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Vorwort

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Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

A Parsimonious Multi-Asset Heston Model: Calibration and Derivative Pricing

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Abstract

We present a parsimonious multi-asset Heston model. All single-asset submodels follow the well-known Heston dynamics and their parameters are typically calibrated on implied market volatilities. We focus on the calibration of the correlation structure between the single-asset marginals in the absence of sufficient liquid cross-asset option price data. The presented model is parsimonious in the sense that $d(d-1)/2$ asset-asset cross-correlations are required for a d-asset Heston model. In order to calibrate the model, we present two general setups corresponding to relevant practical situations: (1) when the empirical cross-asset correlations in the risk neutral world are given by the user and we need to calibrate the correlations between the driving Brownian motions or (2) when they have to be estimated from the historical time series. The theoretical background, including the ergodicity of the multidimensional CIR process, for the proposed estimators is also studied.

Key Words: Heston model, multi-asset, option pricing, calibration, correlation

JEL Classification: G13

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1 Introduction

The Heston model is a derivative pricing framework widely used for financial markets including equity, commodity, and foreign exchange. The single-asset Heston is a two-factor model describing the dynamics of an asset and simultaneously the asset's volatility. In the traditional single-asset model, the parameters are calibrated from plain vanilla option price data, if available.

For valuing derivatives written on more than one asset, say d assets, the Heston model can be extended accordingly to the multi-asset Heston model. For each asset and its corresponding volatility, the parameters can be calibrated in the same fashion as done for the single-asset Heston. However, the cross-correlations asset-asset, asset-volatility, and volatility-volatility cannot be calibrated from single-asset option price data. Note, here we use the expression cross-correlation if the factors belong to different single-asset Heston models.

In the absence of sufficient cross-asset option price data, we propose a method to fill in the missing cross-correlations. The method uses empirical correlations of the assets as input data. Hence, cross-correlations asset-volatility and volatility-volatility are not required as input data. The rationale is based on: (1) cross-correlations involving volatilities are hard to estimate from time series data since volatility is typically not observable and has to be filtered/constructed from market data (volatility implied by option prices or using proxies like volatility indices such as VDAX and VIX). (2) Taking the full set cross of correlations as input data restricts possible choices of asset-volatility correlations for each single-asset Heston model contained in the multi-asset framework. The correlation of an asset and the asset's volatility are not obtained empirically but from option price data. If the calibrated correlation does not satisfy the restrictions imposed by the empirical cross-correlations, the model breaks down.

Our method is based on the two-asset case, and can be extended to the full multi-asset model by considering all two-asset sub-models. Thus the stability of the method for d assets is identical to the simple two-asset case, and the computer run times grow with $(d-1)d/2$ (the number of two-dimensional sub-models). For the two-asset Heston, we

compute the missing continuous time model parameters by a calibration approach. In the two-asset case there are four missing cross-correlations. The cross-correlation asset-asset is determined and the remaining three cross-correlations are expressed as functions of the other parameters. Paths are then generated using Monte-Carlo simulation. These paths are observed at the same frequency as the data on which the empirical correlation is based. From the simulated observations the simulated empirical correlation is computed. Using the bisection methodology, the model cross-correlation asset-asset is then adjusted such that the simulated empirical correlation and the empirical correlation coincide.

2 Model

In this section, the stochastic volatility model of Heston [1993] is reviewed. Subsequently, a parsimonious multi-asset extension is developed and studied for its properties.

2.1 The Single-Asset Heston Model

In the framework of Heston [1993], the risk-neutral dynamics of the asset price process $S(t)$ is given by

$$\frac{\mathrm{d}S(t)}{S(t)} = (r(t) - q(t))\,\mathrm{d}t + \sqrt{\nu(t)}\,\mathrm{d}W_S(t)\,, \quad S(0) > 0\,, \tag{1}$$

where $r(t)$ is risk-free rate, $q(t)$ is the dividend rate, and $W_S(t)$ is a Wiener process. The volatility $\sqrt{\nu(t)}$ is a mean reverting process with risk-neutral dynamics

$$\mathrm{d}\nu(t) = \kappa\,(\overline{\nu} - \nu(t))\,\mathrm{d}t + \eta\,\sqrt{\nu(t)}\,\mathrm{d}W_\nu(t)\,, \quad \nu(0) > 0\,. \tag{2}$$

Thus, $\nu(t)$ describes a mean-reverting process with reversion rate $\kappa > 0$, mean level $\overline{\nu}$, "volatility" $\eta > 0$ and initial value $\nu(0)$. The process driving $\nu(t)$ is a Wiener process $W_\nu(t)$. To allow for possible leverage effects, $W_S(t)$ and $W_\nu(t)$ can be correlated with the coefficient $\rho_{S\,\nu}$, with $|\rho_{S\,\nu}| < 1$. We can write $\mathrm{d}W_S(t)\,\mathrm{d}W_\nu(t) = \rho_{S\,\nu}\,\mathrm{d}t$. The parameters ρ, κ, $\overline{\nu}$, $\nu(0)$, η are assumed to be given, e.g., calibrated from plain vanilla option price data.

2.2 A Parsimonious Multi-Asset Heston Model

Heston can be extended to a multi-asset model in different ways. For example, the Heston model can be extended by generalizing the real-valued variance rate process to a matrix valued version. Wishart processes represent the matrix analogue of the square-root mean-reverting process and are used for volatility modeling by Gourieroux [2006].

We propose a parsimonious multi-asset extension of the single-asset Heston with properties: (1) each single-asset sub-model forms a traditional Heston model; and (2) parameters are single-asset Heston parameters and asset-asset cross-correlations. These properties are desirable since (1) enables using established calibration procedures for the single-asset sub-models parameters, and (2) is parsimonious in the sense that $d(d-1)/2$ asset-asset cross-correlations are required for our d-asset Heston model.

Now consider a system of d assets with price and volatility processes $(S_1(t), \nu_1(t))$, ..., $(S_d(t), \nu_d(t))$. For $i = 1, ..., d$ we can write (1) and (2) in the vectorized form

$$\begin{pmatrix} dS_i(t) \\ d\nu_i^2(t) \end{pmatrix} = \begin{pmatrix} S_i(t)(r(t) - q(t)) \\ \kappa_i \left(\overline{\nu}_i - \nu_i(t)\right) \end{pmatrix} dt + \begin{pmatrix} S_i(t)\sqrt{\nu_i(t)} & 0 \\ 0 & \eta_i \sqrt{\nu_i(t)} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \rho_i & \sqrt{1-\rho_i^2} \end{pmatrix} \begin{pmatrix} dW_i(t) \\ d\widetilde{W}_i(t) \end{pmatrix}, \quad (3)$$

where $W_i(t)$ and $\widetilde{W}_i(t)$ are independent Wiener processes. The parameter describing the bivariate process are collected in $\theta_i = (\rho_i, \kappa_i, \overline{\nu}_i, \nu_i^2(0), \eta_i)$.

The model is completely described except for the dependence structure. So far, the correlations asset-volatility are given for $i = 1, ..., d$, but the cross-correlations asset-asset of $W_i(t)$ and $W_j(t)$, asset-volatility of $W_i(t)$ and $\widetilde{W}_j(t)$, and volatility-volatility of $\widetilde{W}_i(t)$ and $\widetilde{W}_j(t)$, $(i \neq j)$, are not yet specified.

Define the d-dimensional Wiener processes $W(t) = (W_1(t), ..., W_d(t))$ and $\widetilde{W}(t) = (\widetilde{W}_1(t), ..., \widetilde{W}_d(t))$. Our assumptions on the dependence structure are summarized below.

Assumption 2.1. *The Wiener processes $W(t)$ and $\widetilde{W}(t)$ satisfy:*

(1) $W(t)$ has correlation matrix $\Sigma^S = (\rho_{i,j})_{1 \leq i,j \leq d}$, i.e., $dW_i(t)\, dW_j(t) = \rho_{i,j}\, dt$;[1]

[1] A matrix is a correlation matrix if and only if it is symmetric, positive semi-definite, and has ones

(2) $\widetilde{W}(t)$ *has correlation matrix* I_d*, i.e.,* $d\widetilde{W}_i(t)\, d\widetilde{W}_j(t) = \delta_{i,j}\, dt$*;*[2]

(3) $W(t)$ *and* $\widetilde{W}(t)$ *are independent.*

The dependence assumption on $(W(t), \widetilde{W}(t))$ is permissible. To see this, observe that the correlation matrix of $(W(t), \widetilde{W}(t))$ is

$$\Sigma^{(W,\widetilde{W})} \;=\; \begin{pmatrix} \Sigma^S & 0 \\ 0 & I_d \end{pmatrix}.$$

Provided that Σ^S is a correlation matrix, which holds by assumption, the matrix $\Sigma^{(W,\widetilde{W})}$ is also a correlation matrix.

