Fraunhofer Institut
Techno- und
Wirtschaftsmathematik

O. Iliev, I. Rybak, J. Willems

On upscaling heat conductivity for a class of industrial problems

Berichte des Fraunhofer ITWM, Nr. 120 (2007)

ISSN 1434-9973

Bericht 120 (2007)

Die Veröffentlichungen in der Berichtsreihe des Fraunhofer ITWM können bezogen werden über:

Fraunhofer-Institut für Techno- und
Wirtschaftsmathematik ITWM
Fraunhofer-Platz 1

67663 Kaiserslautern
Germany

Telefon: +49(0)631/31600-0
Telefax: +49(0)631/31600-1099
E-Mail: info@itwm.fraunhofer.de
Internet: www.itwm.fraunhofer.de

Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

ON UPSCALING HEAT CONDUCTIVITY FOR A CLASS OF INDUSTRIAL PROBLEMS

O. ILIEV, I. RYBAK, AND J. WILLEMS

ABSTRACT. Calculating effective heat conductivity for a class of industrial problems is discussed. The considered composite materials are glass and metal foams, fibrous materials, and the like, used in isolation or in advanced heat exchangers. These materials are characterized by a very complex internal structure, by low volume fraction of the higher conductive material (glass or metal), and by a large volume fraction of the air. The homogenization theory (when applicable), allows to calculate the effective heat conductivity of composite media by postprocessing the solution of special cell problems for representative elementary volumes (REV). Different formulations of such cell problems are considered and compared here. Furthermore, the size of the REV is studied numerically for some typical materials. Fast algorithms for solving the cell problems for this class of problems, are presented and discussed.

Keywords: Multiscale problems, effective heat conductivity, numerical upscaling, domain decomposition.

1. Introduction

1.1. Motivation and goals.

The effective heat conductivity of composite materials need to be calculated for a big number of industrial and environmental applications. Here we concentrate on foam, fibrous and the like materials, which are widely used in isolation or in advanced heat exchangers. These materials are characterized by a very complex internal structure, by low volume fraction of the higher conductive material (glass or metal), and by a large volume fraction of the air. The homogenization theory [JKO94, Hor97], when applicable, provides a clear algorithm for calculating the effective heat conductivity as a postprocessing of the solution of cell problems for representative elementary volumes. This theory has rigorous background when the REV is periodic, or statistically homogeneous. There are examples of applying this theory also beyond the proved area of applicability. In such a case, one has to take care numerically, or in another way, to justify the usage of this homogenization approach. In fact, different sets of boundary conditions can be used in conjunction with the cell problems (see, e.g., [WEH02] and literature therein).

It should be mentioned that in the engineering literature there exist a number of formulae, which relate the effective heat conductivity to the volume fraction of the different components of a composite material. More advanced formulae incorporate also information about the shape of the inclusions. The last approach works reasonably well when the shapes of the inclusions are more or less conventional, e.g. spheres, cylinders, cubes, etc. Such formulae are very useful in the case when they are applicable, but they give a very rough approximation in the case of complicated microstructure (complicated connectivity of the higher conductive material) and/or in the case of complicated shape of the inclusions.

Date: October 15, 2007.

There exist a huge literature on analytical and numerical approaches for calculating effective heat conductivity for composite materials, [JKO94, WEH02, Tor02, WZ06] to mention just a few. The current paper aims at contributing to certain aspects of the numerical upscaling, which are still not so well studied. In particular, the aims of the current study are:

• To compare the results obtained with different formulations for the cell problem. In this case we study the four formulations described in [WEH02], namely, i) periodic boundary conditions; ii) linear boundary conditions; iii) linear in the direction of the gradient and zero Neumann in normal direction; iv) linear at x=0 and x=1, combined with oscillatory boundary conditions along other faces.

• To get certain understanding how restrictive is the periodicity assumption, and to study applicability of the different cell problem formulations for non-periodic geometries. It is proven in [WEH02] that the most of the above formulations give equivalent results in the periodic case, our aim is to numerically study how they behave in certain non-periodic cases. One approach to study this, is to consider a big volume, and to solve a boundary volume problem there. In this paper, big geometries with linear boundary conditions are solved as example. Furthermore, the big domain is subdivided into subdomains, which still can be considered as REV. If the subdomains are REVs, the effective heat conductivity tensors, calculated for each of the subdomains, can be used to solve the problem in the whole domain. Calculating in four different ways the effective conductivity tensor in each subdomain, one can monitor which one of the four different formulations provide tensors which give better results when the solution in the whole domain is sought.

• To study numerically the size of the REV for considered classes of problems. There exist data in the literature what should be the size of the REV, e.g. in the case of spherical inclusions. The target here is to numerically study the size of the REV for foam and fibrous geometries under consideration. More precisely, the target is to check the definition for a REV in conjunction with used geometry. To do this, a big geometry will be consecutively reduced to smaller ones by cutting small volumes from each side. It is expected that as long at the cutted volumes remain REVs, they will have one and the same effective heat conductivity tensor. As long as the variation in the tensor will be observed, the, the limit for the size of the REV will be determined. It should be noted that the size of REV depends on many parameters, e.g., the shape and the distribution of the fibers shapes in the considered geometry, the way in which fibers touch/overlap, the length of the fibers, etc. Some representative geometries will be studied here.

• To discuss the advanced approach for fast solving the above mentioned cell problems. In particular, different variants of domain decomposition and multilevel iterative methods are discussed. Asymptotical approaches, using as a small parameter the ratio of the heat conductivities of the different materials, will be also discussed. Results from numerical simulations will be presented to illustrate the drawn conclusions.

1.2. **Efficient approaches for solving multiscale elliptic problems- short survey.** Let us shortly discuss the approaches for solving

$$Ax = b,$$

when this system is result of discretization of a scalar elliptic equation with discontinuous scalar or tensor coefficients. The following approaches will be shortly discussed below:
- DD preconditioner with convergence rate which does not depend on the jump of coefficients in the case when the DD resolves the jumps [Nep].
- DD preconditioner which does not depend on the jump of coefficients in the case when the DD does not resolve the jumps [Vas04].
- Algebraic multigrid, AMG
- Full multigrid, FMG and subspace solution

I. DD preconditioner which does not depend on the jump of coefficients in the case when the DD resolves the jumps (Nepomnyashchih, 1992).

The idea of this nonoverlapping DD approach is to introduce Lagrange multipliers on the interfaces. The matrix of the discretized system in this case is written as

$$A = \begin{pmatrix} A_{aa} & \mathbf{0} & A_{al} \\ \mathbf{0} & A_{mm} & A_{ml} \\ A_{la} & A_{lm} & A_{ll} \end{pmatrix},$$

Here the indeces a,m,l stand for air, metal, Lagrange, respectively. The Lagrange multipliers have meaning of temperature on the interface. In the case of voxel geometry and finite volume representation, there is a simple choice of x_l. According to Mishev, Herbin suggested in 2002 the following:

$$x_l = (k_i x_{i+1} + k_{i+1} x_i)/(k_i = k_{i+1}).$$

In fact, this is the same value used by Samarskii, and later by Wesseling in deriving finite volume discretization with harmonic averaging of coefficients.

So, in this case the algorithm looks simple. A_{aa} and A_{mm} are discretized Laplace operators with constant coefficients, what may explain why the convergence of the DD preconditioned conjugate gradients does not depend on the jumps. Furthermore, the blocks A_{aa} and A_{mm} need not be inverted at each iteration, in many cases they can be preconditioned by cheap operators.

II. DD preconditioner which does not depend on the jump of coefficients in the case when the DD does not resolve the jumps (Yu.Vassilevski, 2000).

For details, see, for example, Yu.Vassilevski, Hybrid DD method based on aggregation, NLAA, 2000. The author uses additive Schwarz with minimal overlapping as a smoother, and Galerkin coarse grid operator. However, he has specific coarse grid correction. He uses more reach coarse space. We use one coarse node per a subdomain. His coarse nodes, in addition to this, include all fine nodes which are the part of the overlapping. With our grid and discretization, this means that the coarse grid will contain all near interface fine nodes, plus one coarse node per subdomain. For the coarse nodes which are responsible for a subdomain, he uses agglomeration which should be like $A_{ii}e$, i.e., piecewise constant prolongation and volume averaging restriction The method is well described and can be easily implemented. The above two level DD method, is used as a preconditioner for Krylov subspace iterations. To solve the coarse scale operator, he uses RAMG15. Independence of the convergence rate from the variation of coefficients is proven for continuous coefficients of the PDE, the numerical experiments from the article show also result for the case of discontinuous coefficients.

III. AMG

The SAMG method of Stueben, as well as AMG from Hypre should be robust for our problems. On the other hand, they are general purpose solvers, and like the hierarchical matrices, may require a lot of memory, and the computational time is not optimized exactly for this class of problems.

IV. MG

When efficient calculation of heat conductivity for foam and fibrous materials is a target, it can be achieved in the following way. One can develop special method for the case when the foam or the fibers have much higher conductivity, compare to the air(what is usually the case for glass and metal). In this case, one can use the following two-grid method. Gauss-Seidel or Jacobi is a smoother on the fine scale, and can be implemented also in matrix-free way. The coarse space consists from all the unknowns which are in the foam (fibers), plus a layer of air around them. It should be very easy to extract this information from Geodict files. The coarse problem should be about 10 times smaller (for the benchmark foam - even smaller). To solve at the coarse scale, one can use AMG, or PCG with MILU - should work very well, because there are a lot of Dirichlet b.c.

In this paper, an algorithm which uses as a coarse space only the unknowns in the foam (without a surrounding layer of air), is considered. In this case the condition number of the coarse system should be the same as for Poisson equation (if one considers only one type of fibers). Linear temperature is considered in the surrounding air, and it is used as Dirichlet boundary condition for the equation written for the foam (or fibres) only. Another variant is to use zero Neumann boundary conditions on the lateral side of fibers. A further improvement of the algorithm can be achieved in the case of fibrous materials, if one uses the information about the exact microstructure (i.e. location and size of cylinders forming the fibrous media). This approach is a subject of a forthcoming paper.

1.3. **Structure of the paper.** The paper is organized as follows. The boundary value problem for heat equation with rapidly varying at microscale coefficients, as well as the four different cell problem formulations, are presented in the next section. The third section contains results from numerical experiments, and it has several subsections. First subsection presents results from calculating effective heat conductivity for a periodic foam geometry and for a periodic fiber geometry, each of them considered as having glass or metal as highly conductive material. The emphasize is on the comparison of the different approaches. The second subsection contains similar results, but when fibrous non-periodic geometry is used. The third subsection shows results obtained by essentially solving only in the subspace of the highly conductive material. Glass and metal are considered here for periodic and non-periodic geometries. The fourth subsection discussed the size of the REV. Effective conductivity tensors, calculated in the full and in reduced domains, are presented and discussed. The fifth subsection contains results based on using conductivity tensors, obtained from solving cell problems in subdomains. Different formulations for the subdomain cell problems are considered, comparison with solution obtained in the full domain are presented. The last section is devoted to the conclusions.

2. Upscaling

2.1. **Statement of the problem.** In this paper, we consider steady state heat transfer in highly heterogeneous porous media. Such process is described by elliptic equation for the unknown temperature T

$$-\nabla \cdot (K \nabla T) = f, \quad \text{in } \Omega, \tag{1}$$

subject to the following boundary conditions

$$T = g^D, \quad \text{on } \Gamma_D, \quad K \nabla T \cdot \mathbf{n} = g^N, \quad \text{on } \Gamma_N, \quad \partial\Omega = \Gamma_D \cup \Gamma_N. \tag{2}$$

Here the domain Ω is a parallelepiped with boundaries parallel to the coordinate planes, the set Γ_D is non-empty and had positive surface measure, the heat conductivity tensor is symmetric and uniformly positive definite in Ω. The entries of the heat conductivity tensor K may have jump discontinuities along certain interfaces that are parallel to the coordinate planes and along these interfaces the following conditions are satisfied

$$[T] = 0, \quad [K \nabla T \cdot \mathbf{n}] = 0,$$

where $[\xi] = \xi(x_{int} + 0) - \xi(x_{int} - 0)$ for the interface x_{int}.

In a number of industrial applications it is reasonable to find the effective property of the medium, i.e., instead of K given in every point, find $\tilde{K}$ constant tensor in each grid block ($\tilde{K}$ depends only on heterogeneities and calculated once may be used in different computational scenarios). In this case instead of equation (1) we will have the coarse scale heat conductivity equation

$$-\nabla \cdot (\tilde{K} \nabla T) = f. \tag{3}$$

Homogenization procedures presented here allow the coarse scale equation to be of the same form as equation (1), but with the heat conductivity K replaced by the coarse scale or effective heat conductivity tensor $\tilde{K}$.

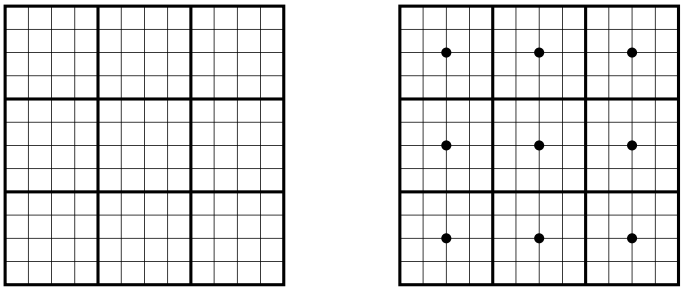


Figure 1. Fine and coarse grids

Different definitions of $\tilde{K}$ have been proposed [CDGW03, WEH02]. Solutions of the local flow problems in each coarse grid block are postprocessed in order to upscale the heat conductivity tensor. The main differences among various formulations are the boundary conditions imposed on the local flow equation and the averaging processes for computing $\tilde{K}$.

2.2. **Four different cell problem formulations.** Consider a cubic grid block V. Following [CDGW03], to define $\tilde{K}$ in V we write the coarse scale "Darcy's law"

$$\langle \mathbf{W} \rangle_V = -\tilde{K} \langle \nabla T \rangle_V \,, \tag{4}$$

where T and $\mathbf{W}$ are fine scale solutions of the problem $\mathbf{W} = -K \cdot \nabla T,\ \nabla \cdot \mathbf{W} = 0$ in the grid block V with appropriate boundary conditions. Note that the source term is set to zero because effective properties should be independent on the source term f and of the boundary conditions posed on the boundary $\partial\Omega$ of the domain of interest. Here $< . >_V$ is the volume average over V:

$$\langle . \rangle_V = \frac{1}{V} \int\limits_V (.) dx.$$

In three-dimensional case, three fine scale flow solutions are necessary in order to determine $\tilde{K}$ from (4) in each grid block

$$\mathbf{W}_i = -K \cdot \nabla T_i,\ \nabla \cdot \mathbf{W}_i = 0,\ i = \overline{1,3}, \ \text{ in } V, \tag{5}$$

provided that the volume averages of the temperature gradients are linearly independent. The subscript of $\mathbf{W}$ and ∇T designates the flow problem (1 corresponds to flow in x direction, 2 to flow in y, and 3 to flow in z).

The solutions of the local flow problems (5) with appropriate boundary conditions are then postprocessed in order to upscale the heat conductivity tensor

$$\langle \mathbf{W}_i \rangle_V = -\tilde{K} \langle \nabla T_i \rangle_V \,, \quad i = \overline{1,3}. \tag{6}$$

A number of local flow boundary conditions are used in practice. Further, the following notations will be used

$$x_1 = x,\ x_2 = y,\ x_3 = z, \quad x_i^{in} = 0,\ x_i^{out} = 1.$$

Periodic conditions can be formulated as

$$T_i = x_i + \varepsilon, \ \text{ periodic on } V. \tag{7}$$

Linear temperature drop conditions are the following

$$T_i = x_i, \ \text{ on } \partial V. \tag{8}$$

Temperature drop no-flow conditions are given by

$$T_i = x_i, \ \text{ on } \partial\Gamma_i, \quad \mathbf{n} \cdot W_i = 0, \ \text{ on } \partial\Gamma_j,\ i \neq j, \tag{9}$$

where Γ_i are the faces of ∂V normal to the unit vector in the ith direction.

Oscillatory boundary conditions are defined in the following way

$$T_i = x_i, \ \text{ on } \partial\Gamma_i, \quad T_i = P(x_i), \ \text{ on } \partial\Gamma_j,\ i \neq j. \tag{10}$$

To build the interpolation operator P, the following two-stage algorithm is used. First, we solve 4 one-dimensional problems on the edges perpendicular to the boundary $\partial\Gamma_i$ in order to develop problem dependent interpolation of the values x_i^{in} and x_i^{out} in the corners of the considered grid block

$$\frac{\partial}{\partial x_i}\left(k_{x_i x_i}\frac{\partial T_i^{edge}}{\partial x_i}\right) = 0, \quad \text{in } \partial\Gamma_j \cap \partial\Gamma_k,\ j,k \neq i, \tag{11}$$

$$T_i^{edge}(x_i^{in}) = x_i^{in},\ T_i^{edge}(x_i^{out}) = x_i^{out}, \quad \text{in } \partial\Gamma_i \cap \partial\Gamma_j \cap \partial\Gamma_k. \tag{12}$$

We discretize problem (11), (12) by means of the finite volume method (harmonic average scheme with 3-point stencil), and solve the discrete problem directly using Thomas algorithm. The solutions of one-dimensional problems (11), (12) give us fine grid pressure values T_i^{edge} along 4 edges of the considered local subdomain.

After that we solve 4 two-dimensional problems on the boundaries $\partial\Gamma_j,\ j \neq i$, of the type

$$\frac{\partial}{\partial x_j}\left(k_{x_j x_j}\frac{\partial T_i}{\partial x_j}\right) + \frac{\partial}{\partial x_k}\left(k_{x_k x_k}\frac{\partial T_i}{\partial x_k}\right) = 0, \quad \text{in } \partial\Gamma_j,\ j \neq i, \tag{13}$$

$$T_i = T_i^{edge}, \quad \text{on} \partial\Gamma_j \cap \partial\Gamma_k. \tag{14}$$

Solutions of the local problems (11), (12) are used as Dirichlet boundary conditions (14) for the boundary value problem (13). We discretize and solve on a fine grid problem (13), (14). The discretization is based on the finite volume approach (harmonic average scheme with 5-point stencil). By solving two-dimensional problems (13), (14), we obtain fine grid pressure on $\partial\Gamma_j,\ j \neq i$ of the local subdomain under consideration.

2.3. **Approaches for calculating the effective heat conductivity.** Upscaled permeability tensor $\tilde{K}$ computed via equations (6) will not in general be symmetric. Various procedures can be applied to enforce symmetry. The simplest approach is to set each of the cross terms equal to $(\tilde{k}_{xy} + \tilde{k}_{yx})/2,\ (\tilde{k}_{xz} + \tilde{k}_{zx})/2,\ (\tilde{k}_{yz} + \tilde{k}_{zy})/2$. The second disadvantage is that the mentioned above approach doesn't guarantee the positive definiteness of the upscaled permeability tensor $\tilde{K}$.

It was shown in [WEH02] that for periodic (7) and linear (8) boundary conditions formula (6) can be simplified as

$$\tilde{K}\boldsymbol{e}_i = \langle W_i \rangle_V. \tag{15}$$

Another nice property of (7) and (8) is that they lead to symmetric and positive definite $\tilde{K}$. In this case we have another way to compute $\tilde{K}$:

$$\boldsymbol{e}_i \cdot \tilde{K}\boldsymbol{e}_j = \langle \nabla p_i \cdot K \nabla p_j \rangle_V, \tag{16}$$

where $\boldsymbol{e}_i$ is the unit vector in the ith direction. Hence, $\tilde{K}$ is symmetric and positive definite. In fact, it gives symmetry up to the round off error in numerical computations.

2.4. **Solving local flow problem in the subspace of the highly conductive material.** In the case the porous media consists of two materials of high and low conductivity, for example, air/glass or air/metal, we solve equations (5) only in the solid volume fraction (the domain is arbitrary). For such problem two types of boundary conditions are possible

- Dirichlet from linear approach;
- Neumann isolation.

In the first case we suppose that the temperature in the air is linear ($T_i = x_i$) and use these values as boundary conditions for the solid subdomain. In the second case we simply pose isolation boundary conditions, i.e., fluxes on the boundary of the solid volume fraction are equal to zero.

3. Numerical experiments

In this section we present results from numerical experiments for periodic and non-periodic porous media. Different approaches for calculating effective heat conductivity are considered.

3.1. **Effective heat conductivity for periodic foam and fiber geometry (file: foamA100.leS).** Consider a periodic foam geometry (Fig. 2) and a periodic fiber geometry (Fig. 3), each of them has metal (contrast 1:1000) or glass (contrast 1:50) as highly conductive material.

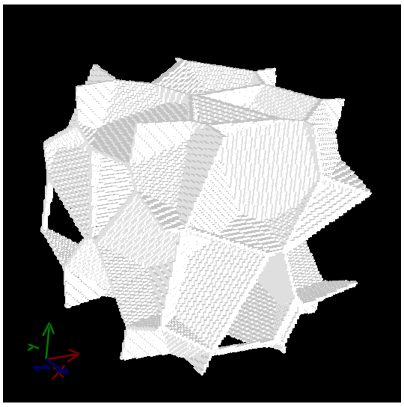


Figure 2. Foam

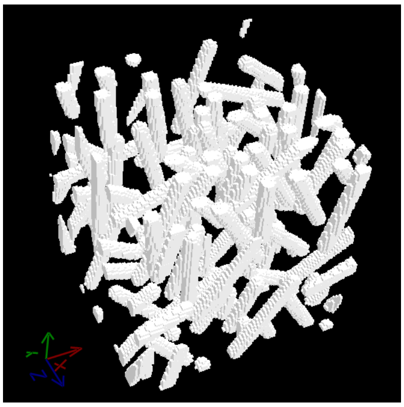


Figure 3. Fiber

To simplify understanding some notations will be used. To define local flow boundary conditions the following stencil is considered $< T^{in} - -T^{out}, BC >$. Here T^{in}, T^{out} are temperature values at x_i^{in}, x_i^{out}, $i = \overline{1,3}$, $< BC >$ is the type of normal local flow boundary conditions, i.e., $< lin. >$ means linear temperature drop boundary conditions (8), $< N. >$ – temperature drop no-flow conditions given by formula (9), $< osc. >$ – oscillatory conditions (10) and finally $< periodic >$ – periodic conditions (7).

Time is given in seconds and this is the full time necessary to find the effective heat conductivity tensor.

EHC stands for MILU preconditioned conjugate gradient solver (details are available, e.g., in [IS00], G stands for the explicit jump FFT solver integrated in the GeoDict software environment (see www.geodict.com for details).

3.1.1. *Periodic foam geometry.* File: foamA100.leS.

Consider a periodic foam geometry with metal and glass as highly conductive materials (Fig. 2). Here we have 8.7 % solid volume fraction. A comparison of different approaches for calculating effective heat conductivity is given in Tab. 1, 2. Note that we solve local flow problems in the whole domain. The results for solving local problems in the space of the highly conductive material only will be given in the next section.

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	92	4.34e+01	8.72e-02	-2.05e-01	8.41e-02	4.34e+01	1.83e-01	-2.05e-01	1.83e-01	4.47e+01
0–1, osc.	104	4.24e+01	9.88e-02	-3.56e-01	-9.30e-02	4.27e+01	6.76e-02	-4.68e-01	2.27e-01	4.37e+01
0–1, N	165	4.04e+01	9.63e-01	-5.45e-01	9.63e-01	3.94e+01	2.67e-01	-2.35e-01	-1.76e-01	4.16e+01
periodic	G	4.28e+01	1.92e-03	-1.58e-01	1.55e-03	4.28e+01	1.55e-01	-1.58e-01	1.55e-01	4.40e+01

TABLE 1. EHC, periodic foam (A100.leS), contrast 1:1000 (8.7 % of solid), whole domain

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	22	3.15e+00	4.01e-03	-8.95e-03	3.87e-03	3.15e+00	7.81e-03	-8.92e-03	7.75e-03	3.21e+00
0–1, osc.	22	3.11e+00	7.15e-03	-1.39e-02	-5.64e-05	3.11e+00	4.12e-03	-1.71e-02	9.52e-03	3.17e+00
0–1, N	42	3.03e+00	3.67e-02	-2.24e-02	3.80e-02	3.01e+00	1.20e-02	-1.01e-02	-5.35e-03	3.09e+00
periodic	G	3.12e+00	2.14e-04	-7.05e-03	2.15e-04	3.12e+00	6.93e-03	-7.05e-03	6.92e-03	3.18e+00

TABLE 2. EHC, periodic foam (A100.leS), contrast 1:50 (8.7 % of solid), whole domain

Remark. In the case of periodic porous media all boundary conditions mentioned above allow to calculate one and the same upscaled permeability tensor.

Remark. Oscillatory boundary conditions give the closest results to the periodic ones (the latter are used in GeoDict).

File: balls211neu_cube_y15z20_steg000_n100_inv.leS.

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	153	6.79e+01	-2.75e-01	-3.57e-01	-1.20e-01	8.75e+01	-7.55e-01	-5.68e-02	-6.14e-01	9.73e+01
0–1, N	D	6.58e+01	-2.51e-01	-1.89e-02	-2.55e-01	8.50e+01	-5.81e-01	4.37e-03	-5.64e-01	9.47e+01
periodic	G	6.70e+01	-8.34e-02	-2.14e-02	-8.34e-02	8.66e+01	-4.18e-01	-2.14e-02	-4.18e-01	9.65e+01

TABLE 3. EHC, periodic foam (balls211neu_cube_y15z20_steg000_n100_inv.leS), contrast 1:1000 (x % of solid), whole domain

3.1.2. *Periodic fiber geometry.* Here the following geometry files are used: fibers_100_10.leS, fibers_100_25.leS.

Consider periodic fiber geometry with 10 % of solid volume fraction (Fig. 3) and 25 % of solid volume fraction with metal and glass as highly conductive materials. Comparison of different approaches for calculating effective heat conductivity is given in Tab. 4, 5 for 10 % SVF and in Tab. 6, 7 for 25 % SVF accordingly. The calculations are performed in the whole domain. Besides the periodicity **it is crucial to know, that the fibers in both geometries are non-touching**. This leads to difficulties calculating the effective thermal conductivity (if it exists) as one can observe from comparing the homogenized tensor obtained with different boundary conditions.

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	102	9.74e+00	7.45e-01	2.62e-01	7.59e-01	1.83e+01	7.73e-02	2.58e-01	5.81e-02	6.74e+00
0–1, osc.	96	4.17e+00	3.99e-01	2.97e-01	1.32e-01	9.96e+00	1.96e-01	2.77e-01	3.66e-01	1.70e+00
0–1, N	151	2.44e+00	-1.64e-01	-1.69e-03	1.33e-01	7.44e+00	-2.47e-04	3.69e-02	5.22e-02	1.31e+00
periodic	G	2.79e+00	1.13e-02	7.27e-02	8.48e-03	5.98e-02	6.27e+00	7.86e-02	4.09e-02	1.34e+00

TABLE 4. EHC, periodic fiber (fibers_100_10.leS), contrast 1:1000 (9.9 % of solid), whole domain

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	24	1.99e+00	-3.02e-02	9.44e-03	-2.94e-02	3.22e+00	2.37e-02	9.65e-03	2.31e-02	1.52e+00
0–1, osc.	23	1.75e+00	-1.79e-02	8.79e-03	-4.33e-02	2.89e+00	3.04e-02	1.54e-02	4.18e-02	1.28e+00
0–1, N	39	1.67e+00	-1.17e-02	-4.52e-04	-2.64e-03	2.80e+00	1.84e-02	4.52e-03	2.22e-02	1.27e+00
periodic	G	1.73e+00	-1.64e-02	1.23e-02	1.64e-02	2.83e+00	2.38e-02	1.22e-02	2.41e-02	1.27e+00

TABLE 5. EHC, periodic fiber (fibers_100_10.leS), contrast 1:50 (9.9 % of solid), whole domain

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	102	3.11e+01	1.33e+00	-3.01e-01	1.36e+00	5.83e+01	9.78e-01	-3.19e-01	9.20e-01	1.67e+01
0–1, osc.	100	1.65e+01	5.56e-01	-5.91e-02	5.31e-01	4.11e+01	1.19e+00	-6.78e-01	1.09e+00	4.83e+00
0–1, N	171	9.78e+00	9.75e-01	-1.38e-01	1.57e+00	3.30e+01	7.45e-01	-1.44e-01	4.72e-01	2.91e+00
periodic	G	1.08e+01	3.09e-01	-1.66e-01	2.92e-01	2.69e+01	5.14e-01	-1.64e-01	4.95e-01	3.32e+00

TABLE 6. EHC, periodic fiber (fibers_100_25.leS), contrast 1:1000 (24.8 % of solid), whole domain

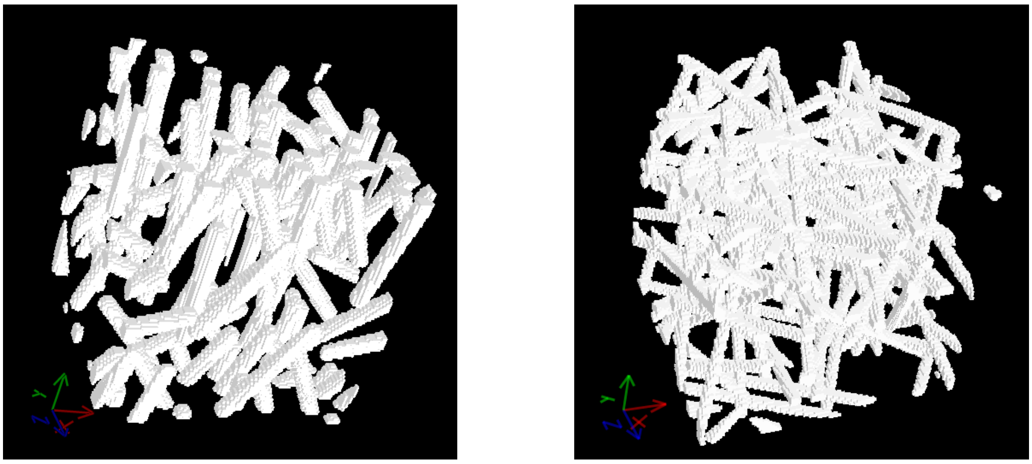
BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	21	3.86e+00	8.67e-02	-1.96e-02	8.82e-02	7.09e+00	5.24e-02	-1.94e-02	5.10e-02	2.53e+00
0–1, osc.	23	3.29e+00	6.57e-02	-5.68e-03	1.02e-01	6.46e+00	6.59e-02	-4.35e-02	5.50e-02	1.99e+00
0–1, N	42	3.17e+00	8.19e-02	-1.84e-02	1.09e-01	6.30e+00	6.08e-02	-2.26e-02	3.97e-02	1.96e+00
periodic	G	3.22e+00	1.23e-01	-2.12e-02	1.23e-01	6.16e+00	4.43e-02	-2.13e-02	4.42e-02	1.96e+02

TABLE 7. EHC, periodic fiber (fibers_100_25.leS), contrast 1:50 (24.8 % of solid), whole domain

Remark. Oscillatory boundary conditions and Neumann boundary conditions give closest results to the periodic boundary conditions (i.e., to Geodict).

3.2. **Fibrous non-periodic geometries.** In this section we consider fibrous non-periodic geometries. A comparison of different approaches for calculating effective heat conductivity is given.

3.2.1. *Two non-periodic fiber geometries with 10% SVP.* Now, we consider two non-periodic fibrous geometries with solid volume fractions of 10%. (Fig. 4). The essential difference between these two geometries is the thickness of the fibers, i.e. the fibers of 070126Fiber10SVP.leS are much thicker than those of 070806fibers_100_10_fine.leS. Results for different high conductive materials (metal, glass) are given in Tab. 8, 9 and Tab. 10, 11.



(a) Fiber geometry with thicker fibers (file: 070126Fiber10SVP.leS).

(b) Fiber geometry with thinner fibers (file: 070806fibers_100_10_fine.leS).

FIGURE 4. Two different fiber geometries with 10% SVP.

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	83	2.83e+01	-3.10e+00	2.59e+00	-3.10e+00	2.87e+01	-6.03e-01	2.60e+00	-5.98e-01	2.86e+01
0–1, osc.	86	2.62e+01	-2.99e+00	2.96e+00	-3.21e+00	2.60e+01	-1.61e+00	2.69e+00	-4.58e-01	2.58e+01
0–1, N	129	2.07e+01	-2.65e+00	3.03e+00	-2.68e+00	2.10e+01	2.49e-03	1.58e+00	9.73e-01	2.09e+01
periodic	G	1.29e+01	-1.02e+00	-1.40e-01	-1.02e+00	1.48e+01	-4.50e-01	-1.40e-01	-4.50e-01	1.21e+01

TABLE 8. EHC, non-periodic fiber (070126Fiber10SVP.leS), contrast 1:1000, whole domain

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	20	2.48e+00	-1.44e-01	1.21e-01	-1.44e-01	2.50e+00	-2.89e-02	1.21e-01	-2.87e-02	2.49e+00
0–1, osc.	20	2.38e+00	-1.46e-01	1.40e-01	-1.49e-01	2.39e+00	-6.99e-02	1.25e-01	-3.65e-02	2.38e+00
0–1, N	34	2.29e+00	-1.33e-01	1.31e-01	-1.29e-01	2.29e+00	-2.13e-02	1.16e-01	-7.91e-03	2.28e+00

TABLE 9. EHC, non-periodic fiber (070126Fiber10SVP.leS), contrast 1:50, whole domain

3.2.2. *Two non-periodic fiber geometries with 25% SVP.* Now, we consider two non-periodic fibrous geometries with solid volume fractions of 25% (Fig. 5). The results for different high conductive materials (metal, glass) are given in Tab. 12, 13 and Tab. 14, 15, respectively. Local problems are solved in the whole domain. As above, the essential difference between the two geometries is the thickness of the

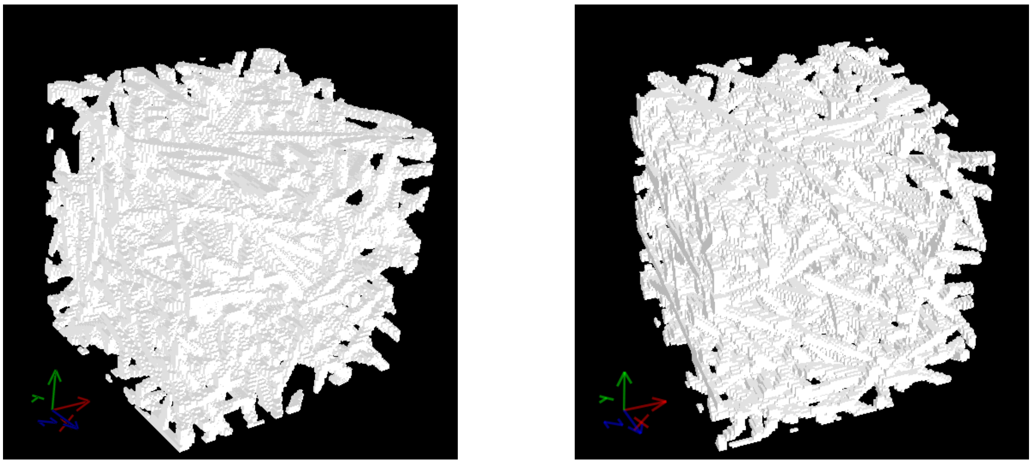
BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0-1, lin.	117	2.70e+01	-1.40e+00	2.52e+00	-1.38e+00	2.81e+01	-3.70e+00	2.54e+00	-3.74e+00	3.03e+01
0-1, osc.	118	2.44e+01	6.48e+00	1.39e+01	-4.57e+00	2.56e+01	-5.17e+00	4.17e+00	-3.35e+00	2.74e+01
0-1, N	205	1.87e+01	5.96e-01	7.68e+00	-8.29e+00	2.13e+01	-8.31e+00	5.37e+00	-4.29e+00	2.24e+01

TABLE 10. EHC, non-periodic fiber (070807fibers_100_10_fine.leS), contrast 1:1000, whole domain (results obtained with accuracy 1e-3 on sculptor, node quad02)

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0-1, lin.	27	2.42e+00	-6.46e-02	1.18e-01	-6.37e-02	2.47e+00	-1.73e-01	1.19e-01	-1.75e-01	2.57e+00
0-1, osc.	28	2.30e+00	1.43e-01	4.00e-01	-1.79e-01	2.36e+00	-1.99e-01	1.84e-01	-1.59e-01	2.44e+00
0-1, N	52	2.20e+00	-2.53e-02	2.19e-01	-2.14e-01	2.27e+00	-2.83e-01	1.92e-01	-1.73e-01	2.34e+00

TABLE 11. EHC, non-periodic fiber (070807fibers_100_10_fine.leS), contrast 1:50, whole domain (results obtained with accuracy 1e-3 on sculptor, node quad02)

fibers used. Here, the fibers of 070126Fiber25SVP.leS are thicker than those of 070806fibers_100_25_fine.leS.



(a) Fiber geometry with thicker fibers (file: 070126Fiber25SVP.leS).

(b) Fiber geometry with thinner fibers (file: 070806fibers_100_25_fine.leS).

FIGURE 5. Two different fiber geometries with 25% SVP.

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	81	7.82e+01	-2.40e+00	8.17e-01	-2.39e+00	8.27e+01	-3.67e+00	8.11e-01	-3.68e+00	8.01e+01
0–1, osc.	84	7.10e+01	-2.19e+00	1.09e+00	-2.29e+00	7.62e+01	-2.77e+00	1.34e+00	-3.70e+00	7.40e+01
0–1, N	167	6.59e+01	-2.41e+00	1.11e+00	-2.83e+00	7.09e+01	-3.81e+00	1.78e-01	-3.58e+00	6.80e+01
periodic	G	5.71e+01	-2.31e+00	8.70e-01	-2.31e+00	5.59e+01	-2.13e+00	8.70e-01	-2.13e+00	5.63e+01

TABLE 12. EHC, non-periodic fiber (070126Fiber25SVP.leS), contrast 1:1000, whole domain

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	21	5.17e+00	-1.08e-01	3.31e-02	-1.08e-01	5.37e+00	-1.66e-01	3.29e-02	-1.67e-01	5.25e+00
0–1, osc.	21	4.85e+00	-9.91e-02	3.83e-02	-1.10e-01	5.07e+00	-1.51e-01	5.19e-02	-1.70e-01	4.97e+00
0-1, N	44	4.74e+00	-1.02e-01	4.52e-02	-1.14e-01	4.94e+00	-1.75e-01	3.47e-02	-1.64e-01	4.83e+00
periodic	G	4.54e+00	-1.00e-01	4.00e-02	-1.00e-01	4.63e+00	-1.40e-01	4.00e-02	-1.40e-01	4.57e+00

Table 13. EHC, non-periodic fiber (070126Fiber25SVP.leS), contrast 1:50, whole domain

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0-1, lin.	119	8.20e+01	-4.51e-01	3.85e+00	-5.73e-01	8.31e+01	-5.56e+00	3.75e+00	-5.53e+00	7.87e+01
0-1, osc.	123	7.56e+01	-9.75e-01	7.74e+00	-4.00e+00	7.62e+01	7.85e+00	8.22e+00	-8.48e+00	7.29e+01
0-1, N	261	6.80e+01	2.74e+00	1.04e+01	4.68e+00	6.99e+01	-8.32e+00	4.10e+00	-1.02e+01	6.59e+01

Table 14. EHC, non-periodic fiber (070807fibers_100_25_fine.leS), contrast 1:1000, whole domain (results obtained with accuracy 1e-3 on sculptor, node quad02)

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0-1, lin.	29	5.34e+00	-2.12e-02	1.75e-01	-2.71e-02	5.40e+00	-2.55e-01	1.69e-01	-2.54e-01	5.19e+00
0-1, osc.	30	5.05e+00	-3.04e-02	3.71e-01	-2.47e-01	5.08e+00	-3.90e-02	3.31e-01	-3.83e-01	4.90e+00
0-1, N	66	4.86e+00	8.96e-02	3.94e-01	1.84e-01	4.93e+00	-3.73e-01	2.75e-01	-4.25e-01	4.72e+00

Table 15. EHC, non-periodic fiber (070807fibers_100_25_fine.leS), contrast 1:50, whole domain (results obtained with accuracy 1e-3 on sculptor, node quad02)

3.3. **Solving only in the subspace of the highly conductive material.** In this section we present the results when we solve local flow problems in the solid volume fraction only and compare them with results for periodic and non-periodic porous media presented above. Glass and metal are considered as highly conductive materials.

3.3.1. *Periodic foam geometry.* Consider the periodic foam from above (Fig. 2) with 8.7 % of solid volume fraction. The results presented in Tab. 16, 17 are quite close to the results for the whole domain (see Tab. 1, 2).

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	6	4.34e+01	5.73e-02	-2.22e-01	9.01e-02	4.34e+01	1.50e-01	-2.02e-01	1.91e-01	4.47e+01
0–1, osc.	8	4.37e+01	2.52e-01	-5.02e-01	6.38e-02	4.26e+01	-3.68e-02	-3.44e-01	8.72e-02	4.23e+01
0–1, N	9	4.39e+01	1.32e+00	-2.59e-01	1.92e-01	4.23e+01	-4.74e-01	-2.48e-01	-9.51e-02	4.26e+01

Table 16. EHC, periodic foam (A100.les), contrast 1:1000 (8.7 % of solid), glass

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	4	3.21e+00	3.05e-03	-7.33e-03	3.18e-03	3.21e+00	5.84e-03	-7.29e-03	6.09e-03	3.27e+00
0–1, osc.	4	3.24e+00	7.73e-03	-1.46e-02	5.99e-03	3.17e+00	5.23e-03	-1.13e-02	7.89e-03	3.17e+00
0-1, N	4	3.25e+00	7.36e-03	-7.17e-03	5.16e-03	3.18e+00	7.26e-03	-9.09e-03	5.85e-03	3.18e+00

Table 17. EHC, periodic foam (A100.les), contrast 1:50 (8.7 % of solid), glass

3.3.2. *Periodic fiber geometry.* File: fibers_100_10.leS.

The geometry is presented in Fig. 3. The results for the whole domain are given in Tab. 4, 5, 6, 7.

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	12	8.28e+00	2.87e-01	2.90e-01	-1.43e-01	3.47e+01	-6.18e-01	3.11e-01	-2.52e-01	1.55e+01
0–1, osc.	12	2.59e+00	7.05e-01	3.84e-01	1.31e-01	2.70e+01	-1.05e+00	3.32e-01	-5.82e-01	9.73e+00
0–1, N	14	3.02e+00	7.95e-01	1.48e-01	7.06e-01	2.84e+01	-6.20e-01	6.85e-01	-3.97e-01	9.96e+00

Table 18. EHC, periodic fiber (fibers_100_10.leS), contrast 1:1000 (9.9 % of solid), glass

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	5	2.05e+00	4.01e-02	4.13e-03	4.10e-02	4.20e+00	-5.74e-02	3.41e-03	-5.89e-02	2.66e+00
0–1, osc.	6	1.81e+00	7.62e-02	3.63e-02	4.46e-02	3.96e+00	-6.48e-02	-8.17e-04	-1.00e-01	2.44e+00
0–1, N	7	1.84e+00	5.65e-02	4.70e-03	4.56e-02	4.08e+00	-5.27e-02	6.37e-03	-4.65e-02	2.48e+00

Table 19. EHC, periodic fiber (fibers_100_10.leS), contrast 1:50 (9.9 % of solid), glass

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	27	2.08e+01	9.99e-01	-2.75e-01	1.19e+00	9.40e+01	1.97e+00	-9.23e-02	1.97e+00	4.34e+01
0–1, osc.	27	7.47e+00	1.21e+00	-5.88e-01	1.59e+00	7.70e+01	1.49e+00	3.00e-01	1.23e+00	2.84e+01
0–1, N	33	8.07e+00	1.00e+00	-1.96e-01	4.68e-01	8.03e+01	2.02e+00	1.57e-01	2.84e+00	3.10e+01

TABLE 20. EHC, periodic fiber (fibers_100_25.leS), contrast 1:1000 (9.9 % of solid), glass

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	10	3.76e+00	4.91e-02	-3.10e-03	4.71e-02	8.95e+00	9.58e-02	-2.84e-03	9.59e-02	5.26e+00
0–1, osc.	11	3.20e+00	6.18e-02	1.12e-03	5.40e-02	8.37e+00	1.00e-01	9.21e-03	7.24e-02	4.70e+00
0–1, N	16	1.84e+00	5.65e-02	4.70e-03	4.56e-02	4.08e+00	-5.27e-02	6.37e-03	-4.65e-02	2.48e+00

TABLE 21. EHC, periodic fiber (fibers_100_25.leS), contrast 1:50 (24.8 % of solid), glass

3.3.3. *Fibrous non-periodic geometry.* In this section we consider fibrous non-periodic geometries with 10 % and 25 % of solid volume fraction (see Fig. 4(a), 5(a)). The results for the corresponding calculations on the whole domain are given in Tab. 8, 9, 12, 13, respectively. Note, that we also perform a far from detailed analysis concerning the choice of appropriate accuracies, i.e. the stopping criterion. The data collected with respect to this issue (see Tables 26, 27, 30, and 31) suggests, that we don't need to solve with a (very) high accuracy, since the choice of boundary conditions seems to affect the computed effective thermal conductivity much more than the required accuracy.

10 % SVP. File: 070126Fiber10SVP.leS

BCs	Form.	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	D	29.11	-0.54	2.56	-0.55	29.29	-3.05	2.54	-3.04	28.85
0–1, osc.	D	27.77	-4.40	-1.60	-3.40	30.94	-8.58	0.50	-6.19	26.27
1–0, N.	D	31.38	-1.12	3.63	-5.15	31.21	-3.35	-1.45	0.98	28.01

TABLE 22. Glass, contrast 1:1000

BCs	Form.	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	D	28.63	-0.50	2.67	-0.51	28.83	-3.01	2.66	-3.01	28.38

TABLE 23. Glass, $acc = 10^{-1}$, contrast 1:1000

BCs	Form.	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	D	28.56	-0.59	2.61	-0.59	28.73	-3.08	2.61	-3.08	28.31

TABLE 24. Glass, $acc = 10^{-2}$, contrast 1:1000

25 % SVP. File: 070126Fiber25SVP.leS

BCs	Form.	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	D	2.74	-0.02	0.10	-0.02	2.75	-0.12	0.10	-0.12	2.73
0–1, osc.	D	2.75	-0.11	0.04	-0.11	2.77	-0.36	0.09	-0.19	2.65
1–0, N.	D	2.96	-0.04	0.17	-0.14	3.16	-0.05	-0.06	0.04	3.06

TABLE 25. Glass, contrast 1:50

BCs	Form.	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	D	2.51	-0.01	0.14	-0.008	2.52	-0.12	0.14	-0.12	2.50

TABLE 26. Glass, $acc = 10^{-1}$, contrast 1:50

BCs	Form.	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	D	2.49	-0.03	0.12	-0.03	2.50	-0.14	0.12	-0.14	2.48

TABLE 27. Glass, $acc = 10^{-2}$, contrast 1:50

BCs	Form.	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	D	81.48	-3.62	0.73	-3.59	84.11	2.37	0.68	-2.35	79.70
0–1, osc.	D	77.19	-16.71	-5.82	4.10	86.84	1.03	8.17	4.17	74.72

TABLE 28. Glass, contrast 1:1000

BCs	Form.	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	D	5.87	-0.13	0.01	-0.13	5.96	-0.08	0.01	-0.08	5.81
0–1, osc.	D	5.69	-0.65	-0.04	0.07	6.21	-0.06	0.31	0.16	5.62

TABLE 29. Glass, contrast 1:50

BCs	Form.	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	D	5.29	-0.11	0.08	-0.11	5.41	-0.05	0.08	-0.06	5.22

TABLE 30. Glass as initial guess ($acc = 10^{-1}$), contrast 1:50

BCs	Form.	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0–1, lin.	D	5.25	-0.16	0.03	-0.16	5.36	-0.10	0.03	-0.10	5.17

TABLE 31. Glass as initial guess ($acc = 10^{-2}$), contrast 1:50

3.3.4. *Non-periodic Fiber geometry.* For completeness, we now give the effective conductivities obtained by solving in the highly conductive parts for the non-periodic fiber geometries with thin fibers (see Figures 4(b) (file: 070806fibers_100_10_fine.leS) and 5(b) (file: 070806fibers_100_25_fine.leS)).

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0-1, lin.	54	2.70e+01	-1.26e+00	2.68e+00	-1.21e+00	2.82e+01	-3.55e+00	2.59e+00	-3.64e+00	3.03e+01
0-1, osc.	68	2.42e+01	6.64e+00	1.41e+01	-4.42e+00	2.55e+01	-5.04e+00	4.23e+00	-3.27e+00	2.71e+01

TABLE 32. EHC, non-periodic fiber (070807fibers_100_10_fine.leS), contrast 1:1000, glass only (corresponding to Table 10) (results obtained with accuracy 1e-4 on sculptor, node quad02)

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0-1, lin.	53	2.35e+00	-6.06e-02	1.28e-01	-5.85e-02	2.41e+00	-1.71e-01	1.25e-01	-1.75e-01	2.51e+00
0-1, osc.	46	2.23e+00	1.52e-01	4.17e-01	-1.73e-01	2.30e+00	-1.97e-01	1.91e-01	-1.61e-01	2.38e+00

TABLE 33. EHC, non-periodic fiber (070807fibers_100_10_fine.leS), contrast 1:50, glass only (corresponding to Table 11) (results obtained with accuracy 1e-4 on sculptor, node quad02)

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0-1, lin.	112	8.21e+01	-2.46e-01	4.07e+00	-2.86e-01	8.31e+01	-5.33e+00	4.01e+00	-5.38e+00	7.86e+01
0-1, osc.	112	7.50e+01	-7.71e-01	7.90e+00	-3.77e+00	7.55e+01	8.03e+00	8.48e+00	-8.32e+00	7.22e+01

TABLE 34. EHC, non-periodic fiber (070807fibers_100_25_fine.leS), contrast 1:1000, glass only (corresponding to Table 14) (results obtained with accuracy 1e-4 on sculptor, node quad02)

BCs	time	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0-1, lin.	128	5.15e+00	-1.18e-02	1.95e-01	-1.40e-02	5.20e+00	-2.56e-01	1.92e-01	-2.59e-01	4.99e+00
0-1, osc.	100	4.84e+00	-1.94e-02	4.00e-01	-2.44e-01	4.87e+00	-2.95e-02	3.55e-01	-3.94e-01	4.69e+00

TABLE 35. EHC, non-periodic fiber (070807fibers_100_25_fine.leS), contrast 1:50, glass only (corresponding to Table 15) (results obtained with accuracy 1e-4 on sculptor, node quad02)

3.4. **Discuss the size of the REV.** Effective conductivity tensors, calculated in the full and in reduced domains (90% of full domains), are presented and discussed.

Vol.	BCs	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
90 %	lin.	2.43	0.05	0.12	0.05	2.55	-0.15	0.12	-0.15	2.48
100 %	lin.	2.49	-0.03	0.12	-0.03	2.50	-0.14	0.12	-0.14	2.48

TABLE 36. Effective permeability for 070126fiber10svp.les, contrast1:50

Vol.	BCs	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
90 %	lin.	5.37	-0.22	0.01	-0.22	5.51	-0.09	0.01	-0.09	5.25
100 %	lin.	5.25	-0.17	0.03	-0.17	5.37	-0.12	0.03	-0.12	5.17

TABLE 37. Effective permeability for 070126fiber25svp.les, contrast1:50

3.5. **Results based on using conductivity tensors, obtained from solving cell problems in subdomains.** Different formulations for the subdomain cell problems are considered and comparisons with solutions obtained in the full domains are presented. The last section is devoted to conclusions.

Here is the effective permeability tensor for example 070126fiber25svp.les (contrast 1:1000) when we consider 8 subdomains. First, we find the effective permeability in each subdomain, then solve the coarse grid equation with full tensors on 8^3 and 20^3 grids, respectively, and then find the effective permeability tensor for the entire domain under consideration.

Note, that the number of coarse grid points doesn't influence the results very much. Here are the results for the example with 8 subdomains, when the effective

Coarse grid blocks	BCs	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
8^3	lin.	85.26	-3.86	1.43	-3.48	88.01	-2.03	0.29	-2.24	83.53
20^3	lin.	85.33	-3.57	1.02	-3.42	88.03	-2.21	0.56	-2.29	83.58
whole domain	lin.	80.09	-3.68	0.81	-3.67	82.69	-2.39	0.82	-2.40	78.23

TABLE 38. Effective permeability for 070126fiber25svp.les, 8 subdomains, contrast1:1000

permeability in each subdomain is calculated by Geodict.

Vol.	BCs	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
subdomains	lin.	46.44	-1.51	-0.04	-1.36	50.02	-3.42	-0.06	-3.01	48.30
whole domain	lin.	57.08	-2.31	0.87	-2.31	55.93	-2.13	0.87	-2.13	56.31

TABLE 39. Effective permeability for 070126fiber25svp.les, contrast1:1000

Coarse grid blocks	BCs	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
whole domain	G	6.70e+01	-8.34e-02	-2.14e-02	-8.34e-02	8.66e+01	-4.18e-01	-2.14e-02	-4.18e-01	9.65e+01
whole domain	lin.	6.79e+01	-2.75e-01	-3.57e-01	-1.20e-01	8.75e+01	-7.55e-01	-5.68e-02	-6.14e-01	9.73e+01
2^3	lin.	6.80e+01	-2.54e-01	4.75e-02	-2.54e-01	8.23e+01	-1.54e-01	4.58e-02	-1.54e-01	9.44e+01

Table 40. EHC, periodic foam (balls211neu_cube_y15z20_steg000_n100_inv.leS), contrast 1:1000, 2 step procedure

BCs	Time (s)	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0-1, lin.	29	4.77e-02	-7.09e-04	1.62e-04	-7.09e-04	7.36e-02	5.31e-04	1.62e-04	5.31e-04	3.78e-02
0-1, osc.	30	4.30e-02	-4.32e-04	1.45e-04	-9.21e-04	6.73e-02	6.77e-04	2.87e-04	9.05e-04	3.30e-02
0-1, N	50	4.16e-02	-2.62e-04	2.13e-06	-1.13e-04	6.58e-02	4.48e-04	9.26e-05	5.11e-04	3.28e-02

Table 41. EHC, periodic fiber (fiber_100_10.leS), contrast 0.026:1, acc $= 10^{-6}$

BCs	Time (s)	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0-1, lin.	375	5.31e-02	1.74e-03	4.25e-04	1.74e-03	7.03e-02	4.41e-04	4.25e-04	4.41e-04	3.76e-02
0-1, osc.	365	4.71e-02	2.35e-03	4.39e-04	1.86e-03	6.34e-02	4.04e-04	4.25e-04	5.13e-04	3.30e-02
0-1, N	826	4.53e-02	1.79e-03	3.25e-04	1.65e-03	6.17e-02	3.70e-04	3.31e-04	3.99e-04	3.28e-02

Table 42. EHC, periodic fiber (fiber_200_10.leS), contrast 0.026:1, acc $= 10^{-6}$

BCs	Time (s)	k^*_{xx}	k^*_{xy}	k^*_{xz}	k^*_{yx}	k^*_{yy}	k^*_{yz}	k^*_{zx}	k^*_{zy}	k^*_{zz}
0-1, lin.	3533	5.40e-02	-1.64e-03	4.25e-04	-1.64e-03	8.17e-02	1.37e-03	4.37e-04	1.38e-03	4.33e-02
0-1, osc.	3225	4.48e-02	-1.36e-03	2.11e-04	-1.36e-03	7.31e-02	2.25e-04	2.16e-04	2.27e-04	3.31e-02

Table 43. EHC, periodic fiber (fiber_400_10.leS), contrast 0.026:1, acc $= 10^{-6}$, subdomains 2^3

4. Conclusions

The results from numerical experiments allow to draw several conclusions:

(1) All four variants of boundary conditions for the cell problems give (almost) the same results, when a periodic REV with well connected inclusions is considered. In this way, simulations with different boundary conditions allow to check if a given volume is an REV (different methods give the same results for an REV).
(2) In this respect it is useful to note, that the differences between effective thermal conductivities computed with different sets of boundary conditions is often larger than the differences due to using lower accuracies as stopping criterions. This means that the significant digits of an effective thermal conductivity tensor can often be computed using a relatively low accuracy resulting in a shorter runtime of the program.
(3) In the case when the inclusions (fibers) do not touch, the results are not reliable, which suggests, that in these cases the considered geometries are not REVs.

(4) A reasonable approach to check if a volume is an REV, is to calculate the effective conductivity for a subvolume (e.g., 90% of the original volume). If the results are (nearly) the same, they are more trustable.
(5) Numerical results show that in the case of connected highly conductive inclusions, the approach for solving only in the inclusions, and taking linear temperature distribution in the air, is reliable. In a forthcoming paper we will give a theoretical justification for this approach.
(6) The approach for a consecutive upscaling also can give good results. However, it should be carefully used, because is is not based on rigorous justifications. In general, it should be successful if we have several scales of heterogeneities, and the subdomains themselves are REVs at one heterogeneity scale, while the ensemble of subdomains constitutes an REV at another heterogeneity scale.
(7) The last two approaches, i.e. solving only in the highly conductive material, and the consecutive upscaling, allow to significantly reduce the computational time.
(8) In the case of high contrast (big ratio between conductivity of the materials), the usage of different boundary conditions leads to the appearance of different boundary and internal layers. In this case it is difficult to judge which method is more trustable. Further investigations are needed in order to provide some recommendations for the practical computations.

Acknowledgement: This work has been supported by EC under the project INTAS-30-50-4395, by Deutscher Akademischer Austausch Dienst (grant A/05/57218) and by the Kaiserslautern Excellence Cluster Dependable Adaptive Systems and Mathematical Modelling, DASMOD.

References

[CDGW03] Y. Chen, L. J. Durlofsky, M. Gerritsen, and X. H. Wen. A coupled local-global upscaling approach for simulating flow in highly heterogeneous formations. *Advances in Water Resources*, 26:1041–1060, 2003.

[Hor97] U. Hornung, editor. *Homogenization and Porous Media*, volume 6 of *Interdisciplinary Applied Mathematics*. Springer, 1st edition, 1997.

[IS00] O. Iliev and M. Schäfer. A numerical study of the efficiency of simple-type algorithms in computing incompressible flows on stretched grids. Griebel, Michael (ed.) et al., Large-scale scientific computations of engineering and environmental problems II. Proceedings of the 2nd workshop, Sozopol, Bulgaria, June 2-6, 1999. Braunschweig: Vieweg. Notes Numer. Fluid Mech. 73, 207-214 (2000)., 2000.

[JKO94] V. V. Jikov, S. M. Kozlov, and O. A. Oleinik. *Homogenization of Differential Operators and Integral Functionals*. Springer, 1st edition, 1994.

[Nep] S.V. Nepomnyaschikh. Application of domain decomposition to elliptic problems with discontinuous coefficients. Fourth international symposium on domain decomposition methods for partial differential equations, Proc. Symp., Moscow/Russ. 1990, 242-251 (1991).

[Tor02] S. Torquato. *Random Heterogeneous Materials. Microstructure and Macroscopic Properties*. Interdisciplinary Applied Mathematics. 16. New York, NY: Springer, 2002.

[Vas04] Y. Vassilevski. A hybrid domain decomposition method based on aggregation. *Numerical Linear Algebra with Applications*, 11(4):327–341, 2004.

[WEH02] X. H. Wu, Y. Efendiev, and T. Y. Hou. Analysis of upscