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A guide on the implementation of the Heath-Jarrow-Morton Two-Factor Gaussian Short Rate Model (HJM-G2++)

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Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Fraunhofer Institut
Techno- und
Wirtschaftsmathematik

A Guide on the Implementation of the Heath-Jarrow-Morton Two-Factor Gaussian Short Rate Model (HJM-G2++)

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1.1 Motivation

In the literature, there are at least two equivalent two-factor Gaussian models for the instantaneous short rate. These are the original two-factor Hull White model (see [3]) and the G2++ one by Brigo and Mercurio (see [1]). Both these models first specify a time homogeneous two-factor short rate dynamics and then by adding a deterministic shift function $\varphi(\cdot)$ fit exactly the initial term structure of interest rates. However, the obtained results are rather clumsy and not intuitive which means that a special care has to be taken for their correct numerical implementation.

On the other side, as noticed by Heath-Jarrow-Morton (1992) (HJM), virtually any exogenous short rate model can be derived within the HJM framework by choosing the class of the forward rate volatilities. By starting within the HJM framework and limiting the volatility of the forward rate to a deterministic, exponentially decaying one, we derive another two-factor Gaussian short rate model which has a very simple and intuitive form. Moreover, no fitting to the initial term structure is required as this is contained in the model by its construction. Additionally, the dynamics of the underlying factors under the forward risk-neutral measure is a very simple one, which facilitates the derivation of closed-form solutions. We call the offered construction HJM-G2++ model due to its similarity to the G2++ model of Brigo and Mercuirio.

In this report, we investigate the offered HJM-G2++ model and derive closed-form solutions to the standard options needed for calibration purposes (caps, floors, bond options and swaptions). Further, we calibrate the model parameters to at-the-monety (ATM) Caps market volatilities and ATM swaption volatility surfaces and make comments on the ability of the model to fit real market data.

Finally, we deal with different methods for the numerical implementation of HJM-G2++ for pricing complex claims with no closed-form solution. For that purpose, we offer a Monte Carlo simulation scheme and a lattice approximation with a construction similar to the one of Li, Ritchken and Sankarasubramanian [4] (for the Cheyette model [2]).

1.2 Definition of the HJM-G2++ model

Let us assume that the instantaneous forward rate follows a two-factor HJM model and its volatility is deterministic and given by

$$\sigma(t,T) = \sigma_1 e^{-\kappa_1(T-t)} + \sigma_2 e^{-\kappa_2(T-t)}$$

with $k_i \in \mathbb{R}^+$, $\sigma_i \in \mathbb{R}^+$, $i = 1, 2$. The forward rate equation for the so-defined volatility can be witten as

$$f(t,T) = f(0,T) + \sum_{i=1}^{2} e^{-\kappa_i(T-t)} \left[X_i(t) + \sum_{k=1}^{2} b_k(t,T) Z_{ik}(0,t) \right] \tag{1}$$

for

$$\begin{aligned} b_1(t,T) &= \frac{1 - e^{-\kappa_1(T-t)}}{\kappa_1}, \quad \kappa_1 \neq 0 \\ b_1(t,T) &= T - t, \quad \kappa_1 = 0; \\ b_2(t,T) &= \frac{1 - e^{-\kappa_2(T-t)}}{\kappa_2}, \quad \kappa_2 \neq 0 \\ b_2(t,T) &= T - t, \quad \kappa_2 = 0; \end{aligned}$$

and cumulative quadrative variation that is defined as

$$\begin{aligned} Z_{11}(u,t) &= \int_u^t \text{cov}(X_1(s), X_1(s)) ds = \int_u^t \sigma_1^2 e^{-2\kappa_1(t-s)} ds = \frac{\sigma_1^2 (1 - e^{-2\kappa_1(t-u)})}{2\kappa_1} \\ Z_{12}(u,t) &= \int_u^t \text{cov}(X_1(s), X_2(s)) ds \\ &= Z_{21}(u,t) = \int_u^t \rho_{1,2}\, \sigma_1 \sigma_2 e^{-(\kappa_1+\kappa_2)(t-s)} ds \\ &= \frac{\rho \sigma_1 \sigma_2}{\kappa_1 + \kappa_2} \left(1 - e^{-(\kappa_1+\kappa_2)(t-u)} \right) \\ Z_{22}(u,t) &= \int_u^t \text{cov}(X_2(s), X_2(s)) ds = \int_u^t \sigma_2^2 e^{-2\kappa_2(t-s)} ds = \frac{\sigma_2^2 (1 - e^{-\kappa_2(t-u)})}{2\kappa_2} \end{aligned}$$

Using $r(t) = f(t,t)$ we can find from (1) the short rate to be

$$r(t) = f(0,t) + X_1(t) + X_2(t) \tag{2}$$

where following Cheyette's model (see [2]) the state variables $X_1(t)$ and $X_2(t)$ are respectively defined under an Equivalent Martingale Measure Q as

$$dX_1(t) = \left(-\kappa_1 X_1(t) + \sum_{k=1}^{2} Z_{1,k}(0,t) \right) dt + \sigma_1 d\tilde{W}_1^Q(t), \quad X_1(0) = 0 \tag{3}$$

$$dX_2(t) = \left(-\kappa_2 X_2(t) + \sum_{k=1}^{2} Z_{2,k}(0,t) \right) dt + \sigma_2 d\tilde{W}_2^Q(t), \quad X_2(0) = 0 \tag{4}$$

with correlation $d\langle \tilde{W}^1, \tilde{W}^2 \rangle_t = \rho dt$.

The price of a zero coupon bond $P(t,T)$ at time t can easily be calculated using integration of the forward rate to be

$$\begin{aligned}
P(t,T) &= E\left(e^{-\int_t^T r(s)ds}|\mathcal{F}_t\right) = e^{-\int_t^T f(t,u)du} \\
&= \frac{P(0,T)}{P(0,t)}\exp\Bigg(-b_1(t,T)X_1(t) - b_2(t,T)X_2(t) \\
&\quad -\frac{1}{2}\sum_{i,j=1}^{2} b_i(t,T)b_j(t,T)Z_{ij}(0,t)\Bigg).
\end{aligned}$$

1.3 Closed form solutions

Using the form of the analytical strong solution of a generalized linear stochastic differential equation, we can write for $s \leq t$

$$\begin{aligned}
X_i(t) &= e^{-\kappa_i(t-s)}\Big(X_i(s) + \int_s^t Z_{i1}(s,u)e^{\kappa_i(u-s)}du + \int_s^t Z_{i2}(s,u)e^{\kappa_i(u-s)}du \\
&\quad + \int_s^t \sigma_i e^{\kappa_i(u-s)}d\tilde{W}_i^Q(u)\Big) \\
&= X_i(s)e^{-\kappa_i(t-s)} + \int_s^t Z_{i1}(s,u)e^{-\kappa_i(t-u)}du + \int_s^t Z_{i2}(s,u)e^{-\kappa_i(t-u)}du \\
&\quad + \int_s^t \sigma_i e^{-\kappa_i(t-u)}d\tilde{W}_i^Q(u)
\end{aligned}$$

In specific for $i = 1$ we have

$$\begin{aligned}
X_1(t) &= X_1(s)e^{-\kappa_1(t-s)} + \int_s^t Z_{11}(s,u)e^{-\kappa_1(t-u)}du + \int_s^t Z_{12}(s,u)e^{-\kappa_1(t-u)}du \\
&\quad + \int_s^t \sigma_1 e^{-\kappa_1(t-u)}d\tilde{W}_1^Q(u) \\
&= X_1(s)e^{-\kappa_1(t-s)} + \int_s^t \frac{\sigma_1^2}{2\kappa_1}\left(1-e^{-2\kappa_1(u-s)}\right)e^{-\kappa_1(t-u)}du \\
&\quad + \int_s^t \frac{\rho_{1,2}\sigma_1\sigma_2}{\kappa_1+\kappa_2}\left(1-e^{-(\kappa_1+\kappa_2)(u-s)}\right)e^{-\kappa_1(t-u)}du + \int_s^t \sigma_1 e^{-\kappa_1(t-u)}d\tilde{W}_1^Q(u) \\
&= X_1(s)e^{-\kappa_1(t-s)} + \frac{\sigma_1^2\left(1-e^{-\kappa_1(t-s)}\right)^2}{2\kappa_1^2} \\
&\quad + \frac{\rho_{1,2}\sigma_1\sigma_2}{(\kappa_1+\kappa_2)\kappa_1\kappa_2}\left[\kappa_2 - (\kappa_1+\kappa_2)e^{-\kappa_1(t-s)} + \kappa_1 e^{-(\kappa_1+\kappa_2)(t-s)}\right] \\
&\quad + \int_s^t \sigma_1 e^{-\kappa_1(t-u)}d\tilde{W}_1^Q(u).
\end{aligned}$$

Thus, we have for the short rate for $s \leq t$

$$\begin{aligned}
r(t) &= f(s,t) + X_1(t) + X_2(t) \\
&= X_1(s)e^{-\kappa_1(t-s)} + X_2(s)e^{-\kappa_2(t-s)} + \frac{\sigma_1^2\left(1-e^{-\kappa_1(t-s)}\right)^2}{2\kappa_1^2} + \frac{\sigma_2^2\left(1-e^{-\kappa_2(t-s)}\right)^2}{2\kappa_2^2} \\
&\quad + \frac{\rho_{1,2}\sigma_1\sigma_2}{\kappa_1\kappa_2}\left(1-e^{-\kappa_1(t-s)}\right)\left(1-e^{-\kappa_2(t-s)}\right) \\
&\quad + \int_s^t \sigma_1 e^{-\kappa_1(t-u)} d\tilde{W}_1^Q(u) + \int_s^t \sigma_2 e^{-\kappa_2(t-u)} d\tilde{W}_2^Q(u)
\end{aligned}$$

where

$$\begin{aligned}
E^Q\big(r(t)|\mathfrak{F}_s\big) &= f(s,t) + X_1(s)e^{-\kappa_1(t-s)} + X_2(s)e^{-\kappa_2(t-s)} \\
&\quad + \frac{\sigma_1^2}{2\kappa_1^2}\left(1-e^{-\kappa_1(t-s)}\right)^2 + \frac{\sigma_2^2}{2\kappa_2^2}\left(1-e^{-\kappa_2(t-s)}\right)^2 \\
&\quad + \frac{\rho\sigma_1\sigma_2}{\kappa_1\kappa_2}\left(1-e^{-\kappa_1(t-s)}\right)\left(1-e^{-\kappa_2(t-s)}\right) \\
\text{Var}^Q\big(r(t)|\mathfrak{F}_s\big) &= \frac{\sigma_1^2\left(1-e^{-2\kappa_1(t-s)}\right)}{2\kappa_1} + 2\frac{\rho\sigma_1\sigma_2}{\kappa_1+\kappa_2}\left(1-e^{-(\kappa_1+\kappa_2)(t-s)}\right) + \frac{\sigma_2^2\left(1-e^{-2\kappa_2(t-s)}\right)}{2\kappa_2}.
\end{aligned}$$

where we notice that the short rate process $r(t)$ is conditionally Gaussian.