Assumption (1) allows all possible correlation matrices for the asset prices. In contrast, assumptions (2) and (3) are restrictions stating that the dependence of the volatilities is carried via the correlations of the corresponding asset prices $\rho_{i,j}$, which is then transferred to each volatility by the corresponding Heston parameter asset-volatility correlation ρ_i and ρ_j.

2.2.1 Correlation Structure

The model specification in (3) and Assumption 2.1 is implicit in the correlation structure of the vector process $(S_1(t), \nu_1^2(t), ..., S_d(t), \nu_d^2(t))$. The correlation structure is stated in the following Proposition.

Proposition 2.2. *Let* $(S_1(t), \nu_1(t), ..., S_d(t), \nu_d(t))$ *be the 2d-dimensional process defined in* (3) *and Assumption 2.1 holds. Then for* $1 \le i, j \le d$ *the instantaneous covariances of the components are*

(1) $\mathrm{d}S_i(t)\, \mathrm{d}S_j(t) = \rho_{i,j}\, S_i(t)\, S_j(t)\, \sqrt{\nu_i(t)\, \nu_j(t)}\, \mathrm{d}t$*;*

(2) $\mathrm{d}S_i(t)\, \mathrm{d}\nu_j(t) = \rho_{i,j}\, \rho_j\, S_i(t)\, \eta_j\, \sqrt{\nu_i(t)\, \nu_j(t)}\, \mathrm{d}t$*;*

(3) $$\mathrm{d}\nu_i(t)\, \mathrm{d}\nu_j(t) = \begin{cases} \rho_{i,j}\, \rho_i\, \rho_j\, \eta_i\, \eta_j\, \sqrt{\nu_i(t)\, \nu_j(t)}\, \mathrm{d}t, & \text{for } i \neq j\,, \\ \eta_i\, \eta_j\, \sqrt{\nu_i(t)\, \nu_j(t)}\, \mathrm{d}t, & \text{for } i = j\,. \end{cases}$$

on the main diagonal.

[2] $\delta_{i,j}$ is Kronecker's delta, i.e. $\delta_{i,j} = 1$ if $i = j$, and $\delta_{i,j} = 0$ if $i \neq j$.

Thus the instantaneous correlations of the component are

$$(1)\quad \frac{\mathrm{d}S_i(t)\,\mathrm{d}S_j(t)}{\sqrt{(\mathrm{d}S_i(t))^2\,(\mathrm{d}S_j(t))^2}} = \rho_{i,j};$$

$$(2)\quad \frac{\mathrm{d}S_i(t)\,\mathrm{d}\nu_j(t)}{\sqrt{(\mathrm{d}S_i(t))^2\,(\mathrm{d}\nu_j(t))^2}} = \rho_{i,j}\,\rho_j;$$

$$(3)\quad \frac{\mathrm{d}\nu_i(t)\,\mathrm{d}\nu_j(t)}{\sqrt{(\mathrm{d}\nu_i(t))^2\,(\mathrm{d}\nu_j(t))^2}} = \begin{cases} \rho_{i,j}\,\rho_i\,\rho_j, & \text{for } i \neq j\,, \\ 1, & \text{for } i = j\,. \end{cases}$$

2.2.2 Simulation Algorithm

The multi-asset Heston model developed here can be simulated using the full truncated Euler method for example. For the time step $t \mapsto t+\Delta t$, the Euler scheme for single-asset Heston is given by

$$\begin{pmatrix} \log \widehat{S}(t+\Delta t) \\ \widehat{\nu}^2(t+\Delta t) \end{pmatrix} = \begin{pmatrix} \log \widehat{S}(t) + (r(t)-q(t))\,\Delta t + \sqrt{\widehat{\nu}(t)\,\Delta t}\,\varepsilon^S \\ \widehat{\nu}(t) + \kappa(\theta - \widehat{\nu}(t))\,\Delta t + \eta\sqrt{\widehat{\nu}(t)\,\Delta t}\,\varepsilon^\nu \end{pmatrix} \tag{4}$$

where $(\varepsilon^S, \varepsilon^\nu)$ is a bivariate standard normal random variable with zero mean and unit variances and correlation ρ. Equation (4) leaves aside numerical issues where the variance rate process may become negative. It can be dealt with using e.g. the full truncation scheme from Lord et al. [2008] or the Andersen approach (see Andersen [2007]).

For the multi-asset Heston the correlation structure set out in Assumption 2.1 suggests the following algorithm:

- simulate a normal random vector $\varepsilon^S = (\varepsilon_1^S, ..., \varepsilon_d^S)$ with covariance matrix Σ^S by:
  (a) calculate the Cholesky square root L of $\Sigma^S = (\rho_{i,j})_{1\leq i,j\leq d}$, i.e. L is a lower triangular matrix with $L\,L^\top = \Sigma^S$;
  (b) generate d independent standard normally distributed random numbers z_1, ..., z_d, and set $z = (z_1, ..., z_d)^\top$;
  (c) compute $\varepsilon^S = Lz$.

- simulate a normal random vector $\varepsilon^\nu = (\varepsilon_1^\nu, ..., \varepsilon_d^\nu)$ by:
  (a) generate d independent standard normally distributed random numbers $\widetilde{z}_1$, ...,

$\widetilde{z}_d$, and set $\widetilde{z} = (\widetilde{z}_1, ..., \widetilde{z}_d)^\top$;

(b) calculate $\varepsilon_i^\nu = \rho_i \, \varepsilon_i^S + \sqrt{1-\rho_i^2} \, \widetilde{z}_i$, for $i = 1, ..., d$.

The algorithm follows essentially the construction implicit in Assumption 2.1. One can verify that the correlation matrix of $(\varepsilon^S, \varepsilon^\nu)$ corresponds to the one given in Proposition 2.2. The simulation algorithm can be applied to schemes other than Euler.

Computing the Cholesky square root of a symmetric positive semi-definite matrix of dimension $n \times n$ is a numerical problem of complexity $\mathcal{O}(n^3)$. Using the algorithm suggested here we apply this procedure to a $d \times d$-matrix. Alternatively, one could use a 'brute force' approach and compute the Cholesky decomposition of the correlation matrix given in Proposition 2.2 which is of dimension $2\,d \times 2\,d$. This would increase the complexity by factor 8.

2.3 Empirical Correlations

Our model is designed for markets with little cross-asset option price data complicating the calibration of potential correlation parameters. Assuming that the single-asset Heston parameters are calibrated, the free parameters for correlating the single-asset Heston models are contained in the matrix Σ^S. Thus $(d-1)\,d/2$ additional parameters have to be calibrated to correlate the single-asset models.

In the absence of sufficient cross-asset option price data, we rely on external estimates that we assume to be given as empirical correlations of the asset prices Σ^{emp}. The key idea for calibration is to adjust the model cross-correlations in Σ^S according to the observed empirical correlations Σ^{emp}. It is important to note that Σ^S is the correlation of infinitesimal noise carried by the Wiener process $W(t)$, whereas Σ^{emp} is the correlation of observed discrete-time asset price data. This observation is crucial since the Σ^S is not influenced by an equivalent measure transformations, while the cross-asset correlations may and typically will change. Therefore in one of the estimation setups considered below we will estimate the Σ^S using Σ^{emp} under the historical measure. The switch to the physical measure under the Heston assumption for the shape of the market price of volatility risk is rather straightforward.

Let us now formulate what is exactly meant by the empirical correlations Σ^{emp}. For observation times $(t_k)_{k=0,\dots,K}$ we are given asset price data $(S_1(t_k),\dots,S_d(t_k))_{k=0,\dots,K}$. The asset prices are suitably transformed to returns, log-returns or first differences, i.e.: $X_i = (X_i(t_k))_{k=1,\dots,K}$ with $X_i = F^i(S_i)$.

$\widehat{\Sigma}_K^{\text{emp}}(\Sigma)$ is then the empirical correlation matrix of these $(X_i)_{i=1,\dots,d}$ using the K-step time series.

In particular, the elements of $\widehat{\Sigma}_K^{\text{emp}}(\Sigma) = (\widehat{\rho}_{i,j:K}^{\text{emp}}(\Sigma))_{1\le i,j\le d}$ are defined by

$$\widehat{\rho}_{i,j:K}^{\text{emp}}(\Sigma) = \frac{\widehat{\nu}_{i,j:K}^{\text{emp}}(\Sigma)}{\sqrt{\widehat{\nu}_{i,i:K}^{\text{emp}}(\Sigma)\,\widehat{\nu}_{j,j:K}^{\text{emp}}(\Sigma)}}\,, \tag{5}$$

where

$$\widehat{\nu}_{i,j:K}^{\text{emp}}(\Sigma) = \frac{1}{K-1}\sum_{k=1,\dots,K}(X_i(t_k)-\bar{X}_i)\,(X_j(t_k)-\bar{X}_j)\,, \quad \text{and} \quad \bar{X}_i = \frac{1}{K}\sum_{k=1,\dots,K}X_i(t_k)\,. \tag{6}$$

The transformation applied commonly to asset price data is to form log returns. Other alternatives are possible (see Example 1), though sensible transforms are closely related to scaled first differences.

Example 1. *Transformations typically used include*

(1) first differences: $F(S(t_0),\dots,S(t_K))) = (S(t_1)-S(t_0),\dots,S(t_K)-S(t_{K-1}))$;

(2) returns: $F(S(t_0),\dots,S(t_K))) = (\frac{S(t_1)-S(t_0)}{S(t_0)},\dots,\frac{S(t_K)-S(t_{K-1})}{S(t_{K-1})})$;

(3) log-returns: $F(S(t_0),\dots,S(t_K))) = (\log(S(t_1)/S(t_0)),\dots,\log(S(t_K)/S(t_{K-1})))$.

In the following we will concentrate on the log-returns as the most widely used transforms of the time series. However, we must stress that the results will hold also for the returns and the first differences.

2.3.1 Properties

As already mentioned above we focus on the log-returns and for a discretized asset price $S(t_k)$ in the following we consider

$$X(t_k) := \log\frac{S(t_k)}{S(t_{k-1})}.$$

Theorem 2.3. *Let $(S_i(t), \nu_i(t))_{i=1,\dots,d}$ be the processes defined in* (3) *and Assumption 2.1 holds. Let $\pi_n = (t_{k,n})_{k=1,\dots,K_n(T)}$ be partitions of $[0,T]$, with $mesh(\pi_n) = \max_{i=1,\dots,K_n} |t_{i,n} - t_{i-1,n}| \to 0$, for $n \to \infty$. Then for all $1 \le i, j \le d$ we have*

(1) $\lim_{T\to 0} \lim_{n\to\infty} \widehat{\rho}^{\text{emp}}_{i,j:K_n(T)}(\Sigma) = \rho_{i,j}$ *almost surely*

(2) $\lim_{T\to\infty} \lim_{n\to\infty} \widehat{\rho}^{\text{emp}}_{i,j:K_n(T)}(\Sigma) = \rho_{i,j} \frac{\mathbb{E}\sqrt{\nu_{ij}(\infty)}}{\sqrt{\mathbb{E}\nu_i(\infty)\mathbb{E}\nu_j(\infty)}} = \rho_{i,j} \frac{\mathbb{E}\sqrt{\nu_{ij}(\infty)}}{\sqrt{\bar{\nu}_i \bar{\nu}_j}}$ *almost surely, where $\nu_i(\infty)$, $\nu_j(\infty)$ and $\nu_{ij}(\infty)$ denote random variables having the stationary distributions of the processes $(\nu_i(t))$, $(\nu_j(t))$ and $(\nu_i(t)\nu_j(t))$ respectively.*

Proof. A straightforward transformation of the expression for $\widehat{\rho}^{\text{emp}}_{i,j:K_n(T)}(\Sigma)$ yields

$$\widehat{\rho}^{\text{emp}}_{i,j:K_n(T)}(\Sigma) = \frac{\sum_{k=1}^{K_n(T)} X_i(t_k)X_j(t_k) - K_n(T)\bar{X}_i\bar{X}_j}{\sqrt{\sum_{k=1}^{K_n(T)} X_i^2(t_k) - K_n(T)\bar{X}_i^2}\sqrt{\sum_{k=1}^{K_n(T)} X_j^2(t_k) - K_n(T)\bar{X}_j^2}}. \tag{7}$$

Following the model (3) we have

$$S_i(t) = S_i(0)\exp\left[\int_0^t \sqrt{\nu_i(s)}\mathrm{d}W_i(s) + \int_0^t (r_i(s) - q_i(s))\mathrm{d}s - \frac{1}{2}\int_0^t \nu_i(s)\mathrm{d}s\right]$$

and the very definition of quadratic variation implies that almost surely the following convergence holds:

$$\begin{aligned}\sum_{k=1}^{K_n(T)} X_i(t_k)X_j(t_k) &= \sum_{k=1}^{K_n(T)} (\log S_i(t_k) - \log S_i(t_{k-1})(\log S_j(t_k) - \log S_j(t_{k-1}) \\ &\underset{n\to\infty}{\longrightarrow} \langle \log S_i, \log S_j\rangle_T = \rho_{i,j}\int_0^t \sqrt{\nu_i(s)\nu_j(s)}\mathrm{d}s.\end{aligned} \tag{8}$$

On the other hand we have

$$K_n(T)\bar{X}_i\bar{X}_j = \frac{1}{K_n(T)}\log\frac{S_i(T)}{S_i(0)}\log\frac{S_j(T)}{S_j(0)} \underset{n\to\infty}{\longrightarrow} 0, \quad \mathbb{P}-\text{a.s.} \tag{9}$$

Combining (7), (8) and (9) we obtain

$$\lim_{n\to\infty} \widehat{\rho}^{\text{emp}}_{i,j:K_n(T)}(\Sigma) = \rho_{i,j}\frac{\int_0^T \sqrt{\nu_i(s)\nu_j(s)}\mathrm{d}s}{\sqrt{\int_0^T \nu_i(s)\mathrm{d}s}\sqrt{\int_0^T \nu_j(s)\mathrm{d}s}}. \tag{10}$$

Letting $T \to 0$ in (10) and recalling the continuity of ν_i we obtain the first claim in the theorem.

For the second claim we need the ergodicity of the processes ν_i, ν_j and (ν_i, ν_j). This is well-known for the one dimensional CIR process. In the Appendix we provide the ergodicity of (ν_i, ν_j) as we were not able to find a reference for this fact in the existing literature. Clearly, the two dimensional result implies the ergodicity of the one dimensional marginals. We use the approach as described in Has'minskii [1980]. Once the ergodicity of the above processes is verified, we can apply the ergodic theorem to obtain the following almost sure convergence:

$$\lim_{n\to\infty} \widehat{\rho}^{\,\mathrm{emp}}_{i,j:K_n(T)}(\Sigma) = \rho_{i,j} \frac{\frac{1}{T}\int_0^T \sqrt{\nu_i(s)\nu_2(s)}\mathrm{d}s}{\sqrt{\frac{1}{T}\int_0^T \nu_i(s)\mathrm{d}s}\sqrt{\frac{1}{T}\int_0^T \nu_2(s)\mathrm{d}s}} \overset{T\to\infty}{\longrightarrow} \rho_{i,j} \frac{\mathbb{E}\sqrt{\nu_i(\infty)\nu_2(\infty)}}{\sqrt{\mathbb{E}\nu_i(\infty)}\sqrt{\mathbb{E}\nu_2(\infty)}}\,.$$

Clearly the expectation of the invariant distribution of $\nu_i(t)$ is the mean reversion level $\bar{\nu}_i$ for all $i = 1, \ldots, d$. This completes the proof. □

Remark 2.4.

(1) The first part of Theorem 2.3 states that calculating correlations from high-frequency data observed over a rather short period renders $\widehat{\Sigma}^{\mathrm{emp}}(\Sigma) = \Sigma$. Thus $\Sigma = \Sigma^{\mathrm{emp}}$, and the correlation matrix is therefore not adjusted.

(2) Note, that the first part of Theorem 2.3 is a theoretical statement. High-frequency data -if available at all- is typically subject to other effects, e.g., caused by the market micro-structure. These effects are making the observations noisy and are potentially distorting the correlation estimates. Therefore, we adopt the second statement as our approach for estimating the correlation structure.

Remark 2.5.

(1) The Cauchy-Schwarz inequality immediately gives $|\widehat{\rho}^{\,\mathrm{emp}}_{i,j;n}(\Sigma)| \leq |\rho_{i,j}|$ asymptotically for big n.

