1k_{pu}u_{in}.
\end{align*}
\]  

(4.39)
Assume a new state variable $F_{HY} = P_A A_1 - P_B A_2$. Then the original system (4.39) can be rewritten as

\[
\begin{align*}
\dot{x}_c &= v_c \\
\dot{v}_c &= \frac{1}{m} (F_{HY} - D_v v_c - f_c - F_{load}) \\
\dot{F}_{HY} &= \frac{E_A(P_A)}{V_{10} + A_1 x_c} (-A_1 v_c - Q_{Lin} + K_Q \alpha) \\
&\quad - \frac{E_B(P_B)}{V_{20} - A_2 x_c} (A_2 v_c - \sqrt{k_{q1} (P_B - P_o) + k_{q2} + Q_{Lin}}) \\
\dot{\alpha} &= v_\alpha \\
\dot{v}_\alpha &= -p_1 \alpha - p_2 v_\alpha + p_1 k_{pu} u_{in}.
\end{align*}
\]

It is obvious that new system possesses the strict-feedback structure. For simplicity the dynamics of $F_{HY}$ is given by

\[
\dot{F}_{HY} = f(x_c, v_c) + g(x_c)\alpha,
\]

where $E_A(P_A)$, $E_B(P_B)$ and $\sqrt{k_{q1} (P_B - P_o) + k_{q2}}$ are viewed as known parameters. According to the actual parameters of the testbed, $g(x_c) \neq 0$ can be obtained.

### 4.2.4.2 Reference trajectories

The goal of control is to realize precise tracking position with good transient performance. With the consideration of hardware constraints the reference trajectory of position is designed as

\[
x_r = 5t - 40 \sin(0.125t) + x_0, \quad (mm)
\]

where $x_0$ is the initial position. It is obvious that this trajectory is always differentiable. The corresponding reference velocity is

\[
v_r = 5 - 5 \cos(0.125t) \quad (mm/sec.).
\]

If the cylinder moves along the reference trajectory, it can reach the final position very gently.

### 4.2.4.3 Controller for fault free case

In this section the position tracking controller for fault free case is derived.

**Step 1:** Define

\[
\begin{align*}
e_1 &= x_c - x_r, & e_2 &= v_c - \beta_2, & e_3 &= F_{HY} - \beta_3 \\
e_4 &= \alpha - \beta_4, & e_5 &= v_\alpha - \beta_5.
\end{align*}
\]
with $\beta_i$ ($i = 2, 3, 4, 5$) are the stabilizing functions. Then the Lyapunov function in this step is

$$V_1 = \frac{1}{2} \rho_1 e_1^2 \quad (\rho_1 > 0).$$

(4.45)

Then the derivative of $V_1$ is

$$\dot{V}_1 = \rho_1 e_1 \dot{e}_1 = \rho_1 e_1 (\beta_2 + e_2 - \dot{x}_r).$$

(4.46)

Choose $\beta_2 = -k_1 e_1 + \dot{x}_r$, with $k_1 > 0$. This follows that

$$\dot{V}_1 = -\rho_1 k_1 e_1^2 + \rho_1 e_1 e_2.$$

(4.47)

Step 2: Define the Lyapunov function of this step

$$V_2 = V_1 + \frac{1}{2} \rho_2 e_2^2 \quad (\rho_2 > 0).$$

(4.48)

The derivative of $V_2$ is

$$\dot{V}_2 = -\rho_1 k_1 e_1^2 + \rho_1 e_1 e_2 + \rho_2 e_2 \dot{e}_2$$

$$= -\rho_1 k_1 e_1^2 + \rho_1 e_1 e_2 + \rho_2 e_2 \left(-\frac{D_v}{m} v_c + \frac{1}{m} (\beta_3 + e_3) - \frac{1}{m} F_{load} - \dot{\beta}_2 \right)$$

(4.49)

Assume the $F_{load}$ is known. $\beta_3$ is chosen as

$$\beta_3 = D_v v_c + f_c + F_{load} - \frac{\rho_1}{\rho_2} m e_1 - m k_2 e_2 + m \dot{\beta}_2 \quad (k_2 > 0)$$

This leads to

$$\dot{V}_2 = -\rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2 + \frac{\rho_2}{m} e_2 e_3.$$  

(4.50)

The solution (4.49) is based on the constraint that the external load $F_{load}$ is known. Now this constraint is released by employing parameter adaption.

Define $\theta_1 = F_{load}$ and the estimation error $e_{\theta_1} = \theta_1 - \dot{\theta}_1$. Augment the Lyapunov function $V_2$ as

$$V_{2a} = V_2 + \frac{1}{2} \rho_{\theta_1} e_{\theta_1}^2 \quad \rho_{\theta_1} > 0.$$  

(4.51)

The derivative of $V_{2a}$ is

$$\dot{V}_{2a} = -\rho_1 k_1 e_1^2 + \rho_1 e_1 e_2 + \rho_2 e_2 \left(-\frac{D_v}{m} v_c + \frac{1}{m} (\beta_3 + e_3) - \frac{1}{m} (e_{\theta_1} + \dot{\theta}_1) - \dot{\beta}_2 \right)$$

(4.52)
Then the $\beta_3$ is rewritten as
\[ \beta_3 = D_v v_c + f_c + \hat{\theta}_1 - \frac{p_1}{p_2} m e_1 - m k_2 e_2 + m \dot{\beta}_2 \] (4.53)
with the adaption law of $\hat{\theta}_1$
\[ \dot{\hat{\theta}}_1 = - \frac{p_2}{m \rho_3} e_2 \] (4.54)

**Step 3:** Define the Lyapunov function of this step
\[ V_3 = V_2 + \frac{1}{2} \rho_3 e_3^2 \quad (\rho_3 > 0). \] (4.55)
The derivative is
\[ \dot{V}_3 = - \rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2 + \frac{\rho_2}{m} e_2 e_3 + \rho_3 e_3 \left[ f(x_c, v_c) + g(x_c)(\beta_4 + e_4) \right]. \] (4.56)
Because $g(x_c) \neq 0$, the virtual control signal $\beta_4$ can be designed as
\[ \beta_4 = - \frac{1}{g(x_c)} \left( - f(x_c, v_c) - \frac{\rho_2}{\rho_3 m} e_2 - k_3 e_3 + \dot{\beta}_3 \right) \quad (k_3 > 0). \] (4.57)
This leads to
\[ \dot{V}_3 = - \rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2 - \rho_3 k_3 e_3^2 + \rho_3 g(x_c) e_3 e_4. \] (4.58)