Denoting

$$\begin{aligned}
\mu_r := E^Q\big(r(t)|\mathfrak{F}_0\big) &= f(0,t) + \frac{\sigma_1^2}{2\kappa_1^2}\left(1-e^{-\kappa_1 t}\right)^2 + \frac{\sigma_2^2}{2\kappa_2^2}\left(1-e^{-\kappa_2 t}\right)^2 \\
&\quad + \frac{\rho\sigma_1\sigma_2}{\kappa_1\kappa_2}\left(1-e^{-\kappa_1 t}\right)\left(1-e^{-\kappa_2 t}\right) \\
\sigma_r := \text{Var}^Q\big(r(t)|\mathfrak{F}_0\big) &= \frac{\sigma_1^2\left(1-e^{-2\kappa_1 t}\right)}{2\kappa_1} + 2\frac{\rho\sigma_1\sigma_2}{\kappa_1+\kappa_2}\left(1-e^{-(\kappa_1+\kappa_2)t}\right) + \frac{\sigma_2^2\left(1-e^{-2\kappa_2 t}\right)}{2\kappa_2}
\end{aligned}$$

and using that the short rate is normally distributed with mean μ_r and variance σ_r we can easily estimate the risk-neutral probability for the short rate to become negative to be

$$Q(r(t)<0) = E_Q\left(\mathbb{1}_{\{r(t)<0\}}\right) = E_Q\left(\mathbb{1}_{\left\{\frac{r(t)-\mu_r}{\sigma_r} < -\frac{\mu_r}{\sigma_r}\right\}}\right) = \varphi\left(-\frac{\mu_r}{\sigma_r}\right) > 0$$

where $\varphi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. We remark here that although the upper probability is in most cases negligibly small, this is a definite drawback of the model.

Additionally, notice that the mean and the variance of the short rate in the HJM-G2++ and in the G2++ are the same (see [1], p.147) which means that the two models produce equivalent dynamics of the short rate. This will be used later in the proof of the derivation of the closed-from solutions of caplets and bond options. However, the dynamics of the underlying factors $X_1(t)$ and $X_2(t)$ are slightly different which leads also to a slightly different closed-form solution for swaptions. For this reason, we will present in more details the derivation of the swaption formula.

1.3.1 The price of a European option on a zero-coupon bond

Theorem 1.1. *Using the already specified short rate dynamics, the price $\mathbf{ZBC}(t,T,S,K)$ at time t of a* ***European zero-bond call option*** *with maturity $T > t$ and strike K on a zero-coupon bond with maturity $S > T$ is calculated to*

$$\begin{aligned}\mathbf{ZBC}(t,T,S,K) &= P(t,S)\Phi\left(\frac{ln\left(\frac{P(t,S)}{KP(t,T)}\right)+\frac{1}{2}\Sigma_p(t,T,S)^2}{\Sigma_p(t,T,S)}\right)\\ &\quad -P(t,T)K\,\Phi\left(\frac{ln\left(\frac{P(t,S)}{KP(t,T)}\right)-\frac{1}{2}\Sigma_p(t,T,S)^2}{\Sigma_p(t,T,S)}\right)\end{aligned}$$

where $\Phi(\cdot)$ denotes the cumulative standard normal distribution function and

$$\Sigma_p^2(t,T,S) := b_1(T,S)^2 Z_{11}(t,T) + b_2(T,S)^2 Z_{22}(t,T) + 2b_1(T,S)b_2(T,S)Z_{12}(t,T).$$

The price $\mathbf{ZBP}(t,T,S,K)$ of a European bond put option is given by

$$\begin{aligned}\mathbf{ZBP}(t,T,S,K) &= -P(t,S)\,\Phi\left(-\frac{ln\left(\frac{P(t,S)}{KP(t,T)}\right)+\frac{1}{2}\Sigma_p(t,T,S)^2}{\Sigma_p(t,T,S)}\right)\\ &\quad +P(t,T)K\,\Phi\left(-\frac{ln\left(\frac{P(t,S)}{KP(t,T)}\right)-\frac{1}{2}\Sigma_p(t,T,S)^2}{\Sigma_p(t,T,S)}\right).\end{aligned}$$

Proof:
Due to the equivalence of the distribution of the short rate process $r(t)$ in the 2FHW and in the G2++ model, we refer for the proof to [1].

1.3.2 The price of a caplet

Theorem 1.2. *Using the already specified short rate dynamics, the price $\mathbf{Caplet}(t,T,S,K)$ at time t of a* ***caplet*** *resetting at time $T > t$, with payoff at time $S > T$*

and strike K is calculated to

$$\begin{aligned}\textbf{Caplet}(t,T,S,K) &= P(t,T)\Phi\left(\frac{\ln\left(\frac{K^*P(t,T)}{P(t,S)}\right)+\frac{1}{2}\Sigma_p^2(t,T,S)}{\Sigma_p(t,T,S)}\right)\\ &\quad -\frac{1}{K^*}P(t,S)\Phi\left(\frac{\ln\left(\frac{K^*P(t,T)}{P(t,S)}\right)-\frac{1}{2}\Sigma_p^2(t,T,S)}{\Sigma_p(t,T,S)}\right)\end{aligned}$$

with $K^*=\frac{1}{1+K(S-T)}$, $\Phi(\cdot)$ *denoting the cumulative standard normal distribution function and*

$$\Sigma_p^2(t,T,S) := b_1(T,S)^2 Z_{11}(t,T) + b_2(T,S)^2 Z_{22}(t,T) + 2b_1(T,S)b_2(T,S)Z_{12}(t,T).$$

Proof: see [1].

1.3.3 The price of a swaption

Consider an European option giving the right at time $t_0 = T$ (maturity) to enter an interest rate swap with payment times $\mathcal{T} = \{t_1, \dots, t_n\}$ (reset times $\{t_0, \dots, t_{n-1}\}$), fixed leg rate X and notional N. The year fraction from t_{i-1} to t_i is denoted as usually by τ_i.

Theorem 1.3. *The arbitrage-free price at time 0 of a European payer/receiver swaption is given by numerically computing the following one-dimensional integral*

$$\begin{aligned}&ES_p(0,T,\mathcal{T},N,K)\\ =\;& NP(0,T)\int_{-\infty}^{+\infty}\frac{e^{-\frac{1}{2}\left(\frac{x_1-\mu_{x_1}}{\sigma_{x_1}}\right)^2}}{\sigma_{x_1}\sqrt{2\pi}}\left[\Phi\left(-h_1(x_1)\right)-\sum_{i=1}^{n}\lambda_i(x_1)e^{\beta_i(x_1)}\Phi\left(-h_2(x_1)\right)\right]dx_1\end{aligned} \tag{5}$$

and

$$\begin{aligned}&ES_r(0,T,\mathcal{T},N,K)\\ =\;& -NP(0,T)\int_{-\infty}^{+\infty}\frac{e^{-\frac{1}{2}\left(\frac{x_1-\mu_{x_1}}{\sigma_{x_1}}\right)^2}}{\sigma_{x_1}\sqrt{2\pi}}\left[\Phi\left(h_1(x_1)\right)-\sum_{i=1}^{n}\lambda_i(x_1)e^{\beta_i(x_1)}\Phi\left(h_2(x_1)\right)\right]dx_1\end{aligned} \tag{6}$$

where

$$
\begin{aligned}
h_1(x_1) &:= \frac{\bar{x_2} - \mu_{x_2}}{\sigma_{x_2}\sqrt{1-\rho^2_{x_1x_2}}} - \frac{\rho_{x_1x_2}(x_1 - \mu_{x_1})}{\sigma_{x_1}\sqrt{1-\rho^2_{x_1x_2}}} \\
h_2(x_1) &:= h_1(x_1) + b_2(T,t_i)\sigma_{x_2}\sqrt{1-\rho^2_{x_1x_2}} \\
\lambda_i(x_1) &:= c_i A(T,t_i) e^{-b_1(T,t_i)x_1} \\
\beta_i(x_1) &:= -b_2(T,t_i)\left[\mu_{x_2} - \frac{1}{2}\left(1-\rho^2_{x_1x_2}\right)\sigma^2_{x_2} b_2(T,t_i) + \rho_{x_1x_2}\sigma_{x_2}\frac{x_1-\mu_{x_1}}{\sigma_{x_1}}\right] \\
A(T,t_i) &:= e^{-\int_T^{t_i} f(0,u)du - \frac{1}{2}\sum_{k,j=1}^{2} b_k(T,t_i)b_j(T,t_i)Z_{kj}(0,T)}.
\end{aligned}
$$

and $c := \{c_1, \ldots, c_n\}$ *such that* $c_i := X\tau_i$ *for* $i = 1, \ldots, n-1$ *and* $c_n := 1 + X\tau_n$.

In addition $\bar{x}_2 = \bar{x}_2(x_1)$ *is the unique solution of the following equation (found via numerical methods)*

$$
\sum_{i=1}^{n} c_i A(T,t_i) e^{-b_1(T,t_i)x_1 - b_2(T,t_i)x_2} = 1 \tag{7}
$$

and

$$
\begin{aligned}
\mu_{x_1} &= \mu_{x_2} = 0 \\
\sigma^2_{x_1} &:= \int_0^T \sigma_1^2 e^{-2\kappa_1(T-u)}du = Z_{11}(0,T) \\
\sigma^2_{x_2} &:= \int_0^T \sigma_2^2 e^{-2\kappa_2(T-u)}du = Z_{22}(0,T) \\
\rho_{x_1x_2} &:= \frac{\rho_{1,2}\int_0^T \sigma_1\sigma_2 e^{-(\kappa_1+\kappa_2)(T-u)}du}{\sigma_{x_1}\sigma_{x_2}} = \frac{Z_{12}(0,T)}{\sqrt{Z_{11}(0,T)Z_{22}(0,T)}}
\end{aligned}
$$

Proof:
We will consider only the case of a European payer swaption and the one for the receiver will follow by analogy.

