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Vorwort

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Pricing American Call Options under the Assumption of Stochastic Dividends - An Application of the Korn-Rogers-Model

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Abstract. In financial mathematics stock prices are usually modelled directly as a result of supply and demand and under the assumption that dividends are paid continuously. In contrast economic theory gives us the dividend discount model assuming that the stock price equals the present value of its future dividends. These two models need not to contradict each other - in their paper Korn and Rogers (2005) introduce a general dividend model preserving the stock price to follow a stochastic process and to be equal to the sum of all its discounted dividends. In this paper we specify the model of Korn and Rogers in a Black-Scholes framework in order to derive a closed-form solution for the pricing of American Call options under the assumption of a known next dividend followed by several stochastic dividend payments during the option's time to maturity.

Key words: option pricing, American options, dividends, dividend discount model, Black-Scholes model

JEL Classification: G13

Mathematics Subject Classification (2000): 60G44, 60H30

Introduction

In finance stock prices are typically modeled directly and are assumed to follow a geometric Brownian motion – or more generally a semi-martingale – without referring to the economic value of the payments obtained by possessing the stock. Even more, sometimes the existence of dividend payments is simply ignored and, in particular, in option pricing the validity of certain key results depend in a crucial way on the absence of dividends – a prominent example is the price equality between European and American calls on a non-dividend paying stock in the presence of a non-negative interest rate. On the one hand, ignoring the dividend payments can lead to serious pricing errors for derivatives. On the other hand, we could interpret this as modeling the stock price evolution only as a result of supply and demand. From an economic point of view, however, the price of a share of a company should be equal to the present value of the future dividend payments. In general the two modeling approaches need not to contradict each other. But there are situations, where an explicit consideration of dividend payments is necessary, since otherwise the price evolution of the share is not modeled in an adequate way. The typical situation is the time span between the announcement of the next dividend payment and its actual payment date. Then the share price dynamics contains a certain component which is deducted from the share price at the dividend payment time. Thus, the dynamics of the share price has to differ from that during times of no explicit dividend announcement.

The case of known dividends and the valuation of European options as well as American call options has been widely discussed in the literature. Roll, Geske and Whaley – see [8], [9], [10], [15], [17] and [18] – have solved the pricing problem of an American call on a stock with one known dividend payment during its time to maturity. Sterk [16] has verified the fit of the Roll-Geske-Whaley formula to American call prices. In mathematical finance Geske [7] was the first to consider uncertain dividends leading to a closed-form solution for European option prices in an adjusted Black-Scholes framework. Following this introduction of an unknown dividend Broadie et al. [3] as well as Chance et al. [4] examined the influence of stochastic dividend payments on the price of an European option. Besides there are a number of publications that are concerned with dividends and the derivation of a market opinion such as for example [5]. Professionals in finance, see for example [1], [2], [6] or [12], still take great interest in the question which model reflects reality the best and offers consistent option pricing – especially with American options. Zhu [19] has tackled the problem of giving a closed-form solution for the price of an American put option.

In order to include both approaches the following facts should be taken into account when constructing a dividend model for stocks:

- The stock price is the present value of all future dividend payments.
- The actual value of the next dividend payment and the stock price are closely related.
- There is no clear relation between the height of interest rates and the dividend payments unless the fact that an investor expects the dividend yield to be higher than the yield of a risk-less bond.
- The closer one gets to the time of the dividend payment, randomness of the dividend reduces.

Based on these facts Korn and Rogers [14] have chosen to model all dividends by a stochastic process and thereby derive the price of a stock $S(t)$ paying dividends $D(t_i)$ at future times $t_i > t$ as

$$S(t) = \mathbb{E}\left[\sum_{i=1}^{\infty} e^{-r(t_i - t)} D(t_i)\right]. \tag{1}$$

We note that this case also includes that the earlier dividend payments can be known and therefore constant.

In this paper we use the basic idea of Korn and Rogers [14] in a Black-Scholes framework which allows us to price American options on a dividend paying stock in closed-form. The main contributions of our paper are the following:

(i) We transfer the model of Korn and Rogers into a Black-Scholes framework.

(ii) To illustrate the key idea of the proof we give closed-form solutions for just one known respectively stochastic dividend during the option's time to maturity.

(iii) We derive a recursive algorithm for our valuation based on transferring back the n-dimensional case to the $(n-1)$-dimensional case.

(iv) We achieve a closed-form solution to the pricing problem of American Call options with several dividend payments during the time to maturity.

The remainder of the paper is structured as follows:

We use the dividend model in a lognormal framework, which is described in detail in Section 1. To illustrate the idea, we discuss closed-form solutions to American option pricing in Section 2 under the assumption that there is only one dividend payment during the time to maturity of the corresponding option. Furthermore, we distinguish between a stochastic and a known dividend payment. In Section 3 we give closed-form solutions to the pricing of American Call options under the assumption of several dividends, where we distinguish two situations: (1) strictly stochastic dividends, and (2) one known dividend followed by stochastic dividend payments. Of course it is possible to combine these results to one, but for the sake of a better understanding of the situation we present these cases seperately.

1 Stochastic Dividends in the Korn-Rogers Model

From now on we assume that the stock pays its dividends at equidistant times of which the first l dividends $D_1, \cdots, D_l$ with payment dates $0 < t_1, t_1 + h \cdots, t_1 + h(l-1)$ are known and therefore deterministic and the later dividends $D_{l+1}, D_{l+2}, \cdots$ with payment dates $t_1 + hl, t_1 + h(l+1), \cdots$ are stochastic. In their paper [14] Korn and Rogers assume that the stochastic dividends are of the form

$$D_{l+1} = X(t_1 + hl), \cdots, D_n = X(t_1 + h(n-1)) \tag{2}$$

where X is an exponential Lévy process scaled by some positive constant. Furthermore they assume that holds

$$\mathbb{E}\left[X(s)\middle|\,\mathcal{F}_t\right] = e^{\mu(t-s)} \cdot X(t) \tag{3}$$

for some $\mu < r$ and any $0 \leq s \leq t$ where r is the risk-free interest rate. Note that the representation of the stock price as the sum of the present values of its future dividend payments

$$S(t) = \mathbb{E}\left[\sum_{i=1}^{\infty} e^{-r(t_i - t)} D_i\right] \tag{4}$$

leads to the fact that we can write the stock price as

$$S(t) = \begin{cases} \sum_{m=1}^{l} e^{-r(t_1+(m-1)h-t)} D_m & +X(t)\frac{e^{-(r-\mu)(t_1+lh-t)}}{1-e^{-(r-\mu)h}} \\ & \text{for } t \in [0, t_1) \\ \sum_{m=k+1}^{l} e^{-r(t_1+(m-1)h-t)} D_m & +X(t)\frac{e^{-(r-\mu)(t_1+lh-t)}}{1-e^{-(r-\mu)h}} \\ & \text{for } t \in [t_1 + h(k-1), t_1 + hk), \\ & 1 \leq k < l \\ X(t)\frac{e^{-(r-\mu)(t_1+kh-t)}}{1-e^{-(r-\mu)h}} & \text{for } t \in [t_1 + h(k-1), t_1 + hk), \\ & l \leq k \end{cases} \tag{5}$$

since by Equation (3) holds

$$\mathbb{E}\left[\sum_{i>l}^{\infty} e^{-r(t_1+(i-1)h-t)} X(t_1 + (i-1)h)\middle|\mathcal{F}_t\right] = X(t)\frac{e^{-(r-\mu)(t_1+lh-t)}}{1-e^{-(r-\mu)h}} \tag{6}$$

for some $t < t_1 + hl$ as well as

$$\mathbb{E}\left[\sum_{i>k}^{\infty} e^{-r(t_1+(i-1)h-t)} X(t_1+(i-1)h) \middle| \mathcal{F}_t\right] = X(t)\frac{e^{-(r-\mu)(t_1+kh-t)}}{1-e^{-(r-\mu)h}} \tag{7}$$

for $t = t_1 + hk \geq t_1 + hl$.