(2) The empirical correlation is approximately linear in ρ with a positive slope strictly smaller than 1. Varying the model correlation $\rho_{i,j}$ by $\Delta\rho$, we can write

$$\widehat{\rho}^{\,\mathrm{emp}}_{i,j;n}(\Sigma + \Delta\rho) \approx \widehat{\rho}^{\,\mathrm{emp}}_{i,j;n}(\Sigma) + \Delta\rho\, \frac{\mathbb{E}\sqrt{\nu_i(s)\nu_j(\infty)}}{\sqrt{\mathbb{E}\nu_i(\infty)}\sqrt{\mathbb{E}\nu_j(\infty)}}\,.$$

where the distribution of $(\nu_1(\infty), ..., \nu_d(\infty))$ *depends on* Σ.

(3) The range of the possible empirical correlations is strictly smaller than $[-1, 1]$*, except for the case if two single-asset Heston sub-models are perfectly correlated in the asset price and the variance rate process, which we want to exclude as it introduces singularity into the model.*

3 Overview of the proposed calibration procedures

We are given the parameters $\theta_i = (\rho_i, \kappa_i, \overline{\nu}_i^2, \nu_i^2(0), \eta_i)$ for each single-asset Heston model obtained by single-asset Heston calibration routines. These parameters describe the dynamics of the model under the equivalent martingale measure. It remains to calibrate the correlation structure between the one-dimensional models. The ideal situation would be if we had a liquid market of correlation sensitive cross-asset derivatives which can be used for calibration. Unfortunately, the existence of such markets is rather an exception and therefore we focus on the alternative to use the historical asset time series to determine the correlations, which is done by calculating the historical empirical cross-asset correlations and extracting the Σ out of them via passing to the physical measure. Observe that Σ is invariant under equivalent measure change. The input of the algorithm are the historical time series of the assets.

In some situations it is sensible to assume that the cross-asset empirical correlations *under the equivalent martingale measure* are provided and our task is to calibrate the infinitesimal correlations Σ directly under the relevant equivalent martingale measure. In this case we have as an input the expected empirical correlations.

In the second setup, calibrating the yet unknown parameters contained in the correlation matrix Σ^S is then solving the problem

$$\min_{\Sigma \in \mathrm{Cor}(d)} \left\| \mathbb{E}\widehat{\Sigma}^{\mathrm{emp}}(\Sigma) - \Sigma^{\mathrm{emp}} \right\| , \tag{11}$$

where $\mathrm{Cor}(d)$ is the space of $d \times d$-dimensional correlation matrices, i.e., $d \times d$-matrices over $\mathbb{R}$ that are symmetric, positive semi-definite and unit elements on the main diagonal, and $\| \cdot \|$ is a suitable matrix-norm.

4 Calibration under the equivalent martingale measure

We first start by describing the algorithm in the setup where we assume the cross-asset empirical correlations *under the equivalent martingale measure* are provided and we aim to determine Σ. We call the algorithm *calibration with correlation adjustment*. The first setup, where we extract the empirical correlations from the historical data is provided in section 5.

4.1 Correlation adjustment algorithm

The input is a $d \times d$-dimensional matrix Σ^{emp}, given by the user, which does not necessarily have to be positive semi-definite. It only has to be symmetric with diagonal entries of value 1. This is then taken as the expected *empirical correlation matrix under the equivalent martingale measure*. From section 2.2 we know, that we do not need to solve a d-dimensional problem, but $d(d-1)/2$ 2-dimensional problems. For these 2-dimensional problems, we propose an algorithm that calibrates the model and returns the desired correlation matrix Σ^s.

First, the two-asset model and its calibration is investigated. With given Heston parameters θ_1 and θ_2 for the two models, we have $\mathbb{E}\widehat{\rho}_{1,2}^{\text{emp}}(\rho_{1,2}) = f_{(\theta_1,\theta_2)}(\rho_{1,2})$, for a strictly increasing and continuous function $f_{(\theta_1,\theta_2)} : [-1,1] \mapsto \mathbb{R}$, parameterized by θ_1 and θ_2. Given that $\rho_{1,2}^{\text{emp}}$ is contained in the range $\{f_{(\theta_1,\theta_2)}(x) : |x| \leq 1\}$, the model can be uniquely calibrated by numerically solving the equation $\mathbb{E}\widehat{\rho}_{1,2}^{\text{emp}}(\rho_{1,2}) = \rho_{1,2}^{\text{emp}}$.[3]

[3]Remember that here $\rho_{1,2}^{\text{emp}}$ is provided by the user.

4.1.1 Two-Asset Calibration

Setting $d = 2$, we can expand (3) to the four-dimensional system:

$$\begin{pmatrix} dS_1(t) \\ d\nu_1^2(t) \\ dS_2(t) \\ d\nu_2^2(t) \end{pmatrix} = \begin{pmatrix} S_1(t)(r(t) - q(t)) \\ \kappa_1 \left(\overline{\nu}_1^2 - \nu_1^2(t)\right) \\ S_2(t)(r(t) - q(t)) \\ \kappa_2 \left(\overline{\nu}_2^2 - \nu_2^2(t)\right) \end{pmatrix} dt \tag{12}$$

$$+diag \begin{pmatrix} S_1(t)\,\nu_1(t) \\ \eta_1 \sqrt{\nu_1^2(t)} \\ S_2(t)\,\nu_2(t) \\ \eta_2 \sqrt{\nu_2^2(t)} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ \rho_1 & \sqrt{1-\rho_1^2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \rho_2 & \sqrt{1-\rho_2^2} \end{pmatrix} \begin{pmatrix} dW_1(t) \\ d\widetilde{W}_1(t) \\ dW_2(t) \\ d\widetilde{W}_2(t) \end{pmatrix}, \tag{13}$$

where $W_1(t)$ and $W_2(t)$ are correlated with $\rho_{1,2}$, and the pairs $W_1(t)$ and $\widetilde{W}_1(t)$, $W_2(t)$ and $\widetilde{W}_2(t)$ and $\widetilde{W}_1(t)$ and $\widetilde{W}_2(t)$ are independent.

For $|\rho_{1,2}| < 1$ we apply the standard Gram-Schmidt-type orthogonalization procedure and define $\overline{W}_1(t) = W_1(t)$ and $\overline{W}_2(t) = (\rho_{1,2}\, W_1(t) - W_2(t))/\sqrt{1-\rho_{1,2}^2}$.[4] Then $(\overline{W}_1(t), \widetilde{W}_1(t), \overline{W}_1(t), \widetilde{W}_2(t))$ is a Wiener process with uncorrelated components. The latter part of (13) then becomes

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ \rho_1 & \sqrt{1-\rho_1^2} & 0 & 0 \\ \rho_{1,2} & 0 & \sqrt{1-\rho_{1,2}^2} & 0 \\ \rho_2\,\rho_{1,2} & 0 & \rho_2\sqrt{1-\rho_{1,2}^2} & \sqrt{1-\rho_2^2} \end{pmatrix}}_{=:\,L} \begin{pmatrix} d\overline{W}_1(t) \\ d\widetilde{W}_1(t) \\ d\overline{W}_2(t) \\ d\widetilde{W}_2(t) \end{pmatrix}.$$

The correlation matrix is then given by

$$C = L\,L^\top = \begin{pmatrix} 1 & \rho_1 & \rho_{1,2} & \rho_{1,2}\,\rho_2 \\ \rho_1 & 1 & \rho_{1,2}\,\rho_1 & \rho_{1,2}\,\rho_1\,\rho_2 \\ \rho_{1,2} & \rho_{1,2}\,\rho_1 & 1 & \rho_2 \\ \rho_{1,2}\,\rho_2 & \rho_{1,2}\,\rho_1\,\rho_2 & \rho_2 & 1 \end{pmatrix}. \tag{14}$$

[4]Note that for $|\rho_{1,2}| = 1$ we have the linear dependence $W_1(t) = \rho_{1,2}\, W_2(t)$, and $W_2(t)$ and its transform $\overline{W}_2(t)$ drop effectively out of the system.

We see that the cross-correlation over the two distinct asset-volatility sub-models is specified by the asset-volatility correlations ρ_1 and ρ_2 of the single-asset Heston models, and the asset-asset cross-correlation $\rho_{1,2}$, i.e.

$$C^{(1,2)} = \rho_{1,2} \begin{pmatrix} 1 & \rho_2 \\ \rho_1 & \rho_1 \rho_2 \end{pmatrix}.$$

Given the single-asset parameters, the dependence structure can be expressed exclusively by a single parameter, the asset-asset cross-correlation $\rho_{1,2}$.

Remark 4.1. *Let $(X_i, Y_i)_{i=1,\dots,K}$ be a sequence of independent identically distributed random variables, where (X_i, Y_i) follows a bivariate normal distribution with correlation ρ. Then the correlation estimator*

$$\widehat{\rho}_K = \frac{\sum_{i=1}^K (X_i - \bar{X}_K)(Y_i - \bar{Y}_K)}{\sqrt{\sum_{i=1}^K (X_i - \bar{X}_K)^2 \sum_{i=1}^K (Y_i - \bar{Y}_K)^2}}$$

is consistent, i.e, $\widehat{\rho}_K \to \rho$ almost surely, for $K \to \infty$, and the variance is

$$\mathrm{var}(\widehat{\rho}_K) = \frac{1 - \rho^2}{K - 2}.$$

Next, we have to numerically find the values ρ which solve the problem

$$\min_{|\rho| \leq 1} |\mathbb{E}\widehat{\rho}_{i,j}^{\mathrm{emp}}(\rho) - \rho^{\mathrm{emp}}|.$$

This can simply be done by some line search method such as bisectioning. Such a bisection procedure adapted to the given problem is stated here:

Input: Lower bound $\underline{\rho} = -1$, upper bound $\overline{\rho} = 1$,[5] accuracy ϵ

Algorithm: (1) Set $\rho_{i,j}^{\mathrm{empA}} = \underline{\rho}$ and $\rho_{i,j}^{\mathrm{empB}} = \overline{\rho} = 1$

(2) Define $\underline{\mathbb{E}} := \mathbb{E}(\widehat{\rho}_{i,j}^{\mathrm{emp}}(\underline{\rho}))$ and $\overline{\mathbb{E}} := \mathbb{E}(\widehat{\rho}_{i,j}^{\mathrm{emp}}(\overline{\rho}))$ and define $\Delta\underline{\mathbb{E}} := \underline{\mathbb{E}} - \rho_{i,j}^{\mathrm{emp}}$.

(3) Update parameters:

$$\begin{aligned} \rho_{i,j}^{\mathrm{empNew}} &:= 0.5 * \rho_{i,j}^{\mathrm{empA}} - \rho_{i,j}^{\mathrm{empB}} \\ \mathbb{E}^* &:= \mathbb{E}(\widehat{\rho}_{i,j}^{\mathrm{emp}}(\rho_{i,j}^{\mathrm{empNew}})) \\ \Delta\mathbb{E} &:= \mathbb{E}^* - \rho_{i,j}^{\mathrm{emp}}. \end{aligned}$$

[5]The choice of these bounds is reasonable, as we are looking for correlations.

(4) If $\Delta\mathbb{E} < \epsilon \rightarrow$ `STOP` and output $\rho_{i,j}^{\text{empNew}}$.
If $(\Delta\underline{\mathbb{E}} \cdot \Delta\mathbb{E}) < 0 \rightarrow \rho_{i,j}^{\text{empB}} = \rho_{i,j}^{\text{empNew}}$ and `GOTO` step 3.
If $(\Delta\underline{\mathbb{E}} \cdot \Delta\mathbb{E}) > 0 \rightarrow \rho_{i,j}^{\text{empA}} = \rho_{i,j}^{\text{empNew}}$ and `GOTO` step 3.

Output: Optimal parameter $\rho_{i,j}^{\text{empNew}}$.

Throughout the whole procedure the calculation of $\mathbb{E}\widehat{\rho}_{i,j}^{\text{emp}}(\rho)$ is needed. This can be done by simple Monte-Carlo simulation as demonstrated in the following algorithm:

Input: Number of Monte-Carlo simulations N and N simulated paths $(P_i^1, P_i^2), 1 \leq i \leq N$.[6]

Algorithm: (1) Define $S := 0$, $i := 1$

(2) Take paths P_i^1 and P_i^2 and calculate their correlation ρ_P using one of the transformation methods described in Example 1.

(3) Update $S = S + \rho_P$ and $i = i + 1$.
If $i < N$ `GOTO` step 2.

(4) Calculate the mean value $M := S/N$

Output: Monte-Carlo estimate M.

Remark 4.2. *Observe that the $\widehat{\rho}_{i,j}^{\text{emp}}(\rho)$ is actually a ratio of time averages and therefore the ergodic theorem states that it will converge to its expected value for large T. Therefore for calculating the expectation it is enough to take a fairly small number of Monte-Carlo runs. In our tests a value of 10 already provided good results.*

This produces a symmetric matrix $\Sigma = (\rho_{i,j})_{1 \leq i,j \leq d}$, which is our candidate $\Sigma^\star$ to solve equation (11).

The calibration is completed if the symmetric matrix Σ is a correlation matrix, i.e. $\Sigma \in \text{Cor}(d)$, as required in (11). If Σ is not a correlation matrix, we propose three ways of transforming Σ such that the transform is in $\text{Cor}(d)$.

[6]Simulation of these paths can be done by Euler discretization possibly using variance reduction methods such as antithetic paths. Note that the paths are simulated with a correlation of ρ.

4.1.2 Multi-Asset Calibration

In the general multi-asset setting, the two-asset procedure can be carried out for all asset pairs. The correlation matrix has two kinds of blocks

$$C^{(i)} = \begin{pmatrix} 1 & \rho_i \\ \rho_i & 1 \end{pmatrix}, \quad \text{and} \quad C^{(i,j)} = \rho_{i,j} \begin{pmatrix} 1 & \rho_j \\ \rho_i & \rho_i \rho_j \end{pmatrix}. \tag{15}$$

Using these blocks, the whole correlation matrix can be respresented. For example C in (14) can be expressed as

$$C = L\,L^\top = \begin{pmatrix} C^{(1)} & C^{(1,2)} \\ C^{(2,1)} & C^{(2)} \end{pmatrix}. \tag{16}$$

By this block representation, we can rearrange the single blocks, such that for any two assets i and k we obtain a matrix

$$C_{i,k} = L_{i,k}\,L_{i,k}^\top = \begin{pmatrix} C^{(i)} & C^{(i,k)} \\ C^{(k,i)} & C^{(k)} \end{pmatrix}. \tag{17}$$

After we have repeated the bisection procedure for all 2-dimensional problems, we get a candidate $\Sigma^\star$ to solve equation (11). To validate this candidate $\Sigma^\star$, we must check whether $\Sigma^\star$ is positive semi-definite or not. If the test succeeds, the calibration is finished. Otherwise, some regularization has to be done (see section 4.2).

4.2 Generate a valid correlation matrix

We investigated three ways of transforming $\Sigma^\star$ such that the resulting matrix is in $\mathrm{Cor}(d)$.

4.2.1 Regularization by Jäckel [2002]

One way out of this problem is a regularization proposed by Jäckel [2002]. The following algorithm briefly demonstrates how this regularization works:

Input: Model correlation matrix $\Sigma^\star$ (not necessarily positive definite).

Algorithm: (1) Do an eigenvalue decomposition of $\Sigma^\star \rightarrow \Sigma^\star = S\Lambda S^T$, with $\Lambda = \mathrm{diag}(\lambda_i)$. Here, λ_i, $i = 1, ..., d$ represents the eigenvalues of $\Sigma^\star$.

(2) Define the diagonal matrix $\Lambda^{'}$ with $\lambda_i^{'} = \begin{cases} \lambda_i : & \lambda_i \geq 0 \\ 0 : & \lambda_i < 0 \end{cases}$

(3) Create the diagonal matrix T with $t_i := [\sum_m s_{im}^2 \lambda_m^{'}]^{-1}$.

(4) Define $B := \sqrt{T} S \sqrt{\Lambda'}$.

(5) $\widehat{\Sigma^\star} := BB^T$

Output: Positive definite correlation matrix $\widehat{\Sigma}^\star$.

4.2.2 Regularization by Mishra [2004]

Another way out of the problem of non-positive definite correlation matrices is a regularization proposed by Mishra [2004]. The following algorithm briefly demonstrates how this regularization works:

Input: Model correlation matrix $\Sigma^\star$ (not necessarily positive definite).

Algorithm: (1) Do an eigenvalue decomposition of $\Sigma^\star \rightarrow \Sigma^\star = S\Lambda S^T$, with $\Lambda = \text{diag}(\lambda_i)$. Here, λ_i, $i = 1, ..., d$ represents the eigenvalues of $\Sigma^\star$.

(2) Define the diagonal matrix $\Lambda^{'}$ with $\lambda_i^{'} = \begin{cases} \lambda_i : & \lambda_i \geq 0 \\ 0 : & \lambda_i < 0 \end{cases}$

(3) Generate d uniformly $(0, 1)$-distributed random numbers and add them to the diagonal elements of Λ. Normalize Λ such that its trace is equal to d.

(4) By random walk methods of optimization find best possible Λ such that $trace(\lambda) = m$ and $\widehat{\Sigma}^\star = S\Lambda S^T$ has positive determinant and is positive definite and closest (wrt. maximum norm) to $\Sigma^\star$.

(5) `IF` all diagonal entries of $\widehat{\Sigma}^\star$ are approximately 1, depending on a tolerance level `THEN STOP`, `ELSE` set diagonal entries to 1 and `GOTO` step 1.

Output: Positive definite correlation matrix $\widehat{\Sigma}^\star$.

Remark 4.3. *By construction, both algorithms are well defined, as they both deliver a positive definite correlation matrix, if they stop. Obviously, the first two steps of Jaeckel's and*

Mishra's algorithms are equal. In the proceeding steps, Jaeckel uses a rather deterministic way to create the positive definite correlation matrix, while Mishra uses a stochastic algorithm. Unfortunately, there is also a chance of non-convergence for the Mishra Algorithm in a case that the intermediate matrix $\widehat{\Sigma}^\star$ in the intermediate step 4 is near-singular.

4.2.3 Regularization by convex combination

If the input matrix Σ^{emp} is already positive definite, but the model matrix $\Sigma^\star$ is not, there is another easy way of creating a positive definite correlation matrix as a convex combination of the original input matrix and the model matrix. The following algorithm briefly demonstrates how this regularization works.

Input: Correlation matrix Σ (positive definite) and model correlation matrix $\Sigma^\star$ (not necessarily positive definite).

Algorithm: By standard line search algorithms find smallest $\lambda \in [0, 1]$ such that $\widehat{\Sigma}^\star := \lambda\Sigma + (1 - \lambda)\Sigma^\star$ is positive definite.

Output: Positive definite correlation matrix $\widehat{\Sigma}^\star$

Remark 4.4. *By construction, the matrix $\widehat{\Sigma}^\star$ is symmetric and positive definite and its diagonal contains only values 1, and is therefore a valid correlation matrix.*

4.2.4 Comparison of the different methods

We tested the regularization procedure of all three methods for one example. As the convex combination only works if the empirical input matrix is already a correlation matrix, we constructed the example such that this is fulfilled. We chose 10 assets with correlation

$$\rho_{ij} = \begin{cases} 0.8 & \text{if} \quad i \neq j \\ 1 & \text{if} \quad i = j \end{cases} \tag{18}$$

Norm	Jäckel	Mishra	Convex
Euclidean norm	5.66%	8.27%	37.63%
maximum norm	1.03%	1.15%	5.25%
row-sum norm	8.50%	10.00%	39.13%

Table 1: Errors of different methods

except for one asset pair (asset 1 and asset 8), for which we defined a correlation of 0.56. This matrix is a valid correlation matrix. The calibrated matrix $\Sigma^\star$ is in this setting

$$\begin{pmatrix}
100.00\% & 84.61\% & 84.61\% & 83.87\% & 84.61\% & 84.61\% & 84.61\% & 84.61\% & 61.60\% & 84.61\% \\
84.61\% & 100.00\% & 83.13\% & 83.13\% & 83.87\% & 83.87\% & 83.13\% & 83.87\% & 83.87\% & 83.13\% \\
84.61\% & 83.13\% & 100.00\% & 83.13\% & 83.87\% & 83.87\% & 83.87\% & 83.87\% & 83.87\% & 83.13\% \\
83.87\% & 83.13\% & 83.13\% & 100.00\% & 83.13\% & 83.13\% & 83.13\% & 83.87\% & 83.87\% & 83.13\% \\
84.61\% & 83.87\% & 83.87\% & 83.13\% & 100.00\% & 84.61\% & 84.61\% & 85.35\% & 85.35\% & 85.35\% \\
84.61\% & 83.87\% & 83.87\% & 83.13\% & 84.61\% & 100.00\% & 84.61\% & 85.35\% & 84.61\% & 84.61\% \\
84.61\% & 83.13\% & 83.87\% & 83.13\% & 84.61\% & 84.61\% & 100.00\% & 84.61\% & 84.61\% & 84.61\% \\
84.61\% & 83.87\% & 83.87\% & 83.87\% & 85.35\% & 85.35\% & 84.61\% & 100.00\% & 84.61\% & 84.61\% \\
61.60\% & 83.87\% & 83.87\% & 83.87\% & 85.35\% & 84.61\% & 84.61\% & 84.61\% & 100.00\% & 84.61\% \\
84.61\% & 83.13\% & 83.13\% & 83.13\% & 85.35\% & 84.61\% & 84.61\% & 84.61\% & 84.61\% & 100.00\%
\end{pmatrix}$$

This matrix has negative eigenvalues and therefore it is not positive semi-definite. After trying the different regularizations we can compute the error in terms of some matrix norm. According to different norms, the errors can be seen in table 1. For the chosen setting, Jäckel clearly performs best. Mishra also produces good results. Although convex combination is the easiest method, it returns the worst results.

To do further investigations, we also calculated an error matrix $\Sigma_{\text{error}} := |\widehat{\Sigma}^\star - \Sigma^\star|$. Figures 1 to 3 graphically show the distribution of these errors in the error matrices for the different methods. To illustrate the differences of the three methods, we applied the same scale to every graph. In figure 1 one clearly sees, that the error is concentrated vertically and horizontally from the critical value, which was the correlation of asset 1 and asset 9. Using Mishra's method, the errors in figure 2 are distributed more equally all over the error matrix, however, the maximum error is worse than for Jäckel. What we can already derive from the numbers in table 1 gets even more obvious when we take a look at figure 3 for the case of convex combination. This method almost completely fails and there are large errors all over the whole matrix.

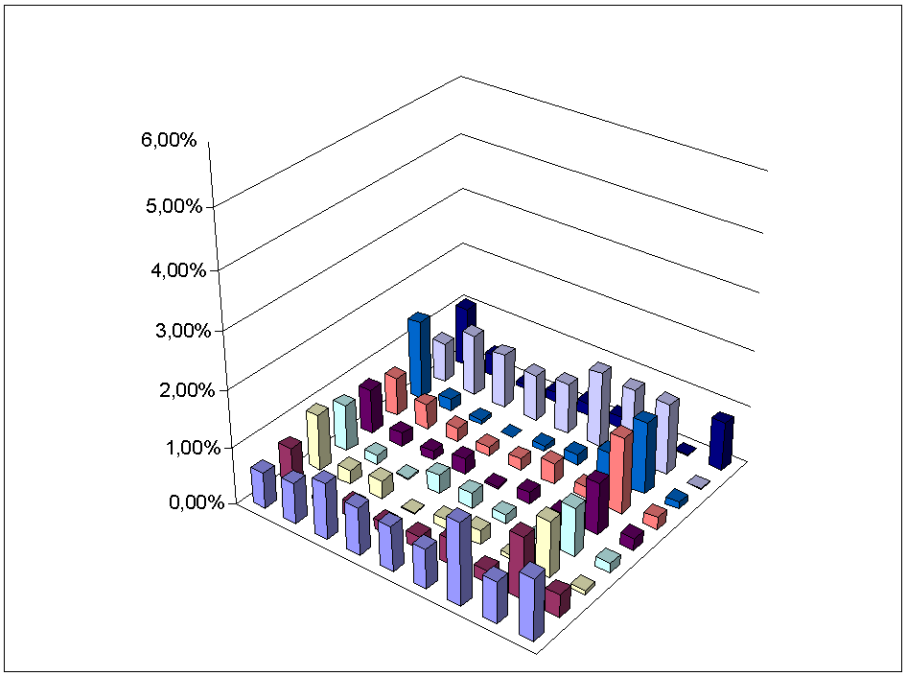

Figure 1: Jäckel regularization

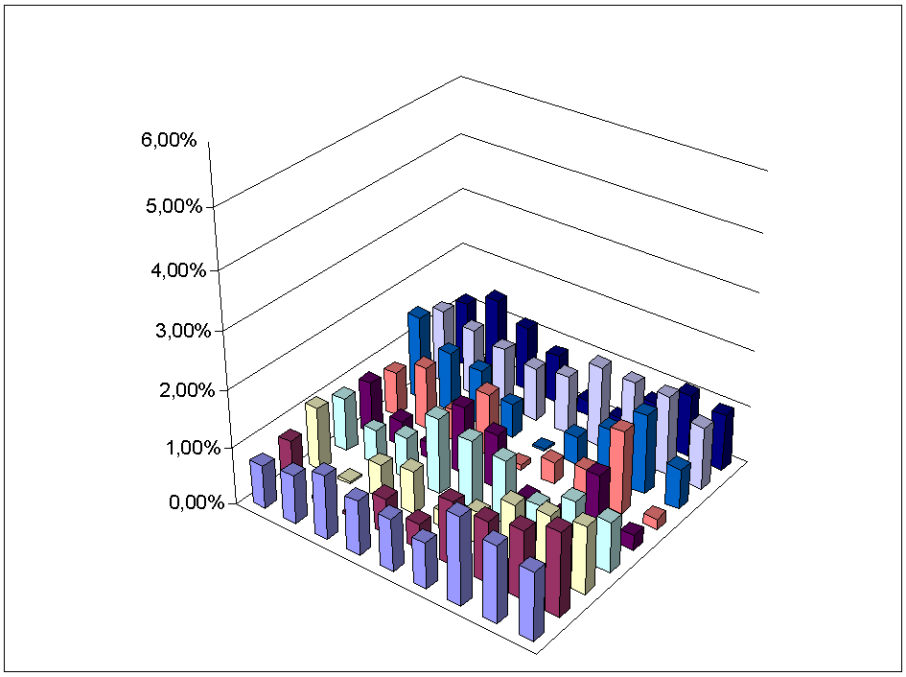

Figure 2: Mishra regularization

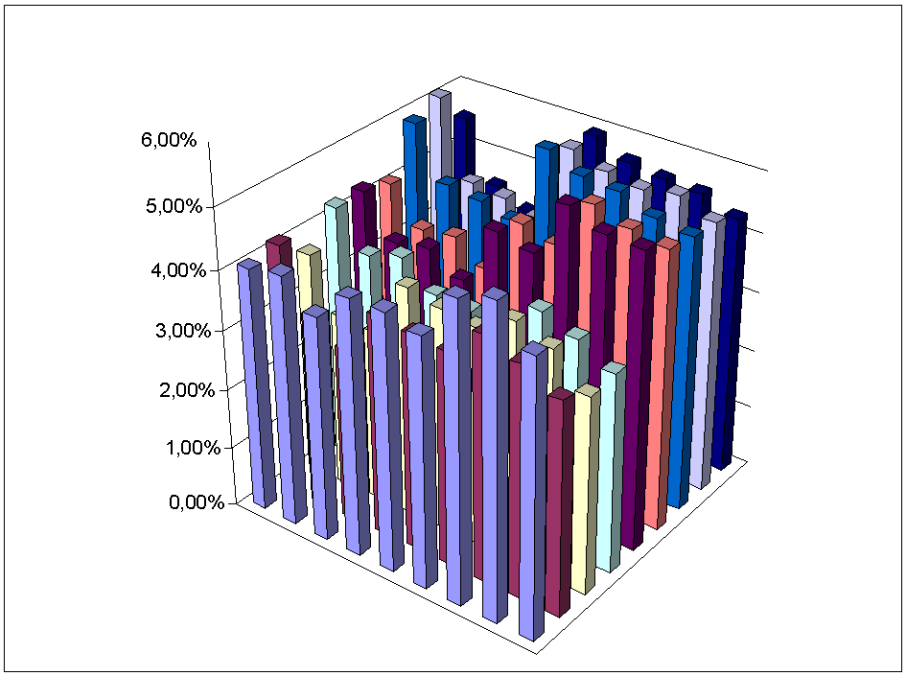

Figure 3: Convex combination regularization

So, in this case the regularization by Jäckel clearly does best.

5 Calibrating the correlations using historical time series

As already mentioned, in case we do not have liquid correlations dependent derivatives and we are not provided with the expected cross-asset empirical correlations under the pricing measure, we are left with the only option to determine the correlations historically. We will treat only the transition to the physical measure and the estimation of the correlations ρ_{ij} in the two asset case. The regularization to a valid correlation matrix is identical to the previous section.

5.1 Algorithm

The input is the d-dimensional historical time series $(S_1(t_k), ..., S_d(t_k))_{k=0,...,K}$ of the assets in interest. They are then transformed into log returns $X_i(t_k) := \log \frac{X_i(t_k)}{X_i(t_{k-1})}$ and the

historical empirical correlations $\widehat{\rho}^{\text{emp}}_{i,j:K}$ are defined as in (5).

The next step is to determine the historical mean reversion level of the volatilities of all single-asset models. Using only the historical mean reversion parameters and retaining Heston's original assumption on the shape of the market price of risk, we can deduce the dynamics of the single-asset models under the physical measure.

At the end we calibrate the correlation matrix Σ using the historical empirical correlation by the correlation adjustment method presented above. The only difference is that the simulations are being performed under the physical measure rather than the equivalent martingale measure.

5.2 Dynamics under the physical measure

According to the original article Heston [1993] the market price of volatility risk has the form $\lambda \cdot \nu$ (we omit the subscripts indicating the assets as we only treat the one dimensional Heston in this section) and consequently the model parameters θ under the pricing measure and the parameters θ^* under the physical measure are connected via the following relations

$$\begin{aligned} \rho &= \rho^*, \;\; \eta = \eta^* \\ \kappa &= \kappa^* + \lambda \\ \bar{\nu} &= \frac{\kappa^* \bar{\nu}^*}{\kappa^* + \lambda}. \end{aligned} \tag{19}$$

From the above equations we immediately conclude that $\kappa^* \cdot \bar{\nu}^* = \kappa \cdot \bar{\nu}$ and therefore it is enough to estimate the mean reversion level $\bar{\nu}^*$ under the physical measure.

We chose the estimator based on the following well-known convergence.

Proposition 5.1. *Let $(S(t), \nu(t))$ be the processes defined in (1) and (2). Let $\pi_n = (t_{k,n})_{k=1,\dots,K_n(T)}$ be partitions of $[0,T]$, with $mesh(\pi_n) = \max_{i=1,\dots,K_n(T)} |t_{i,n} - t_{i-1,n}| \to 0$, for $n \to \infty$. Then for the log returns $X(t_k) := \log \frac{S(t_k)}{S(t_{k-1})}$*

$$\lim_{T \to 0} \lim_{n \to \infty} \frac{1}{T} \sum_{k=1}^{K_n(T)} X^2(t_k) = \bar{\nu}^* \text{ almost surely}$$

Proof. We know that almost surely

$$\sum_{k=1}^{K_n(T)} X^2(t_k) = \sum_{k=1}^{K_n(T)} (\log S(t_k) - \log S(t_{k-1}))^2 \underset{n\to\infty}{\longrightarrow} \langle \log S \rangle_T = \int_0^T \nu(s)\mathrm{d}s$$

and the ergodic theorem implies

$$\frac{1}{T}\int_0^T \nu(s)\mathrm{d}s \underset{T\to\infty}{\longrightarrow} \bar{\nu}\text{a.s.}$$

Combining the above facts proves the statement. □

A straightforward estimator for the mean reversion level under the physical measure is obtained by stipulating that for large T and every $i = 1, \ldots, d$ we have $\sum_{k=1}^{K_n(T)} X_i^2(t_k) \approx T\bar{\nu}_i^*$, i.e. we set

$$\widehat{\bar{\nu}}_i^* := \frac{1}{T}\sum_{k=1}^{K} X_i^2(t_k) \text{ for all } i = 1, \ldots, d. \tag{20}$$

5.3 Calibration of Σ

The two-asset calibration of the correlations $\rho_{i,j}$ can be performed analogously to the procedure in Section 4.1.1, with the technical difference that we do not use the parameters under the risk neutral measure to generate the paths and calculate $\mathbb{E}\widehat{\rho}_{i,j}^{\text{emp}}(\rho_{i,j})$, but we rather use the parameters under the physical measure. Obviously the regularization of the matrix obtained by the successive two-asset calibrations in order to end up with a valid correlation matrix does not depend on the measure under which we perform the calibration and is exactly as in Section 4.2.

6 Conclusion

In this paper we presented a parsimonious multi-asset Heston model. All single-asset sub-models follow the well-known Heston dynamics and their parameters are calibrated as usual on implied market volatilities. Our focus was on the calibration of the correlation structure between the single-asset marginals in the absence of sufficient liquid cross-asset

option price data. We presented two general calibration setups corresponding to relevant practical situations: (1) when the empirical cross-asset correlations in the risk neutral world are given by the user and we need to calibrate the correlations between the driving Brownian motions or (2) when they have to be estimated from the historical time series. The theoretical background, including the ergodicity of the multidimensional CIR process, for the proposed estimators are worked out.

The presented model is parsimonious in the sense that $d(d-1)/2$ asset-asset cross-correlations are required for a d-asset Heston model. This correlation matrix can be used to simulate multi-asset dynamics using the Heston model. We also presented three methods for obtaining a valid correlation matrix, if the empirical matrix given by the user is not or the calibrated cross-asset correlations do not form such a matrix. In addition, we compared the three methods by giving numerical results to state the usefulness of the proposed approaches.

A Ergodicity of the two dimensional CIR process

Theorem A.1. *Let $\nu := (\nu_1, \nu_2)$ be a two dimensional CIR process with parameters $\kappa_1, \kappa_2, \eta_1, \eta_2, \bar{\nu}_1, \bar{\nu}_2 > 0$ and $\rho \in (-1, 1)$, i.e. it is a continuous Markov process with infinitesimal generator*

$$\mathcal{A} = \kappa_1(\bar{\nu}_1 - x)\partial_x + \kappa_2(\bar{\nu}_2 - y)\partial_y + \frac{1}{2}\eta_1^2 x\partial_{xx} + \frac{1}{2}\eta_2^2 y\partial_{yy} + \rho\eta_1\eta_2\sqrt{xy}\partial_{xy}.$$

ν has a stationary probability distribution $\mu_\infty(\mathrm{d}x\mathrm{d}y)$ on $\mathbb{R}^2_+$, and moreover, for every μ-integrable $f : \mathbb{R}^2_+ \to \mathbb{R}$ we have for μ-almost all (x, y)

$$\mathbb{P}_{(x,y)}\left[\lim_{T\to\infty}\frac{1}{T}\int_0^T f(\nu_1(t), \nu_2(t))\mathrm{d}t = \int f(u, v)\mu(\mathrm{d}u\mathrm{d}v)\right] = 1.$$

Remark A.2. *We actually need the above result for the function $f(x, y) = \sqrt{xy}$. The fact that f is μ-integrable is a simple consequence of the Hölder inequality. Indeed*

$$\int \sqrt{xy}\mu(\mathrm{d}x\mathrm{d}y) \le \sqrt{\int x\mu(\mathrm{d}x\mathrm{d}y)\int y\mu(\mathrm{d}x\mathrm{d}y)} = \sqrt{\bar{\nu}_1 \cdot \bar{\nu}_2}\,.$$

Proof. The statement of the theorem is a special case of Theorem 5.1, Chapter IV of Has'minskiĭ. However, we need verify that the following condition holds: there is a bounded domain D in $\mathbb{R}^2_+$ having the following properties:

(1) The smallest eigenvalue of the diffusion matrix of the generator $\mathcal{A}$ is bounded away from zero on D.

(2) For every $(x, y) \in \mathbb{R}^2_+ \setminus D$ the mean time τ at which a trajectory $(\nu_1^x(t), \nu_2^y(t))$ started at (x, y) reaches U is finite, and even $\sup_{(x,y)\in K} \tau < \infty$ for every compact set $K \subset \mathbb{R}^2_+ \setminus D$.

We first present a rather concise solution in the case where both one dimensional CIR processes do not reach the origin (i.e. the stability condition $2\kappa_i\bar{\nu}_i \geq \eta_i^2$ is satisfied). In this case the state space is $E := (0, \infty)^2$ as $P(\nu_i(t) > 0 \forall t) = 1,.i = 1, 2$.

The first condition will obviously be satisfied if the closure of D is contained in E. Indeed, the eigenvalues are given by

$$\lambda_{1,2} = \frac{\bar{\eta}_1^2 x + \bar{\eta}_2^2 y \pm \sqrt{(\bar{\eta}_1^2 x + \bar{\eta}_2^2 y)^2 + 4(\rho^2 - 1)\bar{\eta}_1^2 \bar{\eta}_2^2 xy}}{2},$$

which can be uniformly bounded away from zero on any compact set contained in E as long as $\rho^2 < 1$.

The second condition per definition actually states that the process η has to be D-recurrent (see Chapter III, Section 7 in Has'minskii [1980]). According to Theorem 7.1 in Chapter III in the same source, a sufficient condition for the D-recurrence is the existence of a nonnegative (Lyapunov) function $V(s, x)$ on $R_+ \times D^C$, twice continuously differentiable in the space variable and continuously differentiable in the time variable and such that

$$\mathcal{A}V \leq -\delta \text{ for some positive } \delta > 0 \text{ on } R_+ \times D^C.$$

As ν is a time homogeneous diffusion we consider Lyapunov functions depending only on the state variable. We take

$$V(x, y) = ax \ln x + by \ln y + c : E \to \mathbb{R},$$

where $a, b, c > 0$. V is a C^2 function on E. Since the mapping $x \mapsto x \ln x$ is bounded from below on E for every set of parameters $a, b > 0$ we can achieve $V(x, y) > 0$ on E by choosing c big enough. Now observe that

$$\begin{aligned}\mathcal{A}V(x,y) =& \kappa_1(\bar{\nu}_1 - x)a(1 + \ln x) + \kappa_2(\bar{\nu}_2 - y)b(1 + \ln y) + \frac{1}{2}(\eta_1^2 a + \eta_2^2 b) \\ =& -\kappa_1 a(x + x \ln x) + \kappa_1 a\bar{\nu}_1 \ln x - \kappa_2 b(y + y \ln y) + \kappa_2 b\bar{\nu}_2 \ln y \\ &+ \frac{1}{2}(\eta_1^2 a + \eta_2^2 b) + \kappa_1 a\bar{\nu}_1 + \kappa_2 b\bar{\nu}_2.\end{aligned} \tag{21}$$

Since of course $x \ln x$ and $y \ln y$ outperform the other terms for large x and y, the set M on which $\mathcal{A}V$ is bigger than say -1 is a bounded subset of E. Furthermore, letting $x \to 0$ we have $\mathcal{A}V(x, y) \to -\infty$ for all $y > 0$ and analogously for $y \to 0$ we get $\mathcal{A}V(x, y) \to -\infty$ for arbitrary $x > 0$. Thus we conclude that the closure of M is still contained in E. Indeed, assume e.g. $(x_0, 0)$ is in the closure of M. Then we would have a sequence $M \ni (x_n, y_n) \to (x_0, 0)$ and we would get $-\infty = \lim_{n\to\infty} \mathcal{A}V(x_n, y_n) \geq -1$. Summarizing, M is a bounded subset of E, such that the closure of M is also in E and we have that $\mathcal{A}V \leq -1$ on M^C. The nonnegativity of V on the whole E is achieved by setting $c > 0$ big enough. This verifies the conditions (1) and (2) stated in the beginning of the proof.

We will now sketch the proof in the general case. We need to set $E = [0, \infty)^2$ as the one dimensional CIR process may reach the zero boundary. For small enough $\epsilon > 0$ define function $g^\epsilon : [0, \infty)$ to be the quadratic truncation of $x \ln x$, i.e.

$$g(x) = \begin{cases} x \ln x & \text{if } x > \epsilon \\ a_\epsilon x^2 + b_\epsilon x + c_\epsilon & x \in [0, \epsilon] \end{cases},$$

where the coefficients a_ϵ, b_ϵ and c_ϵ are chosen in such a manner that the function $g(x)$ is C^2. We now consider the Lyapunov function

$$V^\epsilon(x, y) = ag^\epsilon(x) + bg^\epsilon(y) + c : E \to \mathbb{R}$$

with $a, b, c > 0$. The nonnegativity can be achieved as above by choosing c large enough. An analogous leading term argument implies that the set M on which $\mathcal{A}V^\epsilon(x, y)$ is bigger than -1 is bounded. A rather technical calculation shows that by choosing ϵ small enough

we can make $\mathcal{A}V^\epsilon(x,y) \leq -2$ on the boundary of E. Clearly, there is no surprise as we approximate the case, which we have worked out above. This, however, proves that actually the closure of M is contained in the interior of E, as otherwise we would have for example a sequence $M \ni (x_n, y_n) \to (x_0, 0)$ and we would get $-2 \geq \lim_{n\to\infty} \mathcal{A}V^\epsilon(x_n, y_n) \geq -1$. Therefore the set M satisfies the requirements of the conditions (1) and (2) in the beginning of the proof and we are done.

An alternative way would be the case when g^ϵ is the linear rather than quadratic truncation of $x \ln x$. This would actually also formally suffice even if the resulting V^ϵ is only a C^1 rather than C^2 function. The reason that it still works is that the proof of Theorem 8.1 in Chapter III in Has'minskiĭ which we invoke relies on Itô's formula and it still holds for C^1 functions with existing and continuous second derivatives everywhere but on a countable exception set and having bounded second derivatives around the singularities.

□

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