By definition of the payer swaption, we can write its discounted until time zero payoff as

$$
Ne^{-\int_0^T r(u)du}\left(\sum_{i=1}^{n} P(T,t_i)\tau_i\left(F(T,t_{i-1},t_i) - X\right)^+\right)
$$

where the forward Libor rate is defined as

$$
F(T,t_{i-1},t_i) := \frac{1}{\tau_i}\left(\frac{P(T,t_{i-1}) - P(T,t_i)}{P(T,t_i)}\right), \quad i = 1, \ldots, n.
$$

Using the definition of the forward Libor rate we can rewrite the payoff of the payer swaption as

$$
\begin{aligned}
& N e^{-\int_0^T r(u)du}\left(\sum_{i=1}^n P(T,t_{i-1}) - \sum_{i=1}^n P(T,t_i) - \sum_{i=1}^n P(T,t_i)\tau_i X\right)^+ \\
&= N e^{-\int_0^T r(u)du}\left(1 - X\sum_{i=1}^n \tau_i P(T,t_i) - P(T,t_n)\right)^+ \\
&= N e^{-\int_0^T r(u)du}\left(1 - \sum_{i=1}^n c_i P(T,t_i)\right)^+ .
\end{aligned}
$$

Taking expectation under the forward Q^T measure we obtain that the price of the payer swaption is given as

$$
ES_p(0,T,\mathcal{T},N,K) \quad = \quad NP(0,T)E^{Q^T}\left(\left(1 - \sum_{i=1}^n c_i P(T,t_i)\right)^+\right).
$$

where

$$
\begin{aligned}
P(T,t_i) \quad = \quad & e^{-\int_T^{t_i} f(0,u)du}\exp\Bigg(- b_1(T,t_i)X_1(T) - b_2(T,t_i)X_2(T) \\
& -\frac{1}{2}\sum_{i,j=1}^2 b_i(T,t_i)b_j(T,t_i)Z_{ij}(0,T)\Bigg)
\end{aligned}
$$

where the joint density of $X_1(T)$ and $X_2(T)$ under measure $\mathbf{Q}^T$ is denoted by $f(x_1,x_2)$. Further, due to Result (1) in the Appendix, the dynamics of $X_1(t)$ and $X_2(t)$ under the forward $\mathbf{Q}^T$ measure is given by

$$
\begin{aligned}
dX_1(t) \quad &= \quad \left(-\kappa_1 X_1(t) + \sum_{k=1}^2 Z_{1,k}(0,t) - b_1(t,T)\sigma_1^2 - \rho_{1,2}b_2(t,T)\sigma_1\sigma_2\right)dt \\
& \qquad +\sigma_1 dW_1^{Q^T}(t),\ X_1(0) = 0 \\
dX_2(t) \quad &= \quad \left(-\kappa_2 X_2(t) + \sum_{k=1}^2 Z_{2,k}(0,t) - b_2(t,T)\sigma_2^2 - \rho_{1,2}b_1(t,T)\sigma_1\sigma_2\right)dt \\
& \qquad +\rho_{1,2}\sigma_2 dW_1^{Q^T}(t) + \sqrt{1-\rho_{1,2}^2}\sigma_2 dW_2^{Q^T}(t),\ X_2(0) = 0
\end{aligned}
$$

and using Result (2) of the Appendix, we can find conditioning on $\mathcal{F}_0$

$$
\begin{aligned}
X_1(T) \quad &= \quad -b_1(T,T)Z_{11}(0,T) - b_2(T,T)Z_{12}(0,T) + \int_0^T \sigma_1 e^{-\kappa_1(T-u)}dW_1^{Q^T}(u) \\
&= \quad \int_0^T \sigma_1 e^{-\kappa_1(T-u)}dW_1^{Q^T}(u)
\end{aligned}
$$

and

$$X_2(T) = \rho_{1,2}\int_0^T \sigma_2 e^{-\kappa_2(T-u)}dW_1^{Q^T}(u) + \sqrt{1-\rho_{1,2}^2}\int_0^T \sigma_2 e^{-\kappa_2(T-u)}dW_2^{Q^T}(u).$$

Thus,

$$\begin{aligned}
E^{Q^T}(X_1(T)|\mathcal{F}_0) &:= \mu_{x_1} = 0 \\
E^{Q^T}(X_2(T)|\mathcal{F}_0) &:= \mu_{x_2} = 0 \\
Var^{Q^T}(X_1(T)|\mathcal{F}_0) &:= \sigma_{x_1}^2 = Z_{11}(0,T) \\
Var^{Q^T}(X_2(T)|\mathcal{F}_0) &:= \sigma_{x_2}^2 = Z_{22}(0,T) \\
corr(X_1(T), X_2(T)) &:= \rho_{x_1x_2} = \frac{\rho_{1,2}\int_0^T \sigma_1(u)\sigma_2(u)e^{-(\kappa_1+\kappa_2)(T-u)}du}{\sigma_{x_1}\sigma_{x_2}} = \frac{Z_{12}(0,T)}{\sqrt{Z_{11}(0,T)Z_{22}(0,T)}}
\end{aligned}$$

and

$$f(x_1,x_2) = \frac{\exp\left\{-\frac{1}{2(1-\rho_{x_1x_2}^2)}\left[\left(\frac{x_1-\mu_{x_1}}{\sigma_{x_1}}\right)^2 - 2\rho_{x_1x_2}\frac{(x_1-\mu_{x_1})(x_2-\mu_{x_2})}{\sigma_{x_1}\sigma_{x_2}} + \left(\frac{x_2-\mu_{x_2}}{\sigma_{x_2}}\right)^2\right]\right\}}{2\pi\sigma_{x_1}\sigma_{x_2}\sqrt{1-\rho_{x_1x_2}^2}}$$

Hereon, the proof follows the same steps as in [1], p.163 but for completeness we will present it in more details here.

$$ES_p(0,T,\mathcal{T},N,K) = NP(0,T)E^{Q^T}\left(\left(1-\sum_{i=1}^n c_iP(T,t_i)\right)^+\right)$$

$$= NP(0,T)\int^{\mathbb{R}^2}\Bigg(1-\sum_{i=1}^n c_i e^{-\int_T^{t_i} f(0,u)du}$$

$$\exp\Bigg(-b_1(T,t_i)x_1 - b_2(T,t_i)x_2 - \frac{1}{2}\sum_{i,j=1}^2 b_i(T,t_i)b_j(T,t_i)Z_{ij}(0,T)\Bigg)\Bigg)^+ f(x_1,x_2)dx_1dx_2$$

$$= NP(0,T)\int^{\mathbb{R}^2}\left(1-\sum_{i=1}^n c_iA(T,t_i)e^{-b_1(T,t_i)x_1-b_2(T,t_i)x_2}\right)^+ f(x_1,x_2)dx_1dx_2$$

Next, freezing x_1 and integrating over x_2 we choose numerically $\bar{x}_2$ such that

$$\sum_{i=1}^n c_iA(T,t_i)e^{-b_1(T,t_i)x_1-b_2(T,t_i)\bar{x}_2} = 1.$$

Then, denoting

$$\begin{aligned}
\lambda_i(x_1) &:= c_i A(T,t_i) e^{-b_1(T,t_i)x_1}, \quad \gamma := \frac{1}{2\pi\sigma_{x_1}\sigma_{x_2}\sqrt{1-\rho_{x_1x_2^2}}} \\
E(x_1) &:= -\frac{1}{2(1-\rho^2_{x_1x_2})}\left[\frac{x_1-\mu_{x_1}}{\sigma_{x_1}}\right]^2, \quad F(x_1) := \frac{\rho_{x_1x_2}}{(1-\rho^2_{x_1x_2})}\frac{x_1-\mu_{x_1}}{\sigma_{x_1}\sigma_{x_2}} \\
G(x_1) &:= \frac{1}{2(1-\rho^2_{x_1x_2})\sigma^2_{x_2}}
\end{aligned}$$

we have that

$$\begin{aligned}
&ES_p(0,T,\mathcal{T},N,K) = NP(0,T)\int_{-\infty}^{+\infty}\int_{\bar{x}_2}^{+\infty}\left(1-\sum_{i=1}^{n}\lambda_i(x_1)e^{-b_2(T,t_i)x_2}\right) \\
&\gamma e^{E(x_1)+F(x_1)(x_2-\mu_{x_2})-G(x_1)(x_2-\mu_{x_2})^2}dx_2dx_1 \\
&= NP(0,T)\int_{-\infty}^{+\infty}\gamma e^{E(x_1)}\int_{\bar{x}_2}^{+\infty} e^{F(x_1)(x_2-\mu_{x_2})-G(x_1)(x_2-\mu_{x_2})^2}dx_2dx_1 \\
&-NP(0,T)\int_{-\infty}^{+\infty}\gamma e^{E(x_1)-b_2(T,t_i)\mu_{x_2}}\sum_{i=1}^{n}\lambda_i(x_1)\int_{\bar{x}_2}^{+\infty} e^{(F(x_1)-b_2(T,t_i))(x_2-\mu_{x_2})} \\
&e^{-G(x_1)(x_2-\mu_{x_2})^2}dx_2dx_1.
\end{aligned}$$

Using that

$$\int_a^b e^{-Ax^2+Bx} = \frac{\sqrt{\pi}}{\sqrt{A}}e^{\frac{B^2}{4A}}\left[\varphi\left(b\sqrt{2A-\frac{B}{\sqrt{2A}}}\right)-\varphi\left(a\sqrt{2A-\frac{B}{\sqrt{2A}}}\right)\right]$$

where $\varphi(\cdot)$ denotes the cumulative standard distribution function, we obtain

$$\begin{aligned}
& ES_p(0,T,\mathcal{T},N,K) \\
= \; & NP(0,T)\int_{-\infty}^{+\infty}\gamma e^{E(x_1)}\frac{\sqrt{\pi}}{\sqrt{G(x_1)}}e^{\frac{F(x_1)^2}{4G(x_1)}}\left[1-\varphi\left((\bar{x}_2-\mu_{x_2})\sqrt{2G(x_1)}-\frac{F(x_1)}{\sqrt{2G(x_1)}}\right)\right]dx_1 \\
& -NP(0,T)\int_{-\infty}^{+\infty}\gamma e^{E(x_1)}\frac{\sqrt{\pi}}{\sqrt{G(x_1)}}e^{-b_2(T,t_i)\mu_{x_2}+\frac{(F(x_1)-b_2(T,t_i))^2}{fG}} \\
& \sum_{i=1}^{n}\lambda_i(x_1)\left[1-\varphi\left((\bar{x}_2-\mu_{x_2})\sqrt{2G(x_1)}-\frac{F(x_1)-b_2(T,t_i)}{\sqrt{2G(x_1)}}\right)\right].
\end{aligned}$$

■

1.3.4 Implementation of the swaption semi closed-form formula

In this section, we shall devote some attention to the implementation of (semi-)closed formula for the payer (resp. receiver) swaption price given by equation (5) (resp.(6)), since it requires numerical procedures of root finding and integration.

Let us consider the pricing formula of the payer swaption, given by equation (5). According to authors best knowledge; one can compute the integral only numerically.

After intensive tests, see Table 1, we have chosen the 20 point Gaussian quadrature,

$$\int_{-\infty}^{\infty} g(x)dx \approx \sum_{j=0}^{19} w_j g(x_j)$$

where the weights and abscissas are set according to the Gauss-Legendre polynomials, see [6].

The limits of the integration are set according to the fact that the integrand in formula (5) is actually a bounded function against a normal distribution. See Figure 1. Hence, one can truncate the domain of integration to the interval $[\mu_{x_1} - N\sigma_{x_1}, \mu_{x_1} + N\sigma_{x_1}]$, where $N \in \mathbb{N}$.

Since the precision of the integral approximation is affected by the integral bounds, by keeping in mind that the integrand is monotonic at the left and right wings, we have chosen the constant N adaptively, according to the following algorithm.

1. Compute lower and upper bounds for $N = 1$: $l = -N\sigma_{x_1}$, $u = N\sigma_{x_1}$
2. Compute the integrand at l and u
3. If $Integrand(l) < 10^{-8}$, set lower bound as l. Else increase N and go to step 1.
4. If $Integrand(u) < 10^{-8}$, set lower bound as u. Else increase N and go to step 1.

Now, let us consider the equation (7). We shall find the root of this equation. We bracket the root in an interval $[x_2^l, x_2^r]$ and use a hybrid root finding algorithm, which is a combination of Newton-Raphson and bisection algorithms. The numerical procedures that we used can be found in [6][1]. The precision for the root finding algorithm is set to be $10E - 7$., see Table 1.

In the view of Table 1, one observes a trade-off between the precision of the results and the computing time. As a reasonable trade-off, we have chosen the **20 point Gauss-Legendre** and the **precision** $10e - 07$ **for the root finding algorithm**.

[1] Bracketing algorithm (zbrac) is in page 356 and root finding algorithm (rtsafe) is in page 370

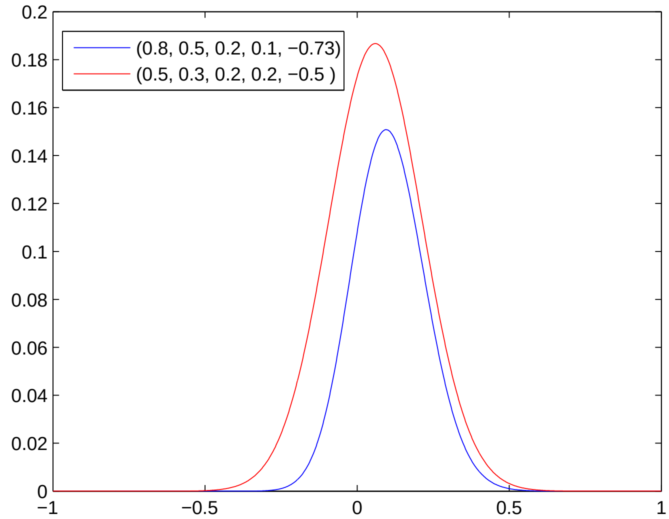


Figure 1 Plot of the integrand in formula (5) for a 1Y-swaption with an underlying 1Y swap and the underlying process has the parameters ($\kappa_1 = 0.8$, $\kappa_2 = 0.5$,$\sigma_1 = 0.2$, $\sigma_2 = 0.1$, $\rho = -0.73$), ($\kappa_1 = 0.5$, $\kappa_2 = 0.3$,$\sigma_1 = 0.2$, $\sigma_2 = 0.2$, $\rho = -0.5$)

	10e-05	10e-07	10e-09	10e-11
10	0.023053435634 00:00:01.79	0.023053435640 00:00:02.34	0.023053435634 00:00:02.90	0.023053435634 00:00:03.46
20	0.023073833332 00:00:02.48	0.023073766462 00:00:03.26	0.023073766455 00:00:04.04	0.023073766455 00:00:04.82
30	0.023073835520 00:00:03.17	0.023073766434 00:00:04.20	0.023073766427 00:00:05.21	0.023073766427 00:00:06.25
40	0.023073834632 00:00:03.87	0.023073766434 00:00:05.15	0.023073766427 00:00:06.39	0.023073766427 00:00:08.18

Table 1 Comparison of the sum of squared errors of the swaption surface calculated by different number of Gauss-Legendre polynomials(on the rows) and different precisions of the root finding algorithm (on the columns).

1.4 Degeneration to a one-factor model

We want to investigate the conditions under which we can derive the **one factor Hull-White** (extended Hull-White, Vasicek, from now on 1FHW) model as a special case of the HJM-G2++ model introduced in Subsection 1.1.

Let us define $\widetilde{X}(t) := X_1(t) + X_2(t)$.

Proposition 1. *Let us assume that the coefficients of the SDEs* (3) *and* (4) *satisfy*

$$\kappa_1 = \kappa_2, \qquad \rho = 0.$$

Then, $\widetilde{X}(t)$ *satisfies the following SDE:*

$$d\widetilde{X}(t) = \left(-\widetilde{\kappa}\widetilde{X}(t) + \widetilde{Z}(0,t)\right) dt + \widetilde{\sigma} dW(t) \tag{8}$$

where $\widetilde{\kappa} = \kappa_1 = \kappa_2$*,* $\widetilde{\sigma} = \sqrt{\sigma_1^2 + \sigma_2^2}$*,* $\widetilde{Z}(0,t) = Z_{11}(0,t) + Z_{22}(0,t)$*, i.e.,* $\tilde{X}$ *is a 1FHW process.*

Proof: Let us consider the strong solutions of SDEs (3) and (4)

$$X_1(t) = X_1(s)e^{-\kappa_1(t-s)} + \int_s^t \left(Z_{11}(s,u) + Z_{12}(s,u)\right) e^{-\kappa_1(t-u)} du + \int_s^t \sigma_1 e^{-\kappa_1(t-u)} d\tilde{W}_1^Q(u),$$

$$X_2(t) = X_2(s)e^{-\kappa_2(t-s)} + \int_s^t \left(Z_{21}(s,u) + Z_{22}(s,u)\right) e^{-\kappa_2(t-u)} du + \int_s^t \sigma_2 e^{-\kappa_2(t-u)} d\tilde{W}_2^Q(u).$$

Then,

$$\begin{aligned}\widetilde{X}(t) \quad := & X_1(s)e^{-\kappa_1(t-s)} + X_2(s)e^{-\kappa_2(t-s)} \\ & + \int_s^t \left(Z_{11}(s,u) + Z_{12}(s,u)\right) e^{-\kappa_1(t-u)} du + \int_s^t \left(Z_{21}(s,u) + Z_{22}(s,u)\right) e^{-\kappa_2(t-u)} du \\ & + \int_s^t \sigma_1 e^{-\kappa_1(t-u)} d\tilde{W}_1^Q(u) + \int_s^t \sigma_2 e^{-\kappa_2(t-u)} d\tilde{W}_2^Q(u).\end{aligned}$$

By substituting the assumptions in the above equation, we get

$$\begin{aligned}\widetilde{X}(t) \quad &= \left(X_1(s) + X_2(s)\right) e^{-\kappa_1(t-s)} + \int_s^t \left(Z_{11}(s,u) + Z_{22}(s,u)\right) e^{-\kappa_1(t-u)} du \\ &\quad + \sqrt{\sigma_1^2 + \sigma_2^2} \int_s^t e^{-\kappa_1(t-u)} d\tilde{W}^Q(u) \\ &= \widetilde{X}(t)e^{-\widetilde{\kappa}(t-s)} + \int_s^t \widetilde{Z}(s,u)e^{-\widetilde{\kappa}(t-u)} du + \widetilde{\sigma} \int_s^t e^{-\widetilde{\kappa}(t-u)} d\tilde{W}^Q(u)\end{aligned}$$

which is the solution of SDE (8). ∎

2 Numerical implementation

Since no closed solutions are available for the prices of complex structured notes, we have to use numerical methods for their approximation. For this purpose, we have chosen to use a Monte Carlo simulation and a lattice approximation. In this section, we shall shortly describe their implementation.

2.1 Monte Carlo simulation

For the Monte Carlo simulation we have chosen to use the classical Euler discretization scheme of the driving stochastic differential equations (3) and (4). Then, if π is an arbitrary discretization of $[0,T]$ such that $0 \leq t_0 < t_1 < \ldots < t_n = T$ we have

$$
\begin{aligned}
X_1^\pi(t_{i+1}) &= X_1^\pi(t_i) + \left(-\kappa_1 X_1^\pi(t_i) + \sum_{k=1}^{2} Z_{1,k}(0,t_i)\right)\Delta t_i^\pi + \sigma_1 \Delta_i^\pi W_1^Q, \\
& \quad X_1(0) = 0 \\
X_2^\pi(t_{i+1}) &= X_2^\pi(t_i) + \left(-\kappa_2 X_2^\pi(t_i) + \sum_{k=1}^{2} Z_{2,k}(0,t_i)\right)\Delta t_i^\pi + \sigma_2 \left(\rho \Delta_i^\pi W_1^Q + \sqrt{1-\rho^2}\Delta_i^\pi W_2^Q\right), \\
& \quad X_2(0) = 0
\end{aligned}
$$

with $\Delta_i^\pi := t_i - t_{i-1}$, $\Delta_i^\pi W_j^Q := W_j^Q(t_{i+1})^\pi - W_j^Q(t_i)^\pi$, $i = 1,\ldots,n$, $j = 1,2$.