So far we did not specify the dynamics of the dividend process. From now on we assume that the dividend process $X(t)$ follows a geometric Brownian motion

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t), \quad \mu < r \tag{8}$$

where r is the risk-free interest rate and σ the volatility of the dividend process X and thereby of the stock price process S.

If we denote by S_0 today's market price of the stock, this is leading to the following representation of the stock price for $t \in [0, t_1)$

$$S(t) = \left(S_0 - \sum_{m=1}^{l} D_m e^{-r(t_1+h(m-1))}\right) e^{(r-\frac{1}{2}\sigma^2)t+\sigma W(t)} + \sum_{m=1}^{l} D_m e^{-r(t_1+h(m-1)-t)} \tag{9}$$

while for $t \in [t_1 + h(k-1), t_1 + hk)$ and $1 \leq k < l$ the ex-dividend stock price equals

$$S(t) = \left(S_0 - \sum_{m=1}^{l} D_m e^{-r(t_1+h(m-1))}\right) e^{(r-\frac{1}{2}\sigma^2)t+\sigma W(t)} + \sum_{m=k+1}^{l} D_m e^{-r(t_1+h(m-1)-t)} \tag{10}$$

while for $t \in [t_1 + h(k-1), t_1 + hk)$, $l \leq k$ holds

$$S(t) = \left(S_0 - \sum_{m=1}^{l} D_m e^{-r(t_1+h(m-1))}\right) e^{-(r-\mu)(k-l)h} e^{(r-\frac{1}{2}\sigma^2)t+\sigma W(t)}. \tag{11}$$

We note that this representation relies on the fact that announcement and payment dates for the dividends coincide. It is possible to include a time difference between the announcement and the payment of dividends, which would lead to higher dimensional distribution functions. Nevertheless due to the fact that the dividend payment date is the important date at which we distinguish between exercising and holding the option we neglect this difference. Our representation contains the case of strictly stochastic dividends ($l = 0$) as well as the case of n known dividends ($l = n$) in some time interval $[0, T]$ with $0 < t_1 < \cdots < t_n = t_1 + (n-1)h < T < t_{n+1} = t_1 + nh$. Furthermore it is consistent with the Black-Scholes formula if we reduce today's stock price with regard to the total as well as proportional dividend payments paid out before the expiry of the option:

Theorem 1.1 *Consider an European call option with strike K and maturity T on a dividend paying stock with market price S_0 and n dividend payments $D_1, \cdots D_n$ during the time to maturity, of which the first l payments at times $t_1, \cdots, t_1 + h(l-1)$ are deterministic and the later dividends $D_{l+1} = X(t_1 + hl), \cdots, D_n = X(t_1 + h(n-1))$ follow a geometric Brownian motion. The price of this option is given by*

$$\tilde{S}_0 N(\tilde{d}_1) - Ke^{-rT} N(\tilde{d}_2) \tag{12}$$

where $N(\cdot)$ is the standard normal cumulative distribution function (cdf),

$$\tilde{d}_1 = \frac{\ln\left(\frac{\tilde{S}_0}{K}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \text{ and } \tilde{d}_2 = \tilde{d}_1 - \sigma\sqrt{T} \tag{13}$$

as well as

$$\tilde{S}_0 = \left(S_0 - \sum_{m=1}^{l} D_m e^{-r(t_1 + h(m-1))}\right) e^{-(r-\mu)(n-l)h}. \tag{14}$$

Proof: Using the representation of the stock price as in (11) we see that the stock price at time to maturity T can written as

$$S(T) = \left(S_0 - \sum_{m=1}^{l} D_m e^{-r(t_1 + h(m-1))}\right) e^{-(r-\mu)(n-l)h} e^{(r - \frac{1}{2}\sigma^2)T + \sigma W(T)}. \tag{15}$$

Hence the rest of the proof goes along the lines of the proof of the Black-Scholes formula. □

2 American Options in the One-Dividend Case

In order to show the basic idea of the proof for the higher dimensional problem we first consider the case of only one dividend during the time to maturity such that $0 < t_1 < T < t_1 + h$.

2.1 The Case of a Stochastic Dividend

In this subsection we focus on the case of American Call options in the presence of completely stochastic dividends with only one dividend payment during the option's time to maturity:

Theorem 2.1 *Assume that the stock pays an unknown dividend $D_1 = X(t_1)$ at time t_1. The price $C_D^{0,1}$ of an American Call with strike K and maturity $T > t_1$ is given by*

$$C_D^{0,1}(S_0, 0, T, K) = S_0 \Pi_1^{0,1}(S_0, 0) - Ke^{-rT} \Pi_2^{0,1}(S_0, 0) \tag{16}$$

where

$$\Pi_1^{0,1}(S_0,0) = N(d_1^1) + e^{-(r-\mu)h} N\left(d_2^1, -d_1^1, -\sqrt{\frac{t_1}{T}}\right) \tag{17}$$

$$\Pi_2^{0,1}(S_0,0) = e^{r(T-t_1)} N(d_1^2) + N\left(d_2^2, -d_1^2, -\sqrt{\frac{t_1}{T}}\right) \tag{18}$$

with

$$d_1^1 = \frac{\ln\left(\frac{S_0 e^{-(r-\mu)h}}{S^*}\right) + \left(r + \frac{1}{2}\sigma^2\right) t_1}{\sigma\sqrt{t_1}} \quad \textit{and} \quad d_1^2 = d_1^1 - \sigma\sqrt{t_1}, \tag{19}$$

$$d_2^1 = \frac{\ln\left(\frac{S_0 e^{-(r-\mu)h}}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}} \quad \textit{and} \quad d_2^2 = d_2^1 - \sigma\sqrt{T}. \tag{20}$$

$N(\cdot)$ is the standard normal cdf, $N(\cdot,\cdot,\rho)$ is the bivariate normal cdf with correlation ρ and S^* is the unique stock price such that holds

$$C_{Black\ Scholes}(S^*, t_1, T, K) = S^* + D^* - K \tag{21}$$

with

$$D^* = S^* \frac{1 - e^{-(r-\mu)h}}{e^{-(r-\mu)h}}. \tag{22}$$

Proof: We note that the only time sensible for an early exercise of the American call is the time t_1 of the dividend payment. Analogously to the Roll-Geske-Whaley formula (as in [13], see also [9], [15] and [17]) we find a "critical" stock price S^* at time t_1 such that the call should be exercised for $S(t_1) > S^*$ or should be hold for $S(t_1) \leq S^*$. S^* is the stock price such that (21) holds. Hence we can write today's option price as

$$\begin{aligned} &\mathbb{E}[e^{-rt_1} \cdot C_D^{0,0}(S(t_1), t_1, T, K) \cdot 1_{\{S(t_1) \leq S^*\}} | \mathcal{F}_0] \\ &+ \mathbb{E}[e^{-rt_1} \cdot (S(t_1) + D(t_1) - K) \cdot 1_{\{S(t_1) > S^*\}} | \mathcal{F}_0] \end{aligned} \tag{23}$$