**Step 4:** Define the Lyapunov function of this step
\[ V_4 = V_3 + \frac{1}{2} \rho_4 e_4^2 \quad (\rho_4 > 0). \] (4.59)
Then the derivative is
\[ \dot{V}_4 = - \rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2 - \rho_3 k_3 e_3^2 + \rho_4 e_4 \left( \beta_5 + e_5 + \dot{\beta}_4 \right). \] (4.60)
Choose $\beta_5$ as
\[ \beta_5 = - \frac{\rho_3}{\rho_4} g(x_c) e_3 - k_4 e_4 + \dot{\beta}_4 \quad (k_4 > 0) \] (4.61)
This leads to
\[ \dot{V}_4 = - \rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2 - \rho_3 k_3 e_3^2 - \rho_4 k_4 e_4^2 + \rho_4 e_4 e_5. \] (4.62)

**Step 5:** Define the Lyapunov function of this step
\[ V_5 = V_4 + \frac{1}{2} \rho_5 e_5^2 \quad (\rho_5 > 0). \] (4.63)
The derivative is
\[
\dot{V}_5 = -\rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2 - \rho_3 k_3 e_3^2 - \rho_4 k_4 e_4^2 + \rho_4 \varepsilon_5 + \rho_5 \varepsilon_5 (-p_1 \alpha - p_2 v_\alpha + p_1 k_{pu} u_in - \dot{\beta}_5). \tag{4.64}
\]

Then the stabilizing control input is
\[
u_{in} = \frac{1}{p_1 k_{pu}} \left( p_1 \alpha + p_2 v_\alpha - \frac{\rho_4}{\rho_5} e_4 - k_5 \varepsilon_5 + \dot{\beta}_5 \right) \quad (k_5 > 0) \tag{4.65}
\]

This leads to
\[
\dot{V}_5 = -\rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2 - \rho_3 k_3 e_3^2 - \rho_4 k_4 e_4^2 - \rho_5 k_5 e_5^2. \tag{4.66}
\]

With designed control input (4.65) the position tracking error will asymptotically converge to zero.

It is noted that for deriving the final control input the derivative of the stabilizing function should be available. It can be calculated analytically. However, this is a fifth order system. The calculation is very cumbersome and is not suitable for real-time implementation. Therefore, the numeric differential is applied. Sometimes the calculated stabilizing function is so large that the value is far away from the limits of the corresponding state variable. Thus, the values of the stabilizing functions are restricted according to the physical constraints.

### 4.2.4.4 The case with leakage fault $Q_{Lin}$

If the internal leakage occurs, the model is different from nominal model. It can be described as
\[
\dot{F}_{HY} = f(x_c, v_c) + \psi(x_c) \theta_2 + g(x_c) \alpha, \tag{4.67}
\]

where $\theta_2$ denotes the leakage $Q_{Lin}$ in model (3.43). $\theta_2$ is an unknown constant and appears linearly in equation (4.67). Thus, the parameter adaption can be utilized to compensate the effect of the leakage.

The new controller design procedure is similar to the one in fault free case. The parameter adaption law is inserted in $V_3$. Define the estimation error $e_{\theta_2} = \theta_2 - \hat{\theta}_2$. The new Lyapunov function is
\[
V_{3a} = V_3 + \frac{1}{2} \rho_{\theta_2} e_{\theta_2}^2 \quad (\rho_{\theta_2} > 0). \tag{4.68}
\]

Then the derivative is
\[
\dot{V}_{3a} = -\rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2 + \frac{\rho_4}{m} e_2 e_3 + \rho_{\theta_2} e_{\theta_2} \dot{e}_{\theta_2}
+ \rho_3 e_3 \left[ f(x_c, v_c) + \psi(x_c)(\dot{\theta}_2 + e_{\theta_2}) + g(x_c)(\beta_4 + e_4) \right]. \tag{4.69}
\]
The stabilizing function is chosen as

\[ \beta_4 = \frac{1}{g(x_c)} \left( -f(x_c, v_c) - \psi(x_c) \dot{\theta}_2 - \frac{\rho_2}{\rho_3 m} e_2 - k_3 e_3 + \dot{\beta}_3 \right) \quad (k_3 > 0), \]  

(4.70)

with the adaption law of \( \dot{\theta}_2 \)

\[ \dot{\theta}_2 = \frac{\rho_3 \psi(x_c)}{\rho_0 g(x_c)} e_3. \]  

(4.71)

This leads to

\[ \dot{V}_{sa} = \dot{V}_3 = -\rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2 - \rho_3 k_3 e_3^2 + \rho_3 g(x_c) e_3 e_4. \]  

(4.72)

The rest of the design is the same as the no fault case. Then the asymptotic convergence of the position tracking error can be guaranteed even with the internal leakage in system.

### 4.2.4.5 The case with the sensor fault \( \Delta P_B \)

For eliminating the effect of \( \Delta P_B \) the system model should be adapted. The simplest method is to assume the dynamics of \( P_B \) is zero at every sampling time. That leads to \( Q_B = A_2 v_c \). Then the \( P_B \) can be approximated as

\[
P_{B_{\text{app}}} = \begin{cases} 
\frac{(A_2 V_c)^2 - k_{ql2}}{k_{q1}} + P_0 & \text{if } (A_2 V_c)^2 - k_{ql2} \geq 0, \\
P_0 & \text{if } (A_2 V_c)^2 - k_{ql2} < 0,
\end{cases}
\]  

(4.73)

where the parameters have the same meaning as in equations (2.9) and (2.10). Furthermore, \( F_{HY} = P_A A_1 + P_{B_{\text{app}}} A_2 \) and \( \dot{F}_{HY} = P_A A_1 \). This approximation can degrade the performance of adaptive backstepping. But the value of pressure \( P_B \) is very small compared to the pressure \( P_A \). Therefore, this impact can be ignored. The rest of design is same as the no fault case.

### 4.2.4.6 The case with the sensor fault \( \Delta P_A \)

The Sensor fault \( \Delta P_A \) can bring more serious consequence than \( \Delta P_B \), especially if the system is running under high pressure. Fortunately, the reconfiguration can be easily accomplished thanking to the structure of adaptive backstepping. The \( \Delta P_A \) affects two parameters of the backstepping controller: error term \( e_3 \) and the bulk modulus \( E_A(P_A) \). Firstly, it can be proved that the sensor offset on error \( e_3 \) can be compensated by the adaption law of \( \dot{\theta}_1 \). Let \( \dot{F}_{HY} = F_{HY} + \Delta F_{HY} \), where \( \Delta F_{HY} = \Delta P_A A_1 \). Design a new stabilizing function \( \beta_{3a} \), which satisfies that \( e_3 = \dot{F}_{HY} - \beta_{3a} \). It leads to

\[ F_{HY} = \beta_{3a} + e_3 - \Delta F_{HY}. \]  

(4.74)
Recall the derivative of $V_{2a}$ (4.52)

$$
\dot{V}_{2a} = -\rho_1 k_1 e_1^2 + \rho_1 e_1 e_2 + \rho \theta_1 e_\theta \dot{\theta}_\theta + \rho_2 e_2 \left( -\frac{D_v}{m} v_c + \frac{1}{m} F_{HY} - \frac{1}{m} \theta_1 + \dot{\beta}_2 \right).
$$

(4.75)

Substitute (4.74) into $\dot{V}_{2a}$. Then

$$
\dot{V}_{2a} = -\rho_1 k_1 e_1^2 + \rho_1 e_1 e_2 + \rho \theta_1 e_\theta \dot{\theta}_\theta + \rho_2 e_2 \left( -\frac{D_v}{m} v_c + \frac{1}{m} (\beta_{3a} + e_3) - \frac{1}{m} (\theta_1 + \Delta F_{HY}) - \dot{\beta}_2 \right).
$$

(4.76)

$\Delta F_{HY}$ is a step-like signal and its derivative can be viewed as zero. So if define $\theta_{1a} = \theta_1 + \Delta F_{HY}$ then it can be estimated by the same adaption law as $\theta_1$. Also $\beta_{3a}$ can be obtained as

$$
\beta_{3a} = D_v v_c + f_c + \hat{\theta}_1 + \Delta \hat{F}_{HY} - \frac{\rho_1}{\rho_2} m e_1 - m k_2 e_2 + m \dot{\beta}_2
$$

(4.77)

It leads to $\hat{F}_{HY} - \beta_{3a} \approx F_{HY} - \beta_3 = e_3$. It can be concluded that the error term $e_3$ is almost unaffected by $\Delta P_A$.

The second affected parameter $E_A(P_A)$ by the fault approximated by a constant. The calculation of $E_A(P_A)$ can not be accurate, when the value of $P_A$ is wrong. Furthermore, if the pressure is high enough, then the bulk modulus can be viewed as a constant.

Thus, the reconfiguration for $\Delta P_A$ is accomplished by setting the bulk modulus $E_A(P_A)$ as constant.

### 4.3 Simulation

The backstepping based FTC for all discussed fault scenarios is simulated in this section. Actually the fault diagnosis system developed in chapter 3 has been integrated in the FTC scheme. But the detailed results about fault diagnosis are omitted here for avoiding presentation of similar contents as chapter 3. In this section the control performance is the highlight and is presented in details. Average errors of position and velocity denoted by $\bar{e}_{xc}$ and $\bar{e}_{vc}$ are introduced for evaluating the control performance.
4.3.1 Scenario 1: fault free

Fig. 4.3 shows excellent tracking performances of proposed backstepping controller in the fault free case. The small “peak” in the velocity trajectory is caused by the variation of load force. The position error converges to zero very soon due to the parameter adaption of the load force. The estimation of $F_{\text{load}}$ by adaptive controller has a bounded error with the actual value. The average errors are $\bar{e}_{x_c} = 0.3086 \text{ (mm)}$ and $\bar{e}_{v_c} = 0.078 \text{ (mm/sec.)}$. 

![Figure 4.3: Tracking control (fault free)](image)

![Figure 4.4: Estimation of $F_{\text{load}}$ (fault free)](image)
4.3 Simulation

4.3.2 Scenario 2: with the internal leakage $Q_{Lin}$

Figure 4.5: Tracking control (with $Q_{Lin}$)

Figure 4.6: Estimation of $F_{load}$ (with $Q_{Lin}$)
The internal leakage $Q_{Lin}$ is activated at $t = 12$ second. It is detected and isolated in about 5 seconds. It can be seen from Fig. 4.6 the control performance is almost same as in the fault free case. The fault $Q_{Lin}$ is tolerated by the reconfigured adaptive controller. The estimation errors of $F_{load}$ and $Q_{Lin}$ are bounded but not converges to zero. The average errors are $\bar{e}_{x_c} = 0.315 \text{ (mm)}$ and $\bar{e}_{v_c} = 0.078 \text{ (mm/sec)}$.

### 4.3.3 Scenario 3: with the sensor offset $\Delta P_B$

Figure 4.7: Estimation of $Q_{Lin}$

Figure 4.8: Tracking control (with $\Delta P_B$)
The sensor fault $\Delta P_B$ is set as 20% of the measurement. It is added in measurement at $t = 12$ second. Although the effect of $\Delta P_B$ is not significant to the system, it is detected in 0.2 seconds. The tracking error is similar to the fault free case with $\bar{e}_{x_c} = 0.310 \, (mm)$ and $\bar{e}_{v_c} = 0.076 \, (mm/sec)$. The estimation of $F_{load}$ is also almost same as in the fault free case, so the figure is not present here.

### 4.3.4 Scenario 4: with the sensor offset $\Delta P_A$

![Figure 4.9: Tracking control (with $\Delta P_A$)](image)

![Figure 4.10: Estimation of $F_{load}$ (with $\Delta P_A$)](image)
In this case the sensor offset $\Delta P_A$ is also set as 20% and added to the measurement at $t = 12$ second. The fault is detected in almost 0.1 second. It can be observed in Fig. 4.10 the effect of $\Delta P_A$ is viewed as the uncertainty of the load by the adaptive controller. Thus control performance is not so affected by the fault. The average errors are $\bar{e}_{x_c} = 0.317 \, (mm)$ and $\bar{e}_{v_c} = 0.095 \, (mm/sec)$.

The extreme situation is simulated to illustrate the robustness of the FTC strategy. The measurement value of $P_A$ is set as zero during the simulation.

Figure 4.11: Tracking control (FTC)

Figure 4.12: Tracking control (adaptive backstepping) (zoomed)

The results in Fig. 4.12 are from the simulation with normal adaptive backstepping con-
4.4 Summary

In this chapter the fault-tolerant control (FTC) scheme is designed. The focus is to find a controller that can perform the position tracking and tolerate the possible faults automatically. Backstepping technology is applied to design the controller. It can handle the nonlinearity and unknown parameter of the system. The asymptotic stability is guaranteed in the fault free case. The considered faults can be accommodated by parameter adaption and model modification. The control performance has only a slight degradation. According to the simulation results it can be concluded that the proposed FTC can achieve the desired control performance even in the presence of the fault.

4.3.5 Conclusion

The simulation study shows wonderful performance of the developed fault diagnosis integrated FTC scheme. The tracking of position and velocity is realized. By employing adaptive backstopping technique the controller can easily eliminate the effect of the variation of external load. The faults can be accommodated by applying pre-designed reconfiguration laws. The control performance is almost same as the nominal state even the system is faulty. The parameter estimation of the adaptive controller has bounded error and can not asymptotically converge to the actual value. It indicates that adaptive controller can not be applied as a residual generator here. An additional FDI system is really necessary for application. In summary it can be concluded that the proposed control scheme has realized the precision control with guaranteed safety and reliability for the electro-hydraulic system.

troller. The position tracking error can be still limited in a small range. But the system begins to oscillate due to the information lost of $P_A$. The proposed FTC performs excellent with the aid of FDI system. The control performance has only a slight degradation with the results $\bar{e}_{x_c} = 0.386$ (mm) and $\bar{e}_{v_c} = 0.114$ (mm/sec).
5 Experimental results

5.1 Practical issues

5.1.1 Hardware and software

The discussed method of fault diagnosis and FTC is implemented on the practical electro-hydraulic testbed. The key hardware of the controller is the a real-time measurement card MODULAR-4 installed in an ISA slot of a PC. The measurement card has a 486 CPU and various I/O units and is delivered with its own real-time multitasking operating system in ROM [Mod]. The card works parallel with the computer. The algorithms are written in C and compiled as executable files. These files are downloaded into the card’s memory as tasks. According to the function of individual task the priority is determined implicitly. The real-time property is guaranteed by the Timer-Interrupt.

The PC is served as a graphic user interface (GUI) for visualization, parameter setting and data storage. These function is realized through the NI software LabWindows/CVI. The computer can access to the card through the drivers. For example, as soon as the buffer area of card is full, the compute will read all saved data. Then the buffer area will be refreshed with the new data. All data can be saved in computer. Hence, the problem of limited memory capacity in card can be well solved. The general description of software structure in computer and measurement card is shown in Fig.5.1.

![Figure 5.1: Structure of the software](image)
5.1.2 Parameters and experiment setting

The sampling time is chosen as $T_s = 5ms$. The system parameters are same as those in system identification. The maximum of load pressure is 120 Bar.

The Controller parameters are chosen as (for standard units)

$$
k_1 = 200, \quad k_2 = 2000, \quad k_3 = 4, \quad k_4 = 100, \quad k_5 = 100, \quad \rho_1 = 1, \quad \rho_2 = 1, \quad \rho_3 = 1, \quad \rho_4 = 100, \quad \rho_5 = 10, \quad \frac{1}{\rho_\theta_1} = 6e7, \quad \rho_\theta_2 = 230.
$$

The faults are set as following:

- $Q_{Lin} \approx 5 \text{ (L/min)}$, if $t \geq t_{rv}$. Here $t_{rv}$ denotes the rising time of the command value, which is a ramp-like signal with the saturation of 5 (L/min).
- $\Delta P_B$ is 20% of the measurement and acts to measurement as step signal.
- $\Delta P_A$ is 20% of the measurement and acts to measurement as step signal.

The setting of the DML is chosen as follows (with the time unit (sec.)).

**DML**

$$DML_{P_B} = \begin{cases} 
1 & t_{r_1} \geq 0.25 \\
0 & \text{otherwise} 
\end{cases} \quad \text{and} \quad DML_{Q_{Lin}} = DML_{Q_P A} = 0 \quad (5.1)$$

$$DML_{P_A} = \begin{cases} 
1 & 0.05 \leq t_{r_2} \leq 0.25 \\
0 & \text{otherwise}
\end{cases} \quad \text{and} \quad DML_{Q_{Lin}} = DML_{Q_P A} = 0 \quad (5.2)$$

$$DML_{Q_{Lin}} = \begin{cases} 
1 & t_{r_3} \geq 0.10 \\
0 & \text{otherwise}
\end{cases} \quad \text{and} \quad DML_{Q_P A} = DML_{Q_P B} = 0 \quad (5.3)$$
5.2 Experiments

5.2.1 Scenario 1: fault free

Figure 5.2: Estimation in fault free case

Figure 5.3: Estimation errors in fault free case
Figure 5.4: Residuals in fault free case

Figure 5.5: The position trajectories in fault free case
There is no fault but the varying external load in system. The fault detection observer possesses good convergence. The residuals are unaffected by the unknown external load force and vary always under the threshold. No false alarm is trigged. The initial errors have significant effect to the estimation. The estimation of pressure $P_B$ is worse than others due to the modeling error. Adaptive backstepping also archives excellent tracking performance. The “peaks” in the position error trajectory shown in Fig. 5.5 are caused by load variation. The average errors are $\bar{e}_{xc} = 0.379 \text{ (mm)}$ and $\bar{e}_{vc} = 0.237 \text{ (mm/sec.)}$. 
The estimation of external load is bounded but had a poor accuracy as shown in Fig. 5.7.

For comparison the results with a PI controller are shown in following.

Figure 5.8: The position trajectories with PI controller (fault free)

Figure 5.9: The velocity trajectories with PI controller (fault free)

It shows that the velocity signal in Fig. 5.9 has began to oscillate due to the variation
of the external load. The average errors are $\bar{e}_x = 1.88\ (mm)$ and $\bar{e}_v = 1.27\ (mm/sec.)$. The conclusion can be drawn that backstepping controller performs better than the conventional PI controller in the fault free case.

### 5.2.2 Scenario 2: with the internal leakage $Q_{Lin}$

![Figure 5.10: Estimation with $Q_{Lin}$](image)

![Figure 5.11: Estimation errors with $Q_{Lin}$](image)
5 Experimental results

Figure 5.12: Residuals with $Q_{Lin}$

Figure 5.13: The decision making logic with $Q_{Lin}$
5.2 Experiments

Figure 5.14: The position trajectories with $Q_{Lin}$

Figure 5.15: The velocity trajectories with $Q_{Lin}$
The leakage $Q_{Lin}$ is introduced into the system at $t \approx 10$ second. The fault is identified about 10 seconds later. The long time delay is caused by fault detection. The residual $r_{Lin}$ can not reflect the fault until the value of leakage flow is settled. Furthermore, $r_{Lin}$ has the maximum when the velocity is fastest. The $DML_{Lin}$ makes a right choice and the second parameter adaption is activated. The position error quickly converges to zero with the reconfigured controller. The estimations of external load and leakage are bounded but not accurate. In spite of the inaccuracy good control performance is still achieved with the average errors $\bar{e}_{x_c} = 0.408 \ (mm)$ and $\bar{e}_{v_c} = 0.237 \ (mm/sec.)$. 

Figure 5.16: Estimation of external load force with fault $Q_{Lin}$

Figure 5.17: Estimation of $Q_{Lin}$
5.2.3 Scenario 3: with the sensor offset $\Delta P_B$

![Estimation with $\Delta P_B$](image1)

**Figure 5.18: Estimation with $\Delta P_B$**

![Estimation errors with $\Delta P_B$](image2)

**Figure 5.19: Estimation errors with $\Delta P_B$**
5 Experimental results

Figure 5.20: Residuals with $\Delta P_B$

Figure 5.21: The decision making logic with $\Delta P_B$
The fault is added at $t \approx 9$ second and is detected about 14 seconds later. The fault is not detected until the value of $P_B$ becomes large. Then the effect of the sensor offset appeared distinctly. The controller reconfiguration is activated as soon as the fault is identified. It can be obtained through the error trajectory of position that the fault $\Delta P_B$ has less impact on system than the load variation. Although the dynamics of $P_B$ is
ignored in controller redesign procedure, the performance degradation is also very small with the average errors $\bar{e}_{xc} = 0.433\ (mm)$ and $\bar{e}_{ve} = 0.280\ (mm/sec.)$. The result of estimation of $F_{load}$ is similar as in scenario 1 and is omitted here.

5.2.4 Scenario 4: with the sensor offset $\Delta P_A$

Figure 5.24: Estimation with $\Delta P_A$

Figure 5.25: Estimation errors with $\Delta P_A$
Figure 5.26: Residuals with $\Delta P_A$

Figure 5.27: The zoomed $r_{P_A}$
5 Experimental results

Figure 5.28: The decision making logic with $\Delta P_A$

Figure 5.29: The position trajectories with $\Delta P_A$
The sensor offset \( \Delta P_A \) is added at \( t \approx 16.63 \) second and is detected in 0.2 second. The offset only caused an impulse-like impact on the residual \( r_{PA} \). Then the residual signal lay under the threshold. Nonetheless, the fault is exactly identified by the DML as shown in Fig. 5.28. Although the residual is over the threshold because of the initial error at the beginning, the DML make a right decision. The DML activated the fault accommodation algorithm. The parameter estimation of \( F_{load} \) is much bigger than actual load. The sensor offset is viewed as load variation by the controller. The average
errors increases slightly compared to the fault free case with the values \( \bar{e}_{x_c} = 0.486 \text{ (mm)} \) and \( \bar{e}_{v_c} = 0.265 \text{ (mm/sec.)} \).

The FTC is also tested in extreme situation, namely the sensor of \( P_A \) has a total failure.

The tracking performance of position is still ensured. But the system has a small oscillation in high speed area. The average errors are \( \bar{e}_{x_c} = 0.601 \text{ (mm)} \) and \( \bar{e}_{v_c} = 0.35 \text{ (mm/sec.)} \). It is should be pointed out that detection and isolation the total failure is simply realized by checking the value of \( r_{PA} \) as explained in chapter 3. The time restrict is released in such situation.
5.2 Experiments

5.2.5 Conclusion

The all experimental results are summarized in the following table, where \( t_f \) denotes the fault occurrence time, \( t_d \) represents the detection time and \( P_A \times \) means sensor of \( P_A \) has a total failure. Although the external load varied a lot. The robust fault detection

<table>
<thead>
<tr>
<th>Controller</th>
<th>PI (no fault)</th>
<th>FTC (no fault)</th>
<th>FTC ((Q_{Lin}))</th>
<th>FTC ((\Delta P_B))</th>
<th>FTC ((\Delta P_A))</th>
<th>FTC ((P_A \times))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{e}_{x_c} ) (mm)</td>
<td>1.88</td>
<td>0.379</td>
<td>0.409</td>
<td>0.433</td>
<td>0.486</td>
<td>0.601</td>
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<tr>
<td>( \bar{e}_{v_c} ) (mm/sec.)</td>
<td>1.27</td>
<td>0.265</td>
<td>0.268</td>
<td>0.283</td>
<td>0.265</td>
<td>0.35</td>
</tr>
<tr>
<td>( t_f ) (sec.)</td>
<td>—</td>
<td>—</td>
<td>10</td>
<td>9</td>
<td>16.63</td>
<td>21.29</td>
</tr>
<tr>
<td>( t_d ) (sec.)</td>
<td>—</td>
<td>—</td>
<td>20</td>
<td>23</td>
<td>16.8</td>
<td>21.3</td>
</tr>
</tbody>
</table>

Table 5.1: The experimental results

and tracking control are realized by the presented FDI integrated FTC. The designed thresholds and the DML ensures zero false alarm in fault free case and correct fault identification, if the considered fault occurs. The residual \( r_{x_c} \) is unaffected by the all considered faults, which is in conformity with the analysis and simulations. The fault \( \Delta P_A \) can be detected by the ad hoc signal checking of the residual. But it has higher risk of missed detection or false alarm than the others. Backstepping based controllers show good tracking performance and robustness. With the parameter adaption of the unknown external load the best results is obtained in fault free case. The control performance degrades just slightly, when the faults are present. Unfortunately, the estimations of parameters are only bounded and have large deviations with true values. Although the FDI for the \( Q_{Lin} \) and \( \Delta P_B \) takes long time, the robustness of the controller can still be kept during the critical period. A theoretical robustness analysis of the system in this time interval is already out of the range of this thesis.
6 Summary and outlook

6.1 Summary

This thesis proposed a model-based methodology of the fault diagnosis integrated fault-tolerant control (FTC) for a nonlinear electro-hydraulic system, whose main components include a servo-pump and hydraulic cylinders. The advantage of such a control scheme is that the pre-specified control performance can be maintained or has only an acceptable degradation with the presence of the considered fault. Meanwhile, the best control performance can be obtained in fault free case. The design procedure was divided in two parts: 1) fault detection and isolation 2) design of a reconfigurable controller. The considered faults were internal leakage $Q_{Lin}$, sensor offsets $\Delta P_A$ and $\Delta P_B$.

Firstly, the mathematical model of the electro-hydraulic testbed, which is the foundation of diagnosis and control, was derived based on the physical relations among the states variables. The unknown parameters, such as parameters of transfer function of the servo pump or friction coefficient of the cylinder, were identified with the help of powerful toolbox of Matlab/Simulink. The established model was validated with the measurements. A good consistency was achieved.

Based on the system model the differential-geometric method was employed for robust fault detection. The solution of fault detection and isolation in terms of differential-geometry is to find an unobservable distribution, which indicates that a system transformation can be carried out for disturbance decoupling or isolation of other faults (sensor fault is not included). Also the transformed system is locally weakly observable. Therefore, it can be applied for decoupling the unknown external load. The construction of the unobservable distribution is realized by establishing the observability codistribution, which is the annihilator of the unobservable distribution. After such a transformation the new system is totally robust to external load and takes variable velocity as an input. The fault detection observer based on the new system model could detect all pre-specified faults. The fault isolation was realized through a special decision making logic. The simulation study showed good results of fault diagnosis. The residuals were independent of the unknown disturbance. Every considered fault could be detected and isolated. The only problem was that the fault diagnosis would cause time delay for the controller reconfiguration.

The backstepping based controller was applied for position tracking. The feedback control law and associated Lyapunov function is designed by recursive approach. The system
states can be viewed as “virtual controls”. In fault free case the parameter adaption was inserted for the unknown external load. Then the tracking error asymptotically converges to zero. As soon as the fault was identified, the backstepping controller can be reconstructed by parameter or model adaption. The fault $Q_{Lin}$ was handled as unknown parameter and could be tolerated by parameter adaption. Accommodation of sensor fault $\Delta P_A$ and $\Delta P_B$ was based on the model adaption. The faulty measurement was replaced by the approximation. Modeling of the faulty system was also simplified. $\Delta P_A$ affected adaptive controller more significantly than $\Delta P_B$. So the reconfigured controller dropped the parameter adaption for unknown external load for preventing from possible oscillation. In addition, numerical differential was applied for simplifying the calculation of derivatives of stabilizing functions. The saturations for the stabilizing functions were introduced according to the physical constraints of system states so that the values of error terms of backstopping controller were limited in a reasonable range.

The proposed fault diagnosis system and backstepping based controller formed a fault diagnosis integrated fault-tolerant control structure. It was successfully implemented in the electro-hydraulic testbed. The on-line robust fault diagnosis and controller reconfiguration were realized. In fault free case the best control performance was obtained. The control performance had only a slight degradation. But the parameter estimation by adaptive controller was only bounded and had a poor accuracy. This also indicates that the parameter estimation of the adaptive can not be used as residual generator.

In conclusion the proposed methodology can perform excellent for precise control of the electro-hydraulic testbed with guaranteed safety and reliability. The system variables are monitored by the fault detection observers. It can be extend to control other electro-hydraulic systems, such as valve controlled hydraulic system.

### 6.2 Outlook

In future’s work it is necessary to analyze the robustness of the system during the critical period. The adaption laws of the backstepping controller should be improved to make a more precise estimation of parameters. Other observers should be tested, like Kalman-Filter, to enhance the sensitivity to the faults. The estimation of the amplitude of the fault should be investigated to archive a complete monitoring for the system. More possible faults of the electro-hydraulic system should be studied and simulated, for example, the leakage in servo-valve or pulsation of the pump.
7 Kurzfassung

Die Grundlagen und Motivation


Eine Regelung, die auch nach dem Auftreten eines Fehlers ihre Funktion erfüllt, heißt fehler tolerant Regelung (Fault tolerant control (FTC)). Die fehler tolerant Regelung basiert auf einer Fehlerdiagnose. Der Regler reagiert auf die Fehler und wird ausgehend von der Fehlerinformation automatisch rekonfiguriert. Auf diese Weise wird die Regelgüte nicht wesentlich verschlechtert.

Die Anwendung der Fehlerdiagnose integrierte fehler tolerant Regelung auf die elektrohydraulischen Systeme ist sehr bedeutend. Außer der Verbesserung der Sicherheit und Zuverlässigkeit werden die Wartungskosten auch wegen der Fehlerfrüherkennung sinken. Und zwar das ganze System funktioniert intelligenter und effizienter. Die
Hauptschwierigkeit zu diesem Thema besteht in folgendes:

- hohe Nichtlinearität und Unsicherheit des elektrohydraulischen Systems
- Robustheit des Fehlerdiagnosesystems
- präzise Regelung
- Rekonfiguration des Reglers im Fehlerfall
- echtzeitig Rechnen

Das Ziel der Arbeit

Motiviert von der obigen Betrachtung wird in dieser Arbeit eine anwendbare Strategie für die Integration der modellbasierten Fehlerdiagnose in die fehlertolerante Regelung elektrohydraulischer Systeme entwickelt. Das Fehlerdiagnosesystem soll die Fehler trotz Systemunsicherheit detektieren und isolieren, die Regelung auch im Fehlerfall präzise funktionieren.

Zusammenfassung der einzelnen Kapitel

In Kapitel 2 wird das Modell der untersuchten elektrohydraulischen Anlage hergeleitet.
Der hydraulische Kreis, wie in Bild 7.1 gezeigt, besteht hauptsächlich aus einer Servopumpe und zwei Zylindern, die jeweils als Arbeitszylinder und Lastzylinder benannt werden.


Die künstliche Realisierung der Fehler wird vorgestellt: eine innere Leckage als Systemfehler und zwei Drucksensorfehler werden als häufig auftretende Fehler ausgewählt. Das Kapitel bildet die Basis für die weitere Arbeit.

In Kapitel 3 wird zunächst ein Überblick zur modelbasierten Fehlerdiagnose gegeben. Der differentialgeometrische Ansatz, der auf eine allgemeine Formulierung des FDI-Problems führt sowie kompakter und transparenter als andere bekannte modellbasierte Ansätze ist, wird ausführlich vorgestellt. Dieser Ansatz zeigt einen systematischen Entwurfprozess für die Bestimmung einer Distribution, die als nicht beobachtbare Distribution (unobservable distribution) deﬁniert wird. Diese Distribution zeigt, wie der Einfluss der wechselnden Last auf das fehlerbeeinflusste System mit Hilfe einer Systemtransformation entkoppelt werden kann. Die Auswirkung des Fehlers wird hierdurch jedoch nicht beeinflusst. Weiterhin ist das transformierte System im fehlerfreien Fall noch lokal schwach beobachtbar.


In Kapitel 4 ist der Entwurf des rekonfigurierbaren Reglers für FTC ausgeführt. Zum Eliminieren des nichtlinearen Systemverhaltens wird ein Backstepping-Verfahren verwendet. Das Backstepping-Verfahren ist ein rekursiver Lyapunov-basierter Ansatz, der das Regelgesetz und die zugehörige Lyapunov-Funktion schrittweise konstruiert. Für das elektrohydraulische System wird ein adaptiver Backstepping-Regler verwendet, um so den Einfluss der unbekannten äußeren Last im fehlerfreien Fall zu kompensieren. Es wird bewiesen, dass die Regelabweichung asymptotisch gegen Null konvergiert und der Parameterschätzungsfehler im fehlerfreien System begrenzt ist. Um die Berechnung des
Reglers zu vereinfachen wird eine numerische Differentiation gefolgt von einem Tiefpass-Filter verwendet.


Ausblick

A Background of geometric theory

Background of geometric theory is detailed described in [Isi95] and [Won85]. Some of them are repeated here.

A.1 Affected/unaffected

Affected/unaffected The $j$th output $y_j(t, x^0, u_1, \ldots, u_k)$ corresponding to an initial condition $x^0$ and to the set of input functions $u_1, \ldots, u_k$ is unaffected by (or invariant under) the $i$th input $u_i$ if, for any initial condition $x^0$ and any collection of admissible input functions $u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_k$, there holds:

$$y_j(t, x^0, u_1, \ldots, u_{i-1}, u_a, u_{i+1}, \ldots, u_k) = y_j(t, x^0, u_1, \ldots, u_{i-1}, u_a, u_{i+1}, \ldots, u_k)$$

for all $t \geq 0$ and any pair $u_a, u_b$. The $j$th output $y_j(t, x^0, u_1, \ldots, u_k)$ is said to be affected by the $u_i$ if it is not unaffected by the $u_i$.

A.2 Vector spaces and subspaces

Vector (linear) spaces consists of an additive group, of elements vectors, together with an underlying filed of scalars. Vector spaces are denoted by script capitals, such as $\mathcal{X}$ and $\mathcal{Y}$. Their elements are written as $x$ and $y$. The dimension of the vector space $\mathcal{X}$ is denoted by $\text{dim}(\mathcal{X})$.

A subspace $\mathcal{M}$ of the vector space $\mathcal{X}$ is a subset of $\mathcal{X}$ that is a vector space under the operations of vector addition and scalar multiplication inherited from $\mathcal{X}$: namely, $\mathcal{M} \subset \mathcal{X}$ and for all $x_1, x_2 \in \mathcal{M}$ and $c_1, c_2 \in \mathbb{R}$ then $c_1 x_1 + c_2 x_2 \in \mathcal{M}$. If the two subspaces $\mathcal{M}, \mathcal{N} \subset \mathcal{X}$, then $\mathcal{M} + \mathcal{N} \subset \mathcal{X}$ and $\mathcal{M} \cap \mathcal{N} \subset \mathcal{X}$. Here $\mathcal{M} + \mathcal{N}$ can be understood as the smallest subspace containing both $\mathcal{M}$ and $\mathcal{N}$, while $\mathcal{M} \cap \mathcal{N}$ is the largest subspace contained in both $\mathcal{M}$ and $\mathcal{N}$. $\mathcal{M}$ and $\mathcal{N}$ are linearly independent if $\mathcal{M} \cap \mathcal{N} = 0$. If $\text{dim}(\mathcal{M}) = d_1$ and $\text{dim}(\mathcal{X}) = d_2$, then the difference of $d_2$ and $d_1$ is defined as the codimension of $\mathcal{M}$ in $\mathcal{X}$ and can be denoted as:

$$\text{codim}(\mathcal{M}) = \text{dim}(\mathcal{X}) - \text{dim}(\mathcal{M}).$$
Let $C : \mathcal{X} \to \mathcal{Y}$ be a linear map. $\mathcal{X}$ is the domain of $C$ and $\mathcal{Y}$ is the codomain. The null space (or kernel) of $C$ is the subspace 

$$KerC = \{ x : x \in \mathcal{X} \& Cx = 0 \} \subset \mathcal{X},$$

while the image (or range) of $C$ is the subspace

$$ImC = \{ Cx : x \in \mathcal{X} \} \subset \mathcal{Y}.$$

The map $C : \mathcal{X} \to \mathcal{Y}$ is an epimorphism (or epic) if $ImC = \mathcal{Y}$. If $C$ is epic there is a right inverse $C_r : \mathcal{Y} \to \mathcal{X}$, such that $CC_r = I_{\mathcal{Y}}$.

The map $C : \mathcal{X} \to \mathcal{Y}$ is a monomorphism (or monic) if $KerC = 0$. If $C$ is monic, there is a left inverse $C_l : \mathcal{Y} \to \mathcal{X}$, such that $C_lC = I_{\mathcal{X}}$.

If $C$ is both epic and monic, $C$ is an isomorphism and this can happen only if $dim(\mathcal{X}) = dim(\mathcal{Y})$. In this case $C_l = C_r = C^{-1}$.

### A.3 Dual spaces and annihilators

Let $\mathcal{X}$ be linear vector space over field $\mathbb{F}$. The set of all linear functionals $x' : \mathcal{X} \to \mathbb{F}$ is denoted by $\mathcal{X}'$. $\mathcal{X}'$ becomes a linear space with the following definitions

$$(x'_1 + x'_2)x = x'_1x + x'_2x; \quad x'_i \in \mathcal{X}', x \in \mathcal{X}$$

$$(ax'_1)x = a(x'_1x); \quad x'_1 \in \mathcal{X}', x \in \mathcal{X}, a \in \mathbb{F}.$$ Then $\mathcal{X}'$ is called the dual space of $\mathcal{X}$ in $\mathbb{F}$. The element of $\mathcal{X}'$ is named as covector.

The annihilator of $M \subset \mathcal{X}$ is usually denoted by $M^\perp$ and is defined as

$$M^\perp = \{ x' : x'M = 0, x' \in \mathcal{X}' \}.$$ 

Therefor $M^\perp \subset \mathcal{X}'$.

### A.4 Factor space

Let $\mathcal{M} \subset \mathcal{X}$. Call vectors $x, y \in \mathcal{X}$ equivalent mod $\mathcal{M}$ if $x - y \in \mathcal{M}$. The factor space (or quotient space) $\mathcal{X}/\mathcal{M}$ is defined as the set of all equivalence classes:

$$\bar{x} := \{ y : y \in \mathcal{X}, y - x \in \mathcal{M} \}, x \in \mathcal{X}.$$ 

For vector spaces the following equation is always true if the factor space exists.

$$dim(\mathcal{X}/\mathcal{M}) = dim(\mathcal{X}) - dim(\mathcal{M})$$
A.5 Invariant subspaces

Let \( A : \mathcal{X} \to \mathcal{X} \) and let \( \mathcal{M} \subset \mathcal{X} \) have the property \( A\mathcal{M} \subseteq \mathcal{M} \) is said to be \( A \)-invariant. Namely, an invariant subspace of this system is a linear subspace \( \mathcal{M} \subset \mathcal{X} \) with the property that every trajectory starting in \( \mathcal{M} \) will stay in \( \mathcal{M} \) in the future. Certainly \( \mathcal{M} \) itself, and the subspace \( 0 \), are trivially invariant subspaces.

For example, there is a linear system

\[
\dot{x} = Ax. \tag{A.1}
\]

where \( x \in \mathbb{R}^n, A \in \mathbb{R}^{n \times n} \). The solution of the differential equation (A.1) \( x \) starts in \( \mathcal{M} \) and ends in \( \mathcal{M} \). The subspace \( \mathcal{M} \subset \mathcal{X} \) is \( A \)-invariant.

A.6 Reachable subspace

Further if there is a linear control system

\[
\begin{align*}
\dot{x} &= Ax + Bu \quad \tag{A.2} \\
y &= Cx \tag{A.3}
\end{align*}
\]

If \( x \in \mathcal{X} \subset \mathbb{R}^n, B = \text{Im}B \), the reachable subspace is:

\[
\langle A|B\rangle = B + AB + \cdots + A^{n-1}B
\]

\( \langle A|B\rangle \) also means minimal \( A \)-invariant subspace containing \( B \).

A subspace \( \mathcal{R} \) is called a reachability subspace if there are matrices \( F \) and \( G \) such that

\[
\langle (A + BF)|\text{Im}BG \rangle.
\]

A.7 Unobservable subspace

The concept of unobservable subspace can be viewed as dual to reachable subspace and is defined as

\[
\langle \text{Ker}C|A \rangle = \text{Ker}(C) \cap \text{Ker}(CA) \cap \cdots \cap \text{Ker}(CA^{n-1}) = \bigcap_{i=1}^{n} \text{Ker}(CA^{i-1}).
\]

It also means maximal \( A \)-invariant subspace contained in \( \text{Ker}C \).
The system (A.2) and (A.3) is observable if and only if
\[ \bigcap_{i=1}^{n} \ker \left( CA^{i-1} \right) = 0 \]

A subspace \( S \) is called unobservability space, if there exist a matrices \( L \) and \( H \) such that \( S \) is the set of unobservable states of the system
\[
\dot{x} = (A + LC)x \\
y = HCx. \tag{A.4}
\]

\[
\dot{x} = (A + LC)x \\
y = HCx. \tag{A.5}
\]

### A.8 Distribution and codistribution

A distribution on \( f_1, f_2, \cdots, f_k \) be smooth vector fields of open set \( D \subset \mathbb{R}^n \) and
\[
\Delta(x) = \text{span} \{ f_1(x), f_2(x), \cdots, f_k(x) \}
\]
be the \( k \)-dimensional subspace spanned by the vectors \( f_1(x), f_2(x), \cdots, f_k(x) \) at any fixed point \( x \in D \). The collection of all vector spaces \( \Delta(x) \) for \( x \in D \) is called a distribution and is noted as
\[
\Delta = \text{span} \{ f_1, f_2, \cdots, f_k \}.
\]
The dimension of the distribution is
\[
\dim(\Delta) = \text{rang} \left[ f_1(x), f_2(x), \cdots, f_k(x) \right] \tag{A.6}
\]

If \( \Delta \) defined on \( D \subset \mathbb{R}^n \) is said to be nonsingular if there is a integer \( d \) such that \( \dim(\Delta) = d \) for \( \forall x \in D \). A distribution \( \dim(\Delta) \) is said to be involutive if two vector fields \( f(\cdot) \) and \( g(\cdot) \) exist such that if \( \forall x \in D, f \in \Delta, g \in \Delta \) then \( [f, g] \in \Delta \).

The codistribution is the dual of distribution. If the elements of \( \mathbb{R}^n \) are column vectors, then denote the dual space as \( \mathbb{R}^{n*} \), whose elements are row vectors. Just as a vector field on \( \mathbb{R}^n \) is a smooth assignment of column vector \( f(x) \) to each point \( x \in \mathbb{R}^n \). Define a covector field to be a smooth assignment of covector \( \omega(x) \) to each point \( x \in \mathbb{R}^n \). Then the definition of codistribuition is
\[
\Omega = \text{span} \{ \omega_1, \omega_2, \cdots, \omega_k \}.
\]
The annihilator of a distribution is its codistribution.
### B Technical data of the electro-hydraulic system

**Servo-pump**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>rotation speed</td>
<td>1000 1/min</td>
</tr>
<tr>
<td>volume flow</td>
<td>28 L/min (max.)</td>
</tr>
<tr>
<td>power</td>
<td>11 kW</td>
</tr>
<tr>
<td>swashplate angle</td>
<td>17.7° (max.)</td>
</tr>
<tr>
<td>control cylinder stroke</td>
<td>17.72 mm (max.)</td>
</tr>
</tbody>
</table>

Table B.1: Mannesmann-Rexroth A10VSO 28 DFE1

**Hydraulic cylinder**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>piston ø</td>
<td>200 mm</td>
</tr>
<tr>
<td>piston rod ø</td>
<td>140 mm</td>
</tr>
<tr>
<td>area ratio</td>
<td>1.96</td>
</tr>
<tr>
<td>piston rod length</td>
<td>1176 mm</td>
</tr>
<tr>
<td>piston rod stroke</td>
<td>500 mm (max.)</td>
</tr>
<tr>
<td>moving mass</td>
<td>166 kg</td>
</tr>
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</table>

Table B.2: Mannesmann-Rexroth CD H3 MF3/200/140/500
Absolute encoder

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>encoder capacity</td>
<td>24 bit (max.)</td>
</tr>
<tr>
<td>steps / revolution</td>
<td>12 bit (4096)</td>
</tr>
<tr>
<td>number of revolution</td>
<td>12 bit (4096)</td>
</tr>
<tr>
<td>resolution</td>
<td>20 μm</td>
</tr>
</tbody>
</table>

Table B.3: TR-Electronic CE-65-SSI

Proportional pressure relief valve

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>volume flow</td>
<td>30 L/min (max.)</td>
</tr>
<tr>
<td>operation pressure</td>
<td>315 Bar (max.)</td>
</tr>
</tbody>
</table>

Table B.4: Mannesmann-Rexroth DBE 62-1X/315
Bibliography


[Mod] *MODULAR-4/486*.


Lebenslauf

Persönliche Daten
Name: Liang Chen
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