Notice that we have decomposed the correlated Brownian motions $\tilde{W_1}^Q$ and $\tilde{W_2}^Q$ into a linear combination of independent Brownian motions W_1^Q and W_2^Q using Cholesky decomposition. In this form, it is straightforward to simulate the forward components using Monte Carlo simulation. If Z_1 and Z_2 are both standard normally distributed independent variables (i. e. $Z_1,\ Z_2 \sim \mathcal{N}(0,1)$) then we can rewrite the simulation scheme as

$$
\begin{aligned}
X_1^\pi(t_{i+1}) &= X_1^\pi(t_i) + \left(-\kappa_1 X_1^\pi(t_i) + \sum_{k=1}^{2} Z_{1,k}(0,t_i)\right)\Delta t_i^\pi + \sigma_1 Z_1 \sqrt{\Delta t_i^\pi}, \\
& \quad X_1(0) = 0 \\
X_2^\pi(t_{i+1}) &= X_2^\pi(t_i) + \left(-\kappa_2 X_2^\pi(t_i) + \sum_{k=1}^{2} Z_{2,k}(0,t_i)\right)\Delta t_i^\pi + \sigma_2 \left(\rho Z_1 \sqrt{\Delta t_i^\pi} + \sqrt{1-\rho^2} Z_2 \sqrt{\Delta t_i^\pi}\right), \\
& \quad X_2(0) = 0.
\end{aligned}
$$

2.2 Tree construction

For shortness of the notations, let us denote the drifts of the two processes by
$\nu_1(t, X_1) := -\kappa_1 X_1(t) + Z_{11}(0,t) + Z_{12}(0,t)$ and
$\nu_2(t, X_2) := -\kappa_2 X_2(t) + Z_{21}(0,t) + Z_{22}(0,t)$.

Next, if we assume that at time t the tree is in state (x_1^a, x_2^a) then at time $t + \Delta t$ it can move to the following four states

$$\begin{array}{ll} \left(x_1^{a^+}, x_2^{a^+}\right) & \text{with probability } p_{uu} \\ \left(x_1^{a^+}, x_2^{a^-}\right) & \text{with probability } p_{ud} \\ \left(x_1^{a^-}, x_2^{a^+}\right) & \text{with probability } p_{du} \\ \left(x_1^{a^-}, x_2^{a^-}\right) & \text{with probability } p_{dd} \end{array}$$

Definition 2.1. *Let us define the up and down jumps of both process by*

$$\begin{aligned} x_1^{a^+} &:= x_1^a + \big(J_1(t, x_1^a) + 1\big)\sqrt{\Delta t}\sigma_1, \quad x_1^{a^-} := x_1^a + \big(J_1(t, x_1^a) - 1\big)\sqrt{\Delta t}\sigma_1 \\ x_2^{a^+} &:= x_2^a + \big(J_2(t, x_2^a) + 1\big)\sqrt{\Delta t}\sigma_2, \quad x_2^{a^-} := x_2^a + \big(J_2(t, x_2^a) - 1\big)\sqrt{\Delta t}\sigma_2 \end{aligned}$$

where

$$J_1(t, x_1^a) := \begin{cases} Z_1 & \text{if } Z_1 \text{ even,} \\ Z_1 + 1 & \text{else.} \end{cases}$$

$$J_2(t, x_2^a) := \begin{cases} Z_2 & \text{if } Z_2 \text{ even,} \\ Z_2 + 1 & \text{else.} \end{cases}$$

for $Z_1 := \text{floor}\left[\frac{\nu_1(t,x_1^a)\sqrt{\Delta t}}{\sigma_1}\right]$ *and* $Z_2 := \text{floor}\left[\frac{\nu_2(t,x_2^a)\sqrt{\Delta t}}{\sigma_2}\right]$.

Remark 2.1. *We want to point out here that we cannot choose exactly*

$$J_1(t, x_1^a) = \frac{\nu_1(t, x_1^a)\sqrt{\Delta t}}{\sigma_1}, \quad J_2(t, x_1^a) = \frac{\nu_2(t, x_2^a)\sqrt{\Delta t}}{\sigma_2}$$

because of two reasons

- *If we want to construct a recombining tree we would like that the absolute value of the up-jump and the absolute value of the down-jump are integer multiples of the same jump heights. Notice that this will not be the case for real values of $J_1(t, x_1^a)$ and $J_2(t, x_2^a)$. Therefore, we need a construction of $J_1(t, x_1^a)$ and $J_2(t, x_2^a)$ that allows them to take only integer values;*

- *On the other side, if we choose simply*

$$J_1(t, x_1^a) = \text{floor}\left[\frac{\nu_1(t, x_1^a)\sqrt{\Delta t}}{\sigma_1}\right], \quad J_2(t, x_2^a) = \text{floor}\left[\frac{\nu_2(t, x_2^a)\sqrt{\Delta t}}{\sigma_2}\right]$$

the tree will be allowed to expand endlessly, hence we will take no account of the mean-reversion in the underlying processes. Finally, if we set

$$J_1(t, x_1^a) := \begin{cases} Z_1 & \text{if } Z_1 \text{ even,} \\ Z_1 + 1 & \text{else.} \end{cases}$$

$$J_2(t, x_2^a) := \begin{cases} Z_2 & \text{if } Z_2 \text{ even,} \\ Z_2 + 1 & \text{else.} \end{cases}$$

for $Z_1 :=$ *floor* $\left[\frac{\nu_1(t,x_1^a)\sqrt{\Delta t}}{\sigma_1}\right]$ *and* $Z_2 :=$ *floor* $\left[\frac{\nu_2(t,x_2^a)\sqrt{\Delta t}}{\sigma_2}\right]$ *we notice that in a "normal case" of not too strong mean reversion, the values of* $J_1(t, x_1)$ *and* $J_2(t, x_2)$ *will most of the time be equal to zero. Thus, we will have a product of two binomial trees in the usual sense. But as soon as* $|\nu_1(t, x_1)\sqrt{\Delta t}| > \sigma_1$ *or in other words as soon as the local mean dominates the local variance, the branching of the tree will be changed and thus* $J_1(t, x_1)$ *and* $J_2(t, x_2)$ *will take the values of* -2 *in the upper part of the tree and of* 2 *in the down part. Thus, we would have both required effects, namely that the local variance will be approximately matched and the effect of mean reversion will be taken into account.*

Theorem 2.1. *If the subsequent probabilities for the* (x_1^a, x_2^a) *node are given by*

$$p_{uu} = \frac{1}{4}\left[\frac{\rho\sigma_1\sigma_2 + (J_1 - 1)(J_2 - 1)\sigma_1\sigma_2 - \nu_1(t, x_1^a)\sqrt{\Delta t}(J_2 - 1)\sigma_2 - \nu_2(t, x_2^a)\sqrt{\Delta t}(J_1 - 1)\sigma_1}{\sigma_1\sigma_2}\right]$$

$$p_{dd} = \frac{1}{4}\left[\frac{\rho\sigma_1\sigma_2 + (J_1 + 1)(J_2 + 1)\sigma_1\sigma_2 - \nu_1(t, x_1^a)\sqrt{\Delta t}(J_2 + 1)\sigma_2 - \nu_2(t, x_2^a)\sqrt{\Delta t}(J_1 + 1)\sigma_1}{\sigma_1\sigma_2}\right]$$

$$p_{ud} = \frac{1}{4}\left[\frac{-\rho\sigma_1\sigma_2 - (J_1 - 1)(J_2 + 1)\sigma_1\sigma_2 + \nu_1(t, x_1^a)\sqrt{\Delta t}(J_2 + 1)\sigma_2 + \nu_2(t, x_2^a)\sqrt{\Delta t}(J_1 - 1)\sigma_1}{\sigma_1\sigma_2}\right]$$

$$p_{du} = \frac{1}{4}\left[\frac{-\rho\sigma_1\sigma_2 - (J_1 + 1)(J_2 - 1)\sigma_1\sigma_2 + \nu_1(t, x_1^a)\sqrt{\Delta t}(J_2 - 1)\sigma_2 + \nu_2(t, x_2^a)\sqrt{\Delta t}(J_1 + 1)\sigma_1}{\sigma_1\sigma_2}\right]$$

then with the choice of jump heights given in Definition 2.1*, the tree matches locally perfectly the first and approximately the second moments of the underlying processes and in addition, up to terms of order* Δt^2 *it matches the local covariance between the underlying processes.*

Proof: Denoting the approximated (with the lattice) local mean and variance respectively with $\widehat{E}(\Delta x_i)$ and $\widehat{\text{var}}(\Delta x_i)$, for $i = 1, 2$ and the true local mean and variance of the underlying process by $E(\Delta x_i)$ and var(Δx_i), for $i = 1, 2$ we notice that matching the first two moments implies that the risk-neutral tree probabilities have to solve

$$\widehat{E}(\Delta x_1) = (p_{uu}+p_{ud})(x_1^{a^+}-x_1^a)+(p_{du}+p_{dd})(x_1^{a^-}-x_1^a) \stackrel{!}{=} E(\Delta x_1)=\nu_1(t,x_1^a)\Delta t \tag{9}$$

$$\begin{aligned}\widehat{E}(\Delta x_1^2) &= (p_{uu}+p_{ud})(x_1^{a^+}-x_1^a)^2+(p_{du}+p_{dd})(x_1^{a^-}-x_1^a)^2\\ &\stackrel{!}{=} E(\Delta x_1^2)=\sigma_1^2\Delta t+\nu_1(t,x_1^a)^2\Delta t^2\end{aligned} \tag{10}$$

$$\widehat{E}(\Delta x_2) = (p_{uu}+p_{du})(x_2^{a^+}-x_2^a)+(p_{ud}+p_{dd})(x_2^{a^-}-x_2^a) \stackrel{!}{=} E(\Delta x_2)=\nu_2(t,x_2^a)\Delta t \tag{11}$$

$$\begin{aligned}\widehat{E}(\Delta x_2^2) &= (p_{uu}+p_{du})(x_2^{a^+}-x_2^a)^2+(p_{du}+p_{dd})(x_2^{a^-}-x_2^a)^2\\ &\stackrel{!}{=} E(\Delta x_2^2)=\sigma_2^2\Delta t+\nu_2(t,x_2^a)^2\Delta t^2\end{aligned} \tag{12}$$

$$\begin{aligned}\widehat{E}(\Delta x_1\Delta x_2) &= \widehat{\text{cov}}(\Delta x_1\Delta x_2)+\widehat{E}(\Delta x_1)\widehat{E}(\Delta x_2)\\ &= p_{uu}(x_1^{a^+}-x_1^a)(x_2^{a^+}-x_2^a)+p_{ud}(x_1^{a^+}-x_1^a)(x_2^{a^-}-x_2^a)\\ &\quad +p_{du}(x_1^{a^-}-x_1^a)(x_2^{a^+}-x_2^a)+p_{dd}(x_1^{a^-}-x_1^a)(x_2^{a^-}-x_2^a)\\ &\stackrel{!}{=} E(\Delta x_1\Delta x_2)=\sigma_1\sigma_2\rho\Delta t+\nu_1(t,x_1^a)\nu(t,x_2^a)\Delta t^2\end{aligned} \tag{13}$$

$$1 = p_{uu}+p_{ud}+p_{du}+p_{dd} \tag{14}$$

where Δx_1 and Δx_2 denote the changes in the respective processes x_1 and x_2 from time t to time $t+\Delta t$. Ignoring the terms of order Δt^2 in the matching of the correlation and substituting with the definitions of $x_1^{a^\pm}$ and $x_2^{a^\pm}$ we solve equations $(9),(11),(13)$ and (14) for the four unknown probabilities and obtain

$$p_{uu}=\frac{1}{4}\left[\frac{\rho\sigma_1\sigma_2+(J_1-1)(J_2-1)\sigma_1\sigma_2-\nu_1(t,x_1^a)\sqrt{\Delta t}(J_2-1)\sigma_2-\nu_2(t,x_2^a)\sqrt{\Delta t}(J_1-1)\sigma_1}{\sigma_1\sigma_2}\right]$$

$$p_{dd}=\frac{1}{4}\left[\frac{\rho\sigma_1\sigma_2+(J_1+1)(J_2+1)\sigma_1\sigma_2-\nu_1(t,x_1^a)\sqrt{\Delta t}(J_2+1)\sigma_2-\nu_2(t,x_2^a)\sqrt{\Delta t}(J_1+1)\sigma_1}{\sigma_1\sigma_2}\right]$$

$$p_{ud}=\frac{1}{4}\left[\frac{-\rho\sigma_1\sigma_2-(J_1-1)(J_2+1)\sigma_1\sigma_2+\nu_1(t,x_1^a)\sqrt{\Delta t}(J_2+1)\sigma_2+\nu_2(t,x_2^a)\sqrt{\Delta t}(J_1-1)\sigma_1}{\sigma_1\sigma_2}\right]$$

$$p_{du}=\frac{1}{4}\left[\frac{-\rho\sigma_1\sigma_2-(J_1+1)(J_2-1)\sigma_1\sigma_2+\nu_1(t,x_1^a)\sqrt{\Delta t}(J_2-1)\sigma_2+\nu_2(t,x_2^a)\sqrt{\Delta t}(J_1+1)\sigma_1}{\sigma_1\sigma_2}\right]$$

Next, let us calculate the local conditional variances for the already found probabilities first for the x_1 process:

$$\begin{aligned}
\widehat{\text{var}}\left(\Delta x_1\right) &= \widehat{E}\left(\Delta x_1^2\right) - \widehat{E}\left(\Delta x_1\right)^2 \\
&= (p_{uu}+p_{ud})(x_1^{a^+}-x_1^a)^2 + (p_{du}+p_{dd})(x_1^{a^-}-x_1^a)^2 - \nu_1(t,x_1^a)^2\Delta t^2 \\
&= \left(\frac{\nu_1(t,x_1^a)\sqrt{\Delta t}-(J_1-1)\sigma_1}{2\sigma_1}\right)(1+J_1)^2\Delta t\sigma_1^2 \\
&\quad + \left(1-\frac{\nu_1(t,x_1^a)\sqrt{\Delta t}-(J_1-1)\sigma_1}{2\sigma_1}\right)(1-J_1)^2\Delta t\sigma_1^2 - \nu_1(t,x_1^a)^2\Delta t \\
&= -J_1^2\Delta t\sigma_1^2 + 2\nu_1(t,x_1^a)\Delta t\sqrt{\Delta t\sigma_1^2}J_1 - \nu_1(t,x_1^a)^2\Delta t + \sigma_1^2\Delta t \\
&= -(J_1\sqrt{\Delta t}\sigma_1 - \nu_1(t,x_1^a)\Delta t)^2 + \sigma_1^2\Delta t
\end{aligned}$$

and since $\text{var}\left(\Delta x_1\right) = \sigma_1^2\Delta t$ we have as in the rotated tree that the local variance is approximately matched due to the right definition of the J_1 process. By symmetry, the same follows for the x_2 process. ■

Notice that although the sum of the probabilities is one, they are not necessarily bounded between 0 and 1. It can however be shown that when we increase the refinement, the probabilities converge to values between 0 and 1 (see [5]). A similar problem is encountered also in the Hull and White [3] 2-factor trinomial tree construction and also by Brigo and Mercurio [1] in their G2++ quadrinomial tree construction. There are (at least) three possible ways to proceed.

The first approach is to change the correlation coefficient at every node where the probabilities are not bounded in $[0,1]$, until we have well-defined probabilities. Brigo and Mercurio claim that this procedure although theoretically not consistent is practically applicable as the correlation has a very negligible contribution to the result.

A second way, (considered by Zvan, Forsyth and Vetzal [7]) is to treat the probabilities of the tree as weights and neglect that they are not properly defined. In this respect, the authors show for a variety of finite difference constructions (although for rather regular parameters) that the meshes allowing for negative coefficients satisfy discrete maximum and minimum principles as the mesh size parameter approaches zero and show no obvious oscillations in the level curves of the priced option values.

Here we follow the second standard approach in the construction of the quadrinomial tree, since at least theoretically we can increase the refinement of the tree until we have well-defined probabilities.

A third way to proceed is to define two new processes, constructed as a linear combination of the original ones and orthogonal to each other (see [5]). The approximating tree for the new orthogonal processes will have then well defined probabilities.

3 Calibration to market data

In this section we will fit the HJM-G2++ model to market ATM caps/floors volatilities and ATM swaption volatility surfaces.

We will calibrate the model parameters by minimizing the squared differences between the market and the model prices. For that reason, let us assume we have n market prices Market price$_1, \ldots,$ Market price$_n$ and with the same specifications for a parameter vector Θ which we want to calibrate, we have n model prices Model price$_1, \ldots,$ Model price$_n$. The objective function is the sum of the squared differences and is defined as

$$\sum_{i=1}^{n} (\text{Market price}_i - \text{Model price}_i)^2 .$$

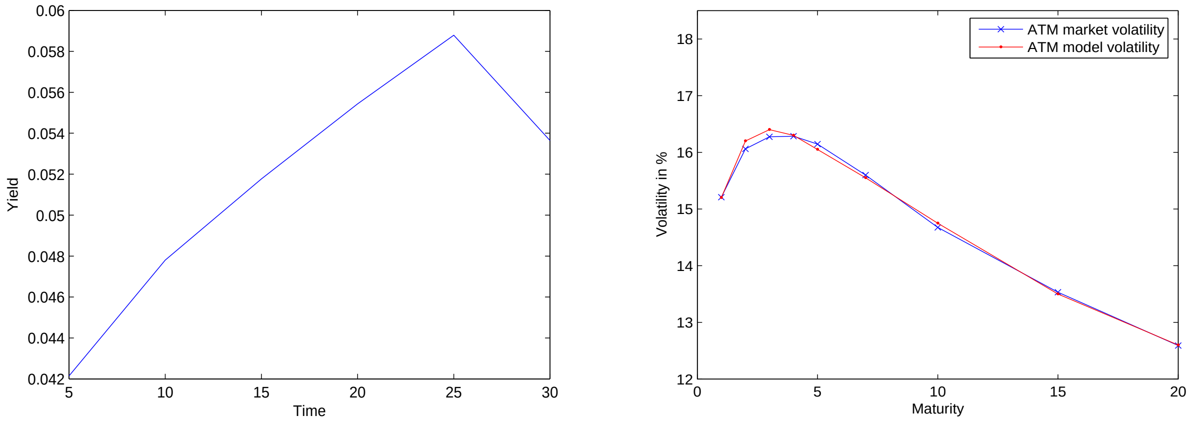
Since most of the market prices are cited in BS volatilities, we define also the **absolute error** between a one market BS volatility and its calibrated model correspondence as

$$\text{absolute error}_i = |\text{Market volatility}_i - \text{Model volatility}_i| \, , \; i = 1, \ldots, n.$$

In the implementation of the calibration of the HJM-G2++ we have the following inputs and outputs:

Input:	**Output:**
Initial yield curve	Calibrated parameter values of $\kappa_1,\ \kappa_2,\ \sigma_1,\ \sigma_2,\ \rho$
Market Caps/Floors or ATM Swaption volatilities or prices	Reports of the calibration results
Type of the used Solver - Local (Simplex) or Global (Hybrid Asa) optimizer	

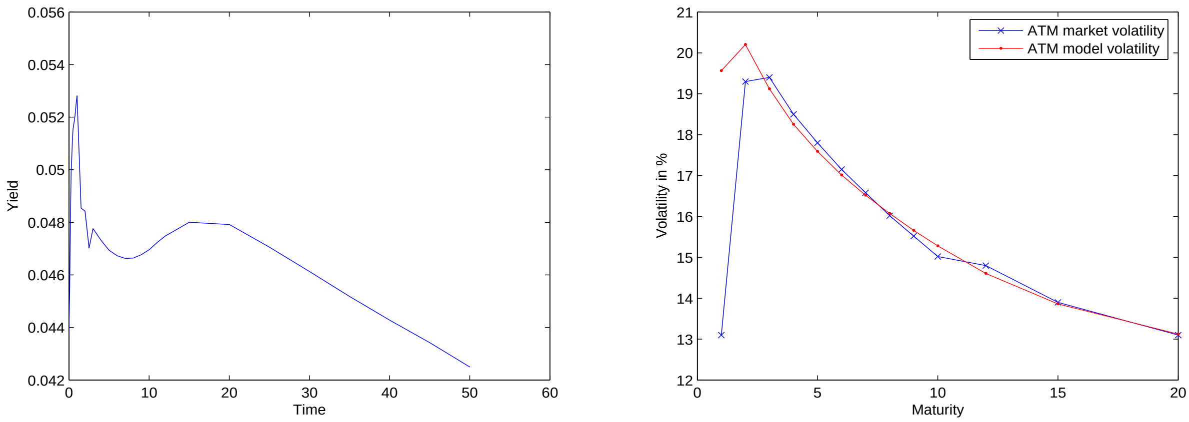
The results of the calibration are delivered in Table 2 until Table 9.



(a) Initial term structure

(b) At the money caps/floors market/model volatilities

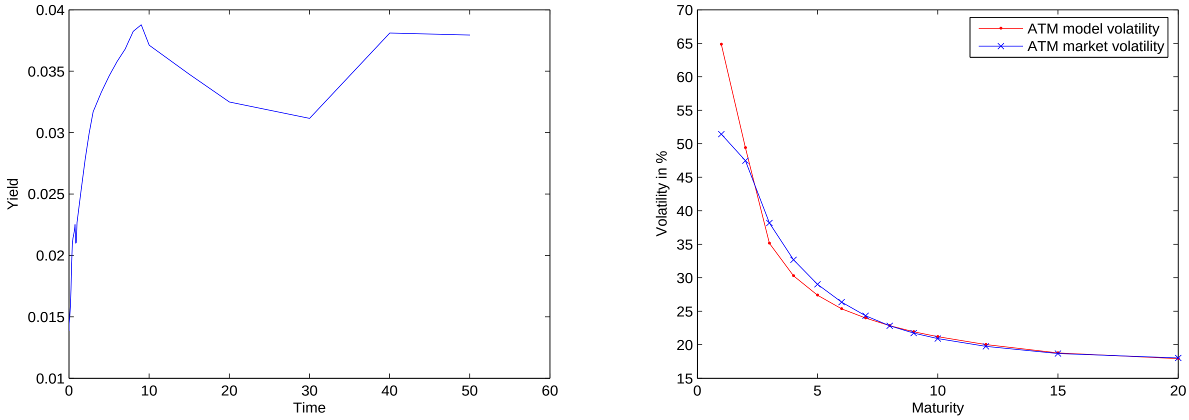
Figure 2 Calibration to ATM Caps - data from 13.02.2001



(a) Initial term structure

(b) At the money caps/floors market/model volatilities

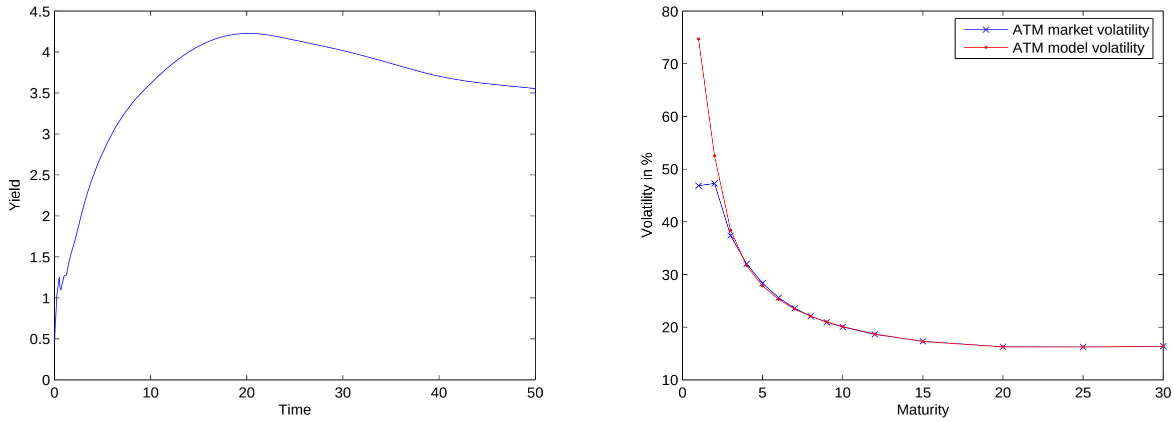
Figure 3 Calibration to ATM Caps - data from 06.08.2008



(a) Initial term structure

(b) At the money caps/floors market/model volatilities

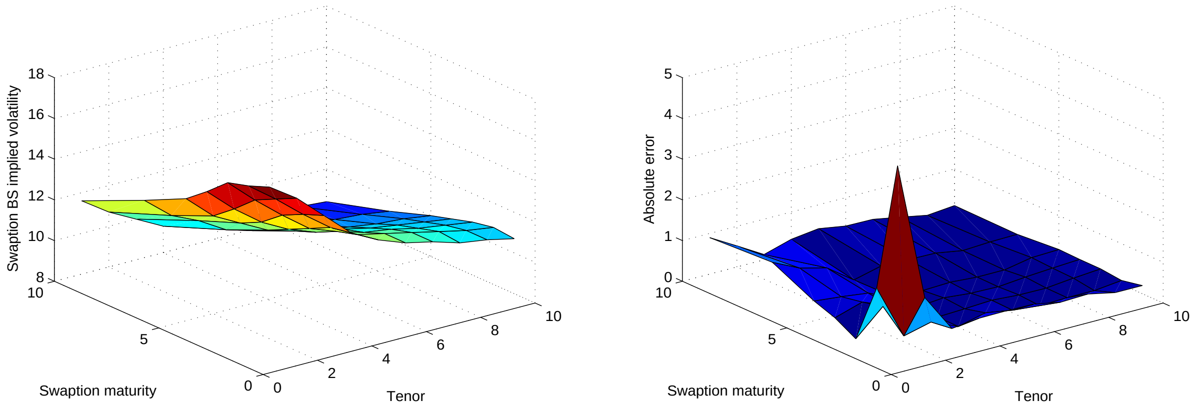
Figure 4 Calibration to ATM Caps - data from 30.04.2009



(a) Initial term structure

(b) At the money caps/floors market/model volatilities

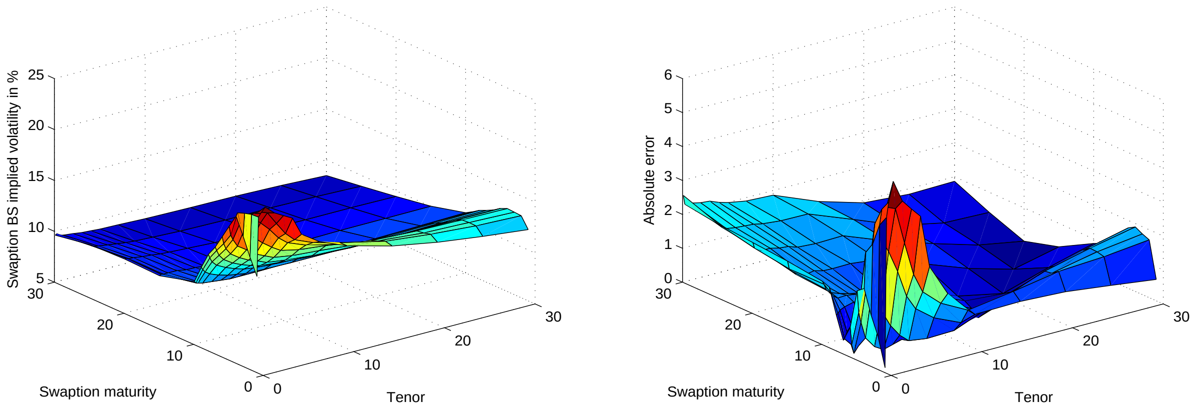
Figure 5 Calibration to ATM Caps - data from 08.07.2009



(a) At the money market swaption volatility surface

(b) Calibration absolute error

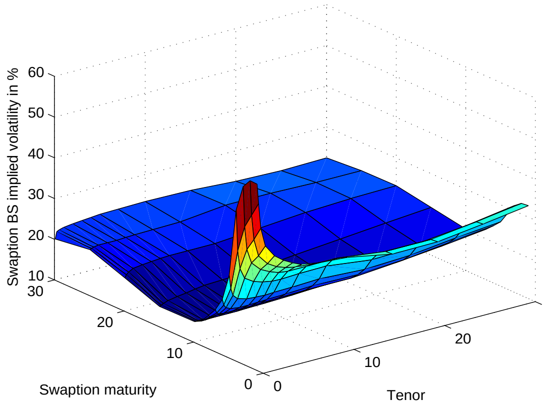
Figure 6 Calibration to ATM Swaptions - data from 13.02.2001



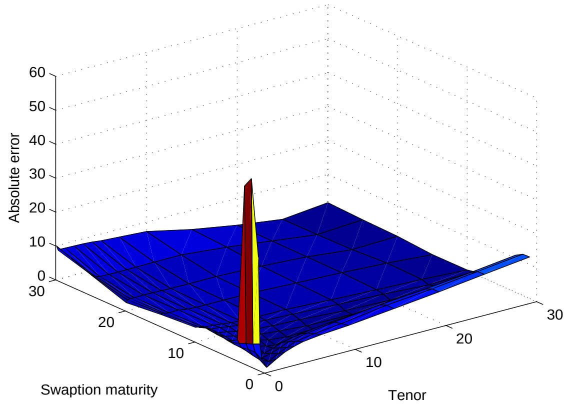
(a) At the money market swaption volatility surface

(b) Calibration absolute error

Figure 7 Calibration to ATM Swaptions - data from 06.08.2008

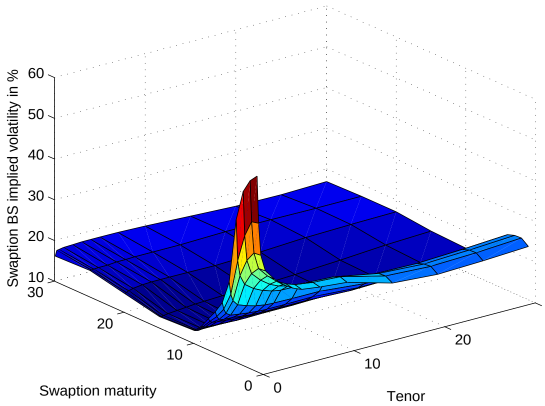


(a) At the money market swaption volatility surface

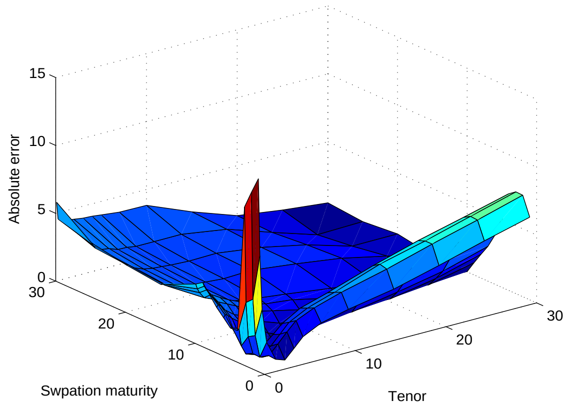


(b) Calibration absolute error

Figure 8 Calibration to ATM Swaptions - data from 30.04.2009



(a) At the money market swaption volatility surface



(b) Calibration absolute error

Figure 9 Calibration to ATM Swaptions - data from 08.07.2009

4 Conclusion

We have presented the HJM-G2++ model which is a two-factor Gaussian short rate model, equivalent to the original two-factor Hull and White model and the G2++ model by Brigo and Mercurio. Although an equivalent representation, the model has (due to its derivation within the HJM framework) a very intuitive form which does not need additional fitting to the initial term structure and is an easier one for numerical implementation.

We have calibrated the HJM-G2++ model to different sets of market data of ATM Caps/Floors volatilities and ATM Swaption volatility surfaces. The obtained results show that due to its gaussian nature, the model is not suitable of fitting strongly humped short-term effects. However, the model delivers satisfactory fitting to options with middle and long-term maturities and is able to reproduce humped volatility structures.

Finally, we have offered a Monte Carlo simulation scheme and a lattice approximation as numerical methods for pricing claims with no closed-form solution. As a more general method for pricing structured notes, we consider here the tree approximation method since it can be easily adapted for pricing derivatives with American features as well as strongly path-dependent options (see [5]). Further, the lattice approximation scheme has a faster convergence than the Monte Carlo methods. However, one can use the Monte Carlo simulation results as a back-testing for the tree approximation.

5 Appendix

Result (1) The equivalent T-forward risk-adjusted measure $\mathbf{Q}^T$ (associated with the bond maturing at time T) is defined as

$$\frac{d\mathbf{Q}^T}{d\mathbf{Q}} = \frac{B(0)P(T,T)}{B(T)P(0,T)},$$

where $B(t)$ denotes the money-market account, driven by

$$dB(t) = B(t)r(t)dt, \quad B(0) = 1.$$

Notice that

$$\begin{aligned}\frac{d\mathbf{Q}^T}{d\mathbf{Q}} &= \frac{\exp\left(-\int_0^T r(u)du\right)}{P(0,T)} = \frac{\exp\left(-\int_0^T \left[f(0,u) + X_1(u) + X_2(u)\right]du\right)}{P(0,T)} \\ &= \exp\left(-\int_0^T \left[X_1(u) + X_2(u)\right]du\right).\end{aligned}$$

Then we find that

$$\begin{aligned}\int_0^T X_1(u)du &= \frac{1}{2}\int_0^T \sigma_1^2 b_1(u,T)^2 du + \int_0^T b_1(u,T)\sigma_1 d\tilde{W}_1^Q(u) \\ &+\rho_{1,2}\int_0^T \sigma_1\sigma_2 \frac{\kappa_2 - (\kappa_1+\kappa_2)e^{-\kappa_1(T-u)} + \kappa_1 e^{-(\kappa_1+\kappa_2)(T-u)}}{\kappa_1\kappa_2(\kappa_1+\kappa_2)}du\end{aligned}$$

and applying the same steps in the integration of the X_2-process, we obtain

$$\begin{aligned}\int_0^T X_1(u)du + \int_0^T X_2(u)du &= \frac{1}{2}\int_0^T b_1(x,T)^2\sigma_1^2 dx + \frac{1}{2}\int_0^T b_2(x,T)^2\sigma_2^2 dx \\ &+\rho_{1,2}\int_0^T b_1(x,T)b_2(x,T)\sigma_1\sigma_2 dx \\ &+\int_0^T b_1(x,T)\sigma_1 d\tilde{W}_1^Q(x) + \int_0^T b_2(x,T)\sigma_2 d\tilde{W}_2^Q(x).\end{aligned}$$

Further, we decompose the Brownian motions $\tilde{W}_1^Q(t)$ and $\tilde{W}_2^Q(t)$ into a sum of two independent Brownian motions $W_1^Q(t)$ and $W_2^Q(t)$ such that

$$\begin{aligned}d\tilde{W}_1^Q(t) &= dW_1^Q(t) \\ d\tilde{W}_2^Q(t) &= \rho_{1,2}dW_1^Q(t) + \sqrt{1-\rho_{1,2}^2}\,dW_2^Q(t)\end{aligned}$$

and then we can write

$$\frac{d\mathbf{Q}^T}{d\mathbf{Q}} = \exp\Big[-\frac{1}{2}\int_0^T (b_1(x,T)\sigma_1 + \rho_{1,2}b_2(x,T)\sigma_2(x))^2\,dx - \frac{1}{2}\int_0^T\Big(\sqrt{1-\rho_{1,2}^2}b_2(x,T)\sigma_2\Big)^2 dx$$
$$-\int_0^T (b_1(x,T)\sigma_1 + \rho_{1,2}b_2(x,T)\sigma_2)\,dW_1^Q(x) - \int_0^T\Big(\sqrt{1-\rho_{1,2}^2}b_2(x,T)\sigma_2\Big)\,dW_2^Q(x)\Big].$$

Next, the Girsanov Theorem for the change of measure implies (referring to [1]) that the Brownian motions $W_1^{Q^T}(t)$ and $W_2^{Q^T}(t)$, under the forward measure $\mathbf{Q}^T$, are given as

$$\begin{aligned} dW_1^{Q^T}(t) &= dW_1^Q(t) + (b_1(t,T)\sigma_1 + \rho_{1,2}b_2(t,T)\sigma_2)\,dt \\ dW_2^{Q^T}(t) &= dW_2^Q(t) + \sqrt{1-\rho_{1,2}^2}b_2(t,T)\sigma_2 dt. \end{aligned}$$

Since $W_1^Q(t)$ and $W_2^Q(t)$ are independent Brownian motions, so are also $W_1^{Q^T}(t)$ and $W_2^{Q^T}(t)$. Then, recalling that the processes $X_1(t)$ and $X_2(t)$ are given under the measure $\mathbf{Q}$ as

$$\begin{aligned} dX_1(t) &= \left(-\kappa_1 X_1(t) + \sum_{k=1}^{2} Z_{1,k}(0,t)\right) dt + \sigma_1 dW_1^Q(t),\ X_1(0) = 0 \\ dX_2(t) &= \left(-\kappa_2 X_2(t) + \sum_{k=1}^{2} Z_{2,k}(0,t)\right) dt + \rho_{1,2}\sigma_2 dW_1^Q(t) + \sqrt{1-\rho_{1,2}^2}\sigma_2 dW_2^Q(t),\ X_2(0) = 0 \end{aligned}$$

we can write their dynamics under the forward $\mathbf{Q}^T$ measure as

$$\begin{aligned} dX_1(t) &= \left(-\kappa_1 X_1(t) + \sum_{k=1}^{2} Z_{1,k}(0,t) - b_1(t,T)\sigma_1^2 - \rho_{1,2}b_2(t,T)\sigma_1\sigma_2\right) dt \\ &\quad + \sigma_1 dW_1^{Q^T}(t),\ X_1(0) = 0 \\ dX_2(t) &= \left(-\kappa_2 X_2(t) + \sum_{k=1}^{2} Z_{2,k}(0,t) - b_2(t,T)\sigma_2^2 - \rho_{1,2}b_1(t,T)\sigma_1\sigma_2\right) dt \\ &\quad + \rho_{1,2}\sigma_2 dW_1^{Q^T}(t) + \sqrt{1-\rho_{1,2}^2}\sigma_2 dW_2^{Q^T}(t),\ X_2(0) = 0. \end{aligned}$$

Result (2): Under measure $\mathbf{Q}^T$, we want to find $X_1(t)$ and $X_2(t)$ for $t \leq T$. We have that

$$\begin{aligned}
X_1(t) &= \int_0^t Z_{11}(0,u)e^{-\kappa_1(t-u)}du + \int_0^t Z_{12}(0,u)e^{-\kappa_1(t-u)}du - \int_0^t \sigma_1^2 e^{-\kappa_1(t-u)} b_1(0,T)du \\
&- \rho_{12}\int_0^t \sigma_1\sigma_2 e^{-(\kappa_1+\kappa_2)(t-0)} b_2(u,T)du + \int_0^t \sigma_1 e^{-\kappa_1(t-u)} dW_1^{Q^T}(u) \\
&= \int_0^t \sigma_1^2 e^{-\kappa_1(t-u)} b_1(0,t)du + \rho_{12}\int_0^t \sigma_1\sigma_2 e^{-(\kappa_1+\kappa_2)(t-0)} b_2(u,t)du \\
&\quad - \int_0^t \sigma_1^2 e^{-\kappa_1(t-u)} b_1(0,T)du - \rho_{12}\int_0^t \sigma_1\sigma_2 e^{-(\kappa_1+\kappa_2)(t-0)} b_2(u,T)du \\
&\quad + \int_0^t \sigma_1 e^{-\kappa_1(t-u)} dW_1^{Q^T}(u) \\
&= -b_1(t,T)Z_{11}(0,t) - b_2(t,T)Z_{12}(0,t) + \int_0^t \sigma_1 e^{-\kappa_1(t-u)} dW_1^{Q^T}(u)
\end{aligned}$$

Thus,

$$\begin{aligned}
E^{Q^T}(X_1(t)|\mathcal{F}_0) &:= -b_1(t,T)Z_{11}(0,t) - b_2(t,T)Z_{12}(0,t) \\
E^{Q^T}(X_2(t)|\mathcal{F}_0) &:= -b_2(t,T)Z_{21}(0,t) - b_2(t,T)Z_{22}(0,t) \\
Var^{Q^T}(X_1(t)|\mathcal{F}_0) &:= \int_0^t \sigma_1^2 e^{-2\kappa_1(t-u)}du = Z_{11}(0,t) \\
Var^{Q^T}(X_2(t)|\mathcal{F}_0) &:= \int_0^t \sigma_2^2 e^{-2\kappa_2(t-u)}du = Z_{22}(0,t) \\
corr(X_1(t), X_2(t)) &:= \frac{\rho_{1,2}\int_0^t \sigma_1\sigma_2 e^{-(\kappa_1+\kappa_2)(t-u)}du}{\sqrt{Z_{11}(0,t)Z_{22}(0,t)}} = \frac{Z_{12}(0,t)}{\sqrt{Z_{11}(0,t)Z_{22}(0,t)}}
\end{aligned}$$

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