where

$$C_D^{0,0}(S(t_1), t_1, T, K) = S(t_1) N(d^1(t_1)) - K e^{-r(T-t_1)} N(d^2(t_1)) \tag{24}$$

with

$$d^1(t_1) = \frac{\ln\left(\frac{S(t_1)}{K}\right) + (r + \frac{1}{2}\sigma^2)(T - t_1)}{\sigma\sqrt{T - t_1}} \tag{25}$$

$$d^2(t_1) = d^1(t_1) - \sigma\sqrt{T - t_1} \tag{26}$$

is the Black-Scholes price of an European option with no dividend payment during the time interval (t_1, T) and

$$D(t_1) = S(t_1)\frac{1-e^{-(r-\mu)h}}{e^{-(r-\mu)h}} \tag{27}$$

is the dividend payment at time t_1. If we split up the above expectation, we first get by using Equation (27)

$$\mathbb{E}[e^{-rt_1} \cdot (S(t_1) + D(t_1) - K) \cdot 1_{\{S(t_1)>S^*\}}|\mathcal{F}_0] = S_0 N(d_1^1) - Ke^{-rt_1}N(d_1^2) \tag{28}$$

with

$$d_1^1 = \frac{\ln\left(\frac{S_0 e^{-(r-\mu)h}}{S^*}\right) + \left(r + \frac{1}{2}\sigma^2\right)t_1}{\sigma\sqrt{t_1}} \text{ and } d_1^2 = d_1^1 - \sigma\sqrt{t_1}. \tag{29}$$

Secondly we derive that

$$\mathbb{E}[e^{-rt_1} \cdot C(S(t_1), t_1, T, K) \cdot 1_{\{S(t_1)\leq S^*\}}|\mathcal{F}_0] \tag{30}$$

equals by plugging in the exact option price (24) at time t_1

$$e^{-(r-\mu)h}S_0 N\left(d_2^1, -d_1^1, -\sqrt{\frac{t_1}{T}}\right) - Ke^{-rT}N\left(d_2^2, -d_1^2, -\sqrt{\frac{t_1}{T}}\right) \tag{31}$$

with

$$d_1^1 = \frac{\ln\left(\frac{S_0 e^{-(r-\mu)h}}{S^*}\right) + \left(r + \frac{1}{2}\sigma^2\right)t_1}{\sigma\sqrt{t_1}} \quad \textit{and} \quad d_1^2 = d_1^1 - \sigma\sqrt{t_1} \tag{32}$$

$$d_2^1 = \frac{\ln\left(\frac{S_0 e^{-(r-\mu)h}}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \text{ and } d_2^2 = d_2^1 - \sigma\sqrt{T} \tag{33}$$

using Lemma A.4, Corollary A.5 and Corollary A.8 for $n = 1$ and taking into account, that we can rewrite $d^1(t_1)$ and $d^2(t_1)$ as

$$d^1(t_1) = \frac{\ln\left(\frac{S(t_1)}{K}\right) + (r + \frac{1}{2}\sigma^2)(T-t_1)}{\sigma\sqrt{T-t_1}} \tag{34}$$

$$= \frac{\ln\left(\frac{S_0 e^{-(r-\mu)h}}{K}\right) + (r + \frac{1}{2}\sigma^2)T - \sigma^2 t_1}{\sigma\sqrt{T-t_1}} + \frac{1}{\sqrt{T-t_1}}W_{t_1} \tag{35}$$

$$= \beta_1^1 + \alpha_1^1\frac{W_{t_1}}{\sqrt{t_1}} \tag{36}$$

$$d^2(t_1) = \frac{\ln\left(\frac{S(t_1)}{K}\right) + (r - \frac{1}{2}\sigma^2)(T-t_1)}{\sigma\sqrt{T-t_1}} \tag{37}$$

$$= \frac{\ln\left(\frac{S_0 e^{-(r-\mu)h}}{K}\right) + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T-t_1}} + \frac{1}{\sqrt{T-t_1}}W_{t_1} \tag{38}$$

$$= \beta_1^2 + \alpha_1^2\frac{W_{t_1}}{\sqrt{t_1}}. \tag{39}$$

□

2.2 The Case of a Known Dividend

In this subsection we assume that there is one known constant dividend during the option's time to maturity. These assumptions are the same as for the Roll-Geske-Whaley formula. In the following we show, that our approach is consistent with this well-known result:

Theorem 2.2 Roll-Geske-Whaley Formula. *Assume that the stock pays a known dividend D_1 at time t_1. The price $C_D^{1,0}$ of an American Call with strike K and maturity $T > t_1$ is given by*

$$\begin{aligned} & C_D^{1,0}(S_0, 0, T, K) \\ & \quad = \left(S_0 - D_1 e^{-rt_1}\right) \Pi_1^{1,0}(S_0, 0) - K e^{-rT} \Pi_2^{1,0}(S_0, 0) + D_1 e^{-rt_1} N(d_1^2) \end{aligned} \tag{40}$$

where $\Pi_1^{1,0}(S_0, 0)$ and $\Pi_2^{1,0}(S_0, 0)$ are given by

$$\Pi_1^{1,0}(S_0, 0) = N(d_1^1) + N\left(d_2^1, -d_1^1, -\sqrt{\frac{t_1}{T}}\right) \tag{41}$$

$$\Pi_2^{1,0}(S_0, 0) = e^{r(T-t_1)} N(d_1^2) + N\left(d_2^2, -d_1^2, -\sqrt{\frac{t_1}{T}}\right) \tag{42}$$

with

$$d_1^1 = \frac{\ln\left(\frac{S_0 - D_1 e^{-rt_1}}{S^*}\right) + \left(r + \frac{1}{2}\sigma^2\right) t_1}{\sigma\sqrt{t_1}} \quad \text{and} \quad d_1^2 = d_1^1 - \sigma\sqrt{t_1}, \tag{43}$$

$$d_2^1 = \frac{\ln\left(\frac{S_0 - D_1 e^{-rt_1}}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2^2 = d_2^1 - \sigma\sqrt{T} \tag{44}$$

where S^* is the unique stock price such that holds

$$C_{Black\ Scholes}(S^*, t_1, T, K) = S^* + D_1 - K. \tag{45}$$

Proof: The proof goes along the lines of the proof to the previous theorem taking into account that under the assumption of a first known dividend the stock price at time t_1 is an ex-dividend stock price such that holds by Equation (11)

$$S(t_1) = \left(S_0 - D_1 e^{-rt_1}\right) e^{(r - \frac{1}{2}\sigma^2)t + \sigma W(t)}. \tag{46}$$

□

3 American Options in the Multi-Dividend Case

We finally focus on American call options on a stock with n dividend payments, such that we have dividend payments at t_1, $t_1 + h$,...,$t_1 + h(n-1)$ until the option's maturity at time T.

3.1 The Case of n Stochastic Dividends

In the previous section we illustrated the structure of deriving a closed-form solution under the assumption of a single dividend during the time to maturity. In the case of n dividends this leads to the recursive calculation of $n-1$ multivariate normal distributions. We note that the number of steps in the calculation process of the option price is of order $O(n^4)$. However, in practice, this remains computationally feasible since the number of dividend payments up to maturity T is usually quite small.

Theorem 3.1 *The price of an American call option with strike K and maturity T on a dividend paying stock with market price S_0 and n stochastic dividends at times $t_1 < t_1 + h < \cdots < t_1 + (n-1)h < T$ during maturity is given by*

$$C_D^{0,n}(S_0, 0, T, K) = S_0 \Pi_1^{0,n}(S_0, 0) - Ke^{-rT}\Pi_2^{0,n}(S_0, 0) \tag{47}$$

where

$$\Pi_1^{0,n}(S_0, 0) = N(d_1^1) + \sum_{i=1}^{n} e^{-(r-\mu)ih} N_{i+1}\left(\mathbf{d^1_{i+1}}; C^{(i+1)}\right) \tag{48}$$

$$\begin{aligned}\Pi_2^{0,n}(S_0, 0) &= e^{r(T-t_1)}N(d_1^2) + \sum_{i=1}^{n-1} e^{r(T-(t_1+ih))} N_{i+1}\left(\mathbf{d^2_{i+1}}; C^{(i+1)}\right) \\ &\quad + N_{n+1}\left(\mathbf{d^2_{n+1}}; C^{(n+1)}\right)\end{aligned} \tag{49}$$

and where for $i = 1, \cdots, n$ and $a = 1, 2$ we define

$$\mathbf{d^a_{i+1}} = (d^a_{i+1}, -d^a_i, \cdots, -d^a_2, -d^a_1) \tag{50}$$

with

$$d_i^1 = \frac{\ln\left(\frac{S_0 e^{-(r-\mu)ih}}{S_i^*}\right) + (r + \frac{1}{2}\sigma^2)(t_1 + (i-1)h)}{\sigma\sqrt{t_1 + (i-1)h}} \tag{51}$$

$$d_i^2 = d_i^1 - \sigma\sqrt{t_1 + (i-1)h} \tag{52}$$

as well as

$$d_{n+1}^1 = \frac{\ln\left(\frac{S_0 e^{-(r-\mu)nh}}{K}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \tag{53}$$

$$d_{n+1}^2 = d_{n+1}^1 - \sigma\sqrt{T} \tag{54}$$

and with S_1^*, $S_2^*, \cdots, S_n^*$ such that for $i = 1, \cdots, n$

$$C_D^{0,n-i}(S_i^*, t_1 + (i-1)h, T, K) = S_i^* + D_i^* - K \tag{55}$$

where D_i^* is the unknown dividend paid at time $t_1 + (i-1)h$ which equals

$$D_i^* = S_i^* \frac{1 - e^{-(r-\mu)h}}{e^{-(r-\mu)h}}. \tag{56}$$

Furthermore $N(\cdot)$ is the standard normal cdf and $N_{i+1}\left(\cdot; C^{(i+1)}\right)$ is the (i+1)-dimensional standard normal cdf with correlation matrix $C^{(i+1)} = \left(c_{kj}^{(i+1)}\right)$, for

$$c_{jk}^{(i+1)} = \begin{cases} \frac{\hat{c}_{jk}^{(i)} \cdot \sqrt{(i-j+1)h}\sqrt{(i-k+1)h + t_1}}{\sqrt{t_1 + (i-j+1)h}\sqrt{t_1 + (i-k+1)h}} & for\ i = 2, \ldots, n-1; j = 1, \ldots, i \\ & and\ k = j+1, \ldots, i \\ -\sqrt{\frac{t_1}{t_1 + (i-j+1)h}} & for\ i = 1, \ldots, n-1; j = 1, \ldots, i \\ & and\ k = i+1 \\ \frac{\hat{c}_{1k}^{(n)} \cdot \sqrt{T - t_1}\sqrt{(n-k+1)h + t_1}}{\sqrt{T}\sqrt{t_1 + (n-k+1)h}} & for\ i = n; j = 1\ and\ k = 2, \ldots, n \\ \frac{\hat{c}_{jk}^{(i)} \cdot \sqrt{(n-j+1)h}\sqrt{(n-k+1)h + t_1}}{\sqrt{t_1 + (n-j+1)h}\sqrt{t_1 + (n-k+1)h}} & for\ i = n; j = 2, \ldots, n \\ & and\ k = j+1, \ldots, n \\ -\sqrt{\frac{t_1}{T}} & for\ i = n; j = 1\ and\ k = n+1 \\ -\sqrt{\frac{t_1}{t_1 + (n-j+1)h}} & for\ i = n; j = 2, \ldots, n\ and\ k = n+1 \end{cases} \tag{57}$$

with $\hat{C}^{(i+1)} = \left(\hat{c}_{kj}^{(i+1)}\right)$ for $i = 1, \ldots, n-1$ being the correlation matrices from the calculation of the American call price in the case of $n-1$ dividends paid at times $t_1 + h < \ldots < t_1 + (n-1)h < T$.

Proof: Obviously the only times sensible for an early exercise of the American call are the times $t_1 < t_1 + h < \cdots < t_1 + (n-1)h$ of the dividend payments during the option's time to maturity. By Theorem 2.1 we know that above statement holds for $n = 1$. Suppose that the above holds for $n-1$. Hence we know that the price of the American option at time t_1 equals

$$C_D^{0,n-1}(S(t_1), t_1, T, K) = S(t_1)\Pi_1^{0,n-1}(S(t_1), t_1) - Ke^{-rT}\Pi_2^{0,n-1}(S(t_1), t_1) \tag{58}$$

with $\Pi_1^{0,n-1}(S(t_1), t_1)$ and $\Pi_2^{0,n-1}(S(t_1), t_1)$ as defined above for $n-1$ dividends paid at times $t_1 + h < \ldots < t_1 + (n-1)h$ such that the multivariate standard normal cdfs are calculated on behalf of the the corresponding correlation matrices $\hat{C}^{(i+1)}$ and at

$$\mathbf{d_{i+1}^a(t_i)} = (d_{i+1}^a(t_i), -d_i^a(t_i), \cdots, -d_2^a(t_i), -d_1^a(t_i)) \tag{59}$$

for $a = 1, 2$ with

$$d_j^1(t_i) = \frac{\ln\left(\frac{S(t_1)e^{-(r-\mu)jh}}{S_{j+1}^*}\right) + (r + \frac{1}{2}\sigma^2)jh}{\sigma\sqrt{jh}} \quad (60)$$

$$d_j^2(t_i) = d_j^1(t_i) - \sigma\sqrt{jh} \quad (61)$$

for $j = 1, \dots, n-1$ as well as

$$d_n^1(t_i) = \frac{\ln\left(\frac{S(t_1)e^{-(r-\mu)(n-1)h}}{K}\right) + (r + \frac{1}{2}\sigma^2)(T - t_1)}{\sigma\sqrt{T - t_1}} \quad (62)$$

$$d_n^2(t_i) = d_n^1(t_i) - \sigma\sqrt{T - t_1}. \quad (63)$$

Furthermore we can find critical stock prices $S_2^*, \cdots, S_n^*$ such that for $i = 2, \cdots, n$ holds

$$C_D^{0,n-i}(S_i^*, t_1 + (i-1)h, T, K) = S_i^* + D_i^* - K. \quad (64)$$

Analogously to the argumentation in Theorem 2.1 we can now find a "critical" stock price S_1^* at time t_1 such that the call should be exercised for $S(t_1) > S_1^*$ or should be hold for $S(t_1) \le S_1^*$. S_1^* is the stock price such that holds

$$C_D^{0,n-1}(S_1^*, t_1, T, K) = S_1^* + D_1^* - K \quad (65)$$

where D_1^* is the unknown dividend to be paid at time t_1. Hence we can write the option price at time $t = 0$ as

$$C_D^{0,n}(S_0, 0, T, K) = \mathbb{E}\left[e^{-rt_1} \cdot \left[\left((S(t_1) + D(t_1) - K) \cdot 1_{\{S(t_1) > S_1^*\}}\right) \middle| \mathcal{F}_0\right]\right.$$
$$+ \mathbb{E}\left[e^{-rt_1} \cdot \left(C_D^{0,n-1}(S(t_1), t_1, T, K) \cdot 1_{\{S(t_1) \le S_1^*\}}\right)\right]\Big|\mathcal{F}_0\Big] \quad (66)$$

where $C_D^{0,n-1}(S(t_1), t_1, T, K)$ is given by Equation (58) and

$$D(t_1) = S(t_1)\frac{1 - e^{-(r-\mu)h}}{e^{-(r-\mu)h}} \quad (67)$$

is the dividend payment at time t_1. It is straightforward that the first term in Equation (66) equals

$$e^{-rt_1} \cdot \mathbb{E}\left[\left((S(t_1) + D(t_1) – K) \cdot 1_{\{S(t_1) > S_1^*\}}\right) \Big] \Big| \mathcal{F}_0 \right] = S_0 N(d_1^1) – K e^{-rt_1} N(d_1^2) \quad (68)$$

with

$$d_1^1 = \frac{\ln\left(\frac{S_0 e^{-(r-\mu)h}}{S_1^*}\right) + (r + \frac{1}{2}\sigma^2)t_1}{\sigma\sqrt{t_1}} \quad \text{and} \quad d_1^2 = d_1^1 - \sigma\sqrt{t_1}. \quad (69)$$

For the calculation of the second term we use the representation of the stock price as in Equation (11) with $l = 0$ and rewrite

$$d_i^1(t_1) = \frac{\ln\left(\frac{S(t_1)e^{-(r-\mu)ih}}{S_{i+1}^*}\right) + \left(r + \frac{1}{2}\sigma^2\right)ih}{\sigma\sqrt{ih}} \tag{70}$$

$$= \frac{\ln\left(\frac{S_0 e^{-(r-\mu)(i+1)h}}{S_{i+1}^*}\right) + (r + \frac{1}{2}\sigma^2)(t_1 + ih) - \sigma^2 t_1}{\sigma\sqrt{ih}} + \frac{1}{\sqrt{ih}}W(t_1) \tag{71}$$

$$= \beta_{n-i}^1 + \alpha_{n-i}^1 \frac{W(t_1)}{\sqrt{t_1}} \tag{72}$$

for $i = 1, ..., n-1$ as well as

$$d_n^1(t_1) = \frac{\ln\left(\frac{S(t_1)e^{-(r-\mu)(n-1)h}}{K}\right) + (r + \frac{1}{2}\sigma^2)(T - t_1)}{\sigma\sqrt{T - t_1}} \tag{73}$$

$$= \frac{\ln\left(\frac{S_0 e^{-(r-\mu)nh}}{K}\right) + (r + \frac{1}{2}\sigma^2)T - \sigma^2 t_1}{\sigma\sqrt{T - t_1}} + \frac{1}{\sqrt{T - t_1}}W(t_1) \tag{74}$$

$$= \beta_1^1 + \alpha_1^1 \frac{W(t_1)}{\sqrt{t_1}}. \tag{75}$$

We note that the terms $d_i^2(t_1)$ and $d_n^2(t_1)$ can be rewritten in a similar way. Finally we apply Lemma A.4 and Corollary A.5 in combination with Corollary A.7 and Corollary A.8 to derive the required probabilities and recursively calculate the correlation matrices $C^{(i)}$ from $\hat{C}^{(i-1)}$ for $i = 2, \ldots, n+1$. Hence we get that the price at time $t = 0$ is given by (47).

□

3.2 The Case of a Known Dividend followed by Stochastic Dividends

In the following theorem we modify the results of Section 3.1 to the realistic case that the first coming dividend payment at t_1 is known and the remaining $n - 1$ dividends are stochastic.

Theorem 3.2 *The price of an American call option with strike K and maturity T on a dividend paying stock with market price S_0 and a deterministic dividend D_1 at time t_1 and $n-1$ stochastic dividends at times $t_1 + h < \cdots < t_1 + (n-1)h < T$ during maturity is given by*

$$C_D^{1,n-1}(S_0, 0, T, K)$$
$$= \left(S_0 - D_1 e^{-rt_1}\right)\Pi_1^{1,n-1}(S_0, 0) - Ke^{-rT}\Pi_2^{1,n-1}(S_0, 0) + D_1 e^{-rt_1} N(d_1^2) \tag{76}$$

where

$$\Pi_1^{1,n-1}(S_0,0) = N(d_1^1) + \sum_{i=1}^{n} e^{-(r-\mu)(i-1)h} N_{i+1}\left(\mathbf{d^1_{i+1}}; C^{(i+1)}\right) \tag{77}$$

$$\begin{aligned}\Pi_2^{1,n-1}(S_0,0) &= e^{r(T-t_1)}N(d_1^2) + \sum_{i=1}^{n-1} e^{r(T-(t_1+ih))} N_{i+1}\left(\mathbf{d^2_{i+1}}; C^{(i+1)}\right) \\ &\quad + N_{n+1}\left(\mathbf{d^2_{n+1}}; C^{(n+1)}\right)\end{aligned} \tag{78}$$

and where for $i = 1, \cdots, n$ and $a = 1, 2$ we define

$$\mathbf{d^a_{i+1}} = (d^a_{i+1}, -d^a_i, \cdots, -d^a_2, -d^a_1) \tag{79}$$

with

$$d_i^1 = \frac{\ln\left(\frac{(S_0 - D_1 e^{-rt_1})e^{-(r-\mu)(i-1)h}}{S_i^*}\right) + (r + \frac{1}{2}\sigma^2)(t_1 + (i-1)h)}{\sigma\sqrt{t_1 + (i-1)h}} \tag{80}$$

$$d_i^2 = d_i^1 - \sigma\sqrt{t_1 + (i-1)h} \tag{81}$$

as well as

$$d_{n+1}^1 = \frac{\ln\left(\frac{(S_0 - D_1 e^{-rt_1})e^{-(r-\mu)(n-1)h}}{K}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \tag{82}$$

$$d_{n+1}^2 = d_{n+1}^1 - \sigma\sqrt{T} \tag{83}$$

and with S_1^*, $S_2^*, \cdots, S_n^*$ such that for $i = 1, \cdots, n$

$$C_D^{0,n-i}(S_i^*, t_1 + (i-1)h, T, K) = S_i^* + D_i^* - K \tag{84}$$

where $D_1^* = D_1$ is the fixed dividend at time t_1 and D_i^*, $i \geq 2$, are the unknown dividends paid at times $t_1 + (i-1)h$ which equal

$$D_i^* = S_i^* \frac{1 - e^{-(r-\mu)h}}{e^{-(r-\mu)h}}. \tag{85}$$

Proof: By Theorem 3.1 we know that the price of the American option at time t_1 equals

$$C_D^{0,n-1}(S(t_1), t_1, T, K) = S(t_1)\Pi_1^{0,n-1}(S(t_1), t_1) - Ke^{-rT}\Pi_2^{0,n-1}(S(t_1), t_1) \tag{86}$$

with $\Pi_1^{0,n-1}(S(t_1), t_1)$ and $\Pi_2^{0,n-1}(S(t_1), t_1)$ as defined by Equation (48) up to Equation (79). Analogously to the argumentation in Theorem 3.1 we can now find a "critical" stock price S_1^* at time t_1 such that the call should be exercised for $S(t_1) > S_1^*$ or should be hold for $S(t_1) \leq S_1^*$. S_1^* is the stock price such that holds

$$C_D^{0,n-1}(S_1^*, t_1, T, K) = S_1^* + D_1 - K \tag{87}$$

where D_1 is the now known dividend to be paid at time t_1. As before we can write the option price at time $t = 0$ as

$$C_D^{1,n-1}(S_0, 0, T, K) = \mathbb{E}\Bigg[e^{-rt_1} \cdot \Big[\big((S(t_1) + D_1) - K\big) \cdot 1_{\{S(t_1) > S_1^*\}}\big) \Big| \mathcal{F}_0\Bigg] + \mathbb{E}\Bigg[e^{-rt_1} \cdot \Big(C_D^{0,n-1}(S(t_1), t_1, T, K) \cdot 1_{\{S(t_1) \leq S_1^*\}}\Big) \Big] \Big| \mathcal{F}_0\Bigg] \tag{88}$$

where $C_D^{0,n-1}(S(t_1), t_1, T, K)$ is given by Equation (86). We note that by using the representation of the ex-dividend stock price at time t_1 as in Equation (11)

$$S(t_1) = \left(S_0 - D_1 e^{-rt_1}\right) e^{(r - \frac{1}{2}\sigma^2)t + \sigma W(t)}. \tag{89}$$

the proof is an analogue to the proof of Theorem 3.1. □

Remark. We note that if we allow several known dividend payments during time to maturity, the option price can not be computed in this iterative way. Alternatively one could use the argument as in [13], that only the last known dividend payment is relevant and use its payment date as time t_1 in the above recursion. Of course one would still have to take all the previous payments to this date into account as in representation (9) to (11).

Conclusion

Since the introduction of the Black-Scholes formula in 1973 the pricing of European options under the assumption that the stock price follows a geometric Brownian motion is quite well understood. Not only has this formula been established as a market standard for these types of options but is also a model most professional market participants are quite familiar with. In this paper we used the general dividend model of Korn and Rogers in a Black-Scholes framework in order to price American Call options in closed-form. We derived closed-form solutions in the case of multiple stochastic dividends during time to maturity of the option which might be following a constant first dividend payment. We also showed that the model is consistent not only with the Black-Scholes price but also with the well-known Roll-Geske-Whaley formula for the price of an American Call in the presence of a known dividend payment.

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A Appendix

We denote by $^\top$ the transpose symbol, by 0_n the n-dimensional null vector, by I_n the $n \times n$ identity matrix and by $N_n(z_1, \ldots, z_n; \hat{C})$ the n-variate (standard) normal cumulative distribution function (cdf) with correlation matrix $\hat{C}$ at values $z_1, \ldots, z_n$.

A.1 Results on the Multivariate Normal Distribution

We first give some useful results on block matrices and multivariate normal distributions:

Lemma A.1 *Let $X \sim N(\mu, \sigma^2)$ be a random variable and let $Y = (Y_1, \ldots, Y_n)^\top \sim N(0, \hat{C})$ be a random vector where $\hat{C}$ is a correlation matrix. Assume that X and Y are independent. Further, denote by f_X the probability density function (pdf) of X and consider a vector of constants $\alpha = (\alpha_1, \ldots, \alpha_n)^\top$. Then holds*

$$\begin{aligned} & \int_{-\infty}^{\gamma} N_n(\alpha_1 x + \beta_1, \ldots, \alpha_n x + \beta_n, \hat{C})\, f_X(x)\, dx \\ &= P(Y_1 \leq \alpha_1 X + \beta_1, \ldots, Y_n \leq \alpha_n X + \beta_n, X \leq \gamma) \\ &= P(Z_1 \leq \beta_1, \ldots, Z_n \leq \beta_n, X \leq \gamma) \end{aligned}$$

where $Z_j = Y_j - \alpha_j X$. Here, the vector $Z = (Z_1, \ldots, Z_n)^\top$ and X are jointly normal with

$$\begin{pmatrix} Z \\ X \end{pmatrix} \sim N \left(\begin{pmatrix} -\mu\alpha \\ \mu \end{pmatrix}, \begin{pmatrix} \hat{C} + \sigma^2\, \alpha\alpha^\top & -\sigma^2\, \alpha \\ -\sigma^2\, \alpha^\top & \sigma^2 \end{pmatrix} \right).$$

Proof: Since X and Y are independent they follow a joint normal distribution

$$\begin{pmatrix} Y \\ X \end{pmatrix} \sim N \left(\begin{pmatrix} 0_n \\ \mu \end{pmatrix}, \begin{pmatrix} \hat{C} & 0_n \\ 0_n^\top & \sigma^2 \end{pmatrix} \right) =: N(\nu, \Sigma)\,.$$

Furthermore we have

$$\begin{pmatrix} Z \\ X \end{pmatrix} = H \begin{pmatrix} Y \\ X \end{pmatrix} \quad \text{with} \quad H = \left(\begin{array}{cc} \mathrm{I}_n & -\alpha \\ 0_n^\top & 1 \end{array} \right).$$

Thus, the vector consisting of Z and X has expection and variance–covariance matrix

$$H\mu = \begin{pmatrix} -\mu\alpha \\ \mu \end{pmatrix} \quad \text{and} \quad H\Sigma H^\top = \left(\begin{array}{cc} \hat{C} + \sigma^2\, \alpha\alpha^\top & -\sigma^2\, \alpha \\ -\sigma^2\, \alpha^\top & \sigma^2 \end{array} \right).$$

With the help of some well-known properties for block matrices (see for example [11], pp. 417–419) we can now derive

$$\begin{aligned} \det(H\Sigma H^\top) &= \sigma^2 \det(\hat{C})\,, \\ (H\Sigma H^\top)^{-1} &= \left(\begin{array}{cc} \hat{C}^{-1} & \hat{C}^{-1}\alpha \\ \alpha^\top \hat{C}^{-1} & \sigma^{-2} + \alpha^\top \hat{C}^{-1}\alpha \end{array} \right). \end{aligned}$$

Let z denote the vector $(z_1, \ldots, z_n)^\top$. The joint density of Z and X is given by

$$f_{Z,X}(z,x) = \frac{1}{\sqrt{\det(2\pi\, H\Sigma H^\top)}} \cdot \exp\left\{ -\frac{1}{2} \begin{pmatrix} z + \mu\alpha \\ x - \mu \end{pmatrix}^\top (H\Sigma H^\top)^{-1} \begin{pmatrix} z + \mu\alpha \\ x - \mu \end{pmatrix} \right\}$$

Since we have that

$$\begin{pmatrix} z + \mu\alpha \\ x - \mu \end{pmatrix}^\top (H\Sigma H^\top)^{-1} \begin{pmatrix} z + \mu\alpha \\ x - \mu \end{pmatrix} = (z + x\alpha)^\top \hat{C}^{-1} (z + x\alpha) + \frac{(x-\mu)^2}{\sigma^2}.$$

it follows

$$\begin{aligned} & P(Z_1 \le \beta_1, \ldots, Z_n \le \beta_n, X \le \gamma) \\ = & \int_{-\infty}^{\gamma} \int_{-\infty}^{\beta_1} \cdots \int_{-\infty}^{\beta_n} f_{Z,X}(z,x)\, dz\, dx \\ = & \int_{-\infty}^{\gamma} f_X(x) \int_{-\infty}^{\beta_1} \cdots \int_{-\infty}^{\beta_n} \frac{1}{\sqrt{\det(2\pi C)}} \exp\left\{ -\frac{1}{2} (z + x\alpha)^\top \hat{C}^{-1} (z + x\alpha) \right\} dz\, dx \\ = & \int_{-\infty}^{\gamma} f_X(x)\, N(\alpha_1 x + \beta_1, \ldots, \alpha_n x + \beta_n, \hat{C})\, dx\,. \end{aligned}$$

□

Corollary A.2 *Under the same assumptions as in Lemma A.1 it follows that*

$$\begin{aligned} & \int_{-\infty}^{\gamma} N_n(\alpha_1 x + \beta_1, \ldots, \alpha_n x + \beta_n, \hat{C})\, e^{\delta + \eta x} f_X(x)\, dx \\ = \;& e^{\delta + \mu\eta + \frac{1}{2}\eta^2\sigma^2} P(Z_1 \le \beta_1,\, \ldots,\, Z_n \le \beta_n,\, \widetilde{X} \le \gamma) \end{aligned}$$

where the vector $Z = (Z_1, \ldots, Z_n)^\top$ *and* $\widetilde{X}$ *are jointly normal with*

$$\begin{pmatrix} Z \\ \widetilde{X} \end{pmatrix} \sim N\left(\begin{pmatrix} -(\mu+\eta\sigma^2)\alpha \\ \mu+\eta\sigma^2 \end{pmatrix}, \begin{pmatrix} \hat{C} + \sigma^2\,\alpha\alpha^\top & -\sigma^2\,\alpha \\ -\sigma^2\,\alpha^\top & \sigma^2 \end{pmatrix} \right).$$

□

Lemma A.3 *Let again* $X \sim N(\mu, \sigma^2)$ *be a random variable and let* $Y = (Y_1, \ldots, Y_n)^\top \sim N(0, \hat{C})$ *be a random vector where* $\hat{C}$ *is a correlation matrix with elements* $\hat{c}_{jk}$*. Assume and* X *and* Y *are independent and let further* U *be the vector consisting of*

$$Y_1 - \alpha_1 X, \ldots, Y_n - \alpha_n X, \text{ and } X\,.$$

Then the correlation matrix of U *is given by the* $(n+1)\times(n+1)$ *matrix* C *which has off-diagonal elements*

$$c_{jk} = \begin{cases} \frac{\hat{c}_{jk}+\alpha_j\alpha_k\sigma^2}{\sqrt{1+\alpha_j^2\sigma^2}\sqrt{1+\alpha_k^2\sigma^2}} & \text{for } j \neq k \text{ and } j,k = 1,\ldots,n, \\ \frac{-\alpha_j\sigma}{\sqrt{1+\alpha_j^2\sigma^2}} & \text{for } k = n+1 \text{ and } j = 1,\ldots,n, \\ \frac{-\alpha_k\sigma}{\sqrt{1+\alpha_k^2\sigma^2}} & \text{for } j = n+1 \text{ and } k = 1,\ldots,n, \end{cases} \tag{90}$$

Proof: Consider again $\alpha = (\alpha_1, \ldots, \alpha_n)^\top$. From the proof of Lemma A.1 we know that

$$Z = (Y_1 - \alpha_1 X, \ldots, Y_n - \alpha_n X)^\top \sim N\left(-\mu\alpha, \hat{C} + \sigma^2\,\alpha\alpha^\top\right)$$

and

$$U = \begin{pmatrix} Z \\ X \end{pmatrix} \sim N\left(\begin{pmatrix} -\mu\alpha \\ \mu \end{pmatrix}, \begin{pmatrix} \hat{C} + \sigma^2\,\alpha\alpha^\top & -\sigma^2\,\alpha \\ -\sigma^2\,\alpha^\top & \sigma^2 \end{pmatrix} \right) =: N(\mu_u, \Sigma_u)\,.$$

Denoting by D_u the diagonal matrix having the elements of Σ_u on the diagonal, we calculate the correlation matrix C by multiplying out $D_u^{-1/2}\Sigma_u D_u^{-1/2}$. □

Lemma A.4 *Let* f_X *be the standard normal pdf and let* $\hat{C}$ *be a correlation matrix. Then holds*

$$\begin{aligned} &\int_{-\infty}^{\gamma} N_n(\alpha_1 x + \beta_1, \ldots, \alpha_n x + \beta_n, \hat{C})\, f_X(x)\, dx \\ &= N_{n+1}\left(\frac{\beta_1}{\sqrt{1+\alpha_1^2}}, \ldots, \frac{\beta_n}{\sqrt{1+\alpha_n^2}}, \gamma, C \right) \end{aligned}$$

where C *is the correlation matrix given in Equation (90).*

Proof: The proof follows immediately by standardizing the Z_j and X from Lemma A.1, their correlation matrix C is given in Lemma A.3. □

Corollary A.5 *Under the same assumptions as in Lemma A.4 it follows that*

$$\int_{-\infty}^{\gamma} N_n(\alpha_1 x + \beta_1, \ldots, \alpha_n x + \beta_n, \hat{C})\, e^{\delta+\eta x}\, f_X(x)\, dx$$

$$= \quad e^{\delta+\frac{1}{2}\eta^2} N_{n+1}\left(\frac{\beta_1 + \eta\alpha_1}{\sqrt{1+\alpha_1^2}}, \ldots, \frac{\beta_n + \eta\alpha_n}{\sqrt{1+\alpha_n^2}}, \gamma - \eta, C\right)$$

where C is the correlation matrix given in Equation (90).

Proof: The proof follows immediately by standardizing the Z_j and $\widetilde{X}$ from Corollary A.2, the correlation matrix C is still that given in Lemma A.3. □

A.2 Recursive Calculation of the Distribution

From Lemma A.3 we see that the $(n+1) \times (n+1)$ correlation matrix C of U is a function of the $n \times n$ correlation matrix $\hat{C}$ and the parameters σ, $\alpha_1, \ldots, \alpha_n$. To implement the n-dividend case, we consider a sequence of time points $t_1, t_2, \ldots, t_{n+1}$ and let the parameters $\sigma = \sigma_n$, $\alpha_j = \alpha_{nj}$ depend on these time points. The following lemma shows how to calculate

$$C^{(n+1)} = \begin{pmatrix} 1 & c_{12}^{(n+1)} & \ldots & c_{1,d+1}^{(n+1)} \\ c_{12}^{(n+1)} & 1 & \ldots & c_{2,n+1}^{(n+1)} \\ \vdots & \vdots & \ddots & \vdots \\ c_{1,n+1}^{(n+1)} & c_{2,n+1}^{(n+1)} & \ldots & 1 \end{pmatrix}$$

in dependence of $\hat{C}^{(n)}$ which can then be used to compute the sequence $\{C^{(n+1)}\}_{n=1,2,\ldots}$ recursively.

Lemma A.6 *For a sequence of time points $t_1 < t_2 < \ldots < t_{m+1}$ define*

$$\sigma_j = \sqrt{t_{m-j+1}} \quad \textit{for} \;\; j = 1, \ldots, m,$$

and

$$\alpha_{jk} = \frac{1}{\sqrt{t_{m-k+2} - t_{m-j+1}}} \qquad \textit{for all} \;\; j = 1, \ldots, m\,; k = 1, \ldots, j \;\textit{and}\; j \geq k\,,$$

and consider the recursion to obtain $C^{(m+1)}$ from $\hat{C}^{(m)}$ according to the $\hat{C} \to C$ mapping from Lemma A.3 starting at $\hat{C}^{(1)} = 1$. Then the off-diagonal elements of the $(m+1) \times (m+1)$ correlation matrix $C^{(m+1)}$ are obtained by:

(a) *case* $m = 1$

$$c_{12}^{(2)} = \frac{-\alpha_{11}\sigma_1}{\sqrt{1+\alpha_{11}^2\sigma_1^2}} = -\sqrt{\frac{t_1}{t_2}}$$

(b) *case* $m > 1$

$$\begin{aligned}
c_{j,m+1}^{(m+1)} &= \frac{-\alpha_{mj}\sigma_m}{\sqrt{1+\alpha_{mj}^2\sigma_m^2}} = -\sqrt{\frac{t_1}{t_{m-j+2}}} \quad \textit{for} \;\; j = 1,\ldots,m \\
c_{jk}^{(m+1)} &= \frac{\hat{c}_{jk}^{(m)} + \alpha_{mj}\alpha_{mk}\sigma_m^2}{\sqrt{1+\alpha_{mj}^2\sigma_m^2}\sqrt{1+\alpha_{mk}^2\sigma_m^2}} \quad \textit{for} \;\; j = 1,\ldots,m; \;\; k = j+1,\ldots,m \\
&= \frac{\hat{c}_{jk}^{(m)}\sqrt{t_{m-j+2}-t_1}\sqrt{t_{m-k+2}-t_1}+t_1}{\sqrt{t_{m-j+2}}\sqrt{t_{m-k+2}}}\,.
\end{aligned}$$

The following two corollaries define the recursive algorithm to calculate the correlation matrices for the n-dividend cases in Theorems 3.1 and 3.2. We here use dividends paid at equidistant time points starting at t_1. Corollary A.7 explains how to compute the matrices $C^{(i+1)}$, $i < n$, whereas Corollary A.8 considers the calculation of $C^{(n+1)}$.

Corollary A.7 *Consider the sequence* $\{t_1, t_2 = t_1 + h, \ldots, t_{i+1} = t_1 + ih\}$, $i < n$. *We then obtain*

$$\begin{aligned}
\sigma_j &= \sqrt{t_1 + (i-j)h} \quad \textit{for} \;\; j = 1,\ldots,i, \\
\alpha_{jk} &= \frac{1}{\sqrt{(j-k+1)h}} \quad \textit{for all} \;\; j = 1,\ldots,i\,; k = 1,\ldots,j \;\textit{and}\; j \geq k\,,
\end{aligned}$$

and the off-diagonal elements of $C^{(i+1)}$ *are given by*

(a) *case* $i = 1$

$$c_{12}^{(2)} = \frac{-\alpha_{11}\sigma_1}{\sqrt{1+\alpha_{11}^2\sigma_1^2}} = -\sqrt{\frac{t_1}{t_1+h}}\,,$$

(b) *case* $i > 1$

$$\begin{aligned}
c_{j,i+1}^{(i+1)} &= \frac{-\alpha_{ij}\sigma_i}{\sqrt{1+\alpha_{ij}^2\sigma_i^2}} = -\sqrt{\frac{t_1}{t_1+(i-j+1)h}} \quad \textit{for} \;\; j = 1,\ldots,i, \\
c_{jk}^{(i+1)} &= \frac{\hat{c}_{jk}^{(i)} + \alpha_{ij}\alpha_{ik}\sigma_i^2}{\sqrt{1+\alpha_{ij}^2\sigma_i^2}\sqrt{1+\alpha_{ik}^2\sigma_i^2}} \quad \textit{for} \;\; j = 1,\ldots,i; \;\; k = j+1,\ldots,i\,. \\
&= \frac{\hat{c}_{jk}^{(i)}\sqrt{(i-j+1)h}\sqrt{(i-k+1)h}+t_1}{\sqrt{t_1+(i-j+1)h}\sqrt{t_1+(i-k+1)h}}\,.
\end{aligned}$$

Corollary A.8 *Consider the sequence* $\{t_1, t_2 = t_1 + h, \ldots, t_n = t_1 + (n-1)h, t_{n+1} = T\}$*. We now have*

$$\begin{aligned}
\sigma_j &= \sqrt{t_1 + (n-j)h} \quad \textit{for} \;\; j = 1, \ldots, n, \\
\alpha_{j1} &= \frac{1}{\sqrt{T - t_1 - (n-j)h}}\,, \\
\alpha_{jk} &= \frac{1}{\sqrt{(j-k+1)h}} \quad \textit{for all} \;\; j = 2, \ldots, n\,; k = 2, \ldots, j \;\; \textit{and} \; j \geq k\,,
\end{aligned}$$

and the off-diagonal elements of $C^{(n+1)}$ *are given by*

(a) case $n = 1$

$$c_{12}^{(2)} = \frac{-\alpha_{11}\sigma_1}{\sqrt{1 + \alpha_{11}^2 \sigma_1^2}} = -\sqrt{\frac{t_1}{T}}\,,$$

(b) case $n > 1$

$$\begin{aligned}
c_{1,n+1}^{(n+1)} &= \frac{-\alpha_{n1}\sigma_n}{\sqrt{1 + \alpha_{n1}^2 \sigma_n^2}} = -\sqrt{\frac{t_1}{T}}\,, \\
c_{1k}^{(n+1)} &= \frac{\hat{c}_{1k}^{(n)} + \alpha_{n1}\alpha_{nk}\sigma_n^2}{\sqrt{1 + \alpha_{n1}^2 \sigma_n^2}\sqrt{1 + \alpha_{nk}^2 \sigma_n^2}} \quad \textit{for} \;\; k = 2, \ldots, n\,. \\
&= \frac{\hat{c}_{jk}^{(n)} \sqrt{T - t_1}\sqrt{(n-k+1)h + t_1}}{\sqrt{T}\sqrt{t_1 + (n-k+1)h}} \\
c_{j,n+1}^{(n+1)} &= \frac{-\alpha_{nj}\sigma_n}{\sqrt{1 + \alpha_{nj}^2 \sigma_n^2}} = -\sqrt{\frac{t_1}{t_1 + (n-j+1)h}} \quad \textit{for} \;\; j = 2, \ldots, n, \\
c_{jk}^{(n+1)} &= \frac{\hat{c}_{jk}^{(n)} + \alpha_{nj}\alpha_{nk}\sigma_n^2}{\sqrt{1 + \alpha_{nj}^2 \sigma_n^2}\sqrt{1 + \alpha_{nk}^2 \sigma_n^2}} \quad \textit{for} \;\; j = 2, \ldots, n; \;\; k = j+1, \ldots, n\,. \\
&= \frac{\hat{c}_{jk}^{(n)} \sqrt{(n-j+1)h}\sqrt{(n-k+1)h + t_1}}{\sqrt{t_1 + (n-j+1)h}\sqrt{t_1 + (n-k+1)h}}\,.
\end{aligned}$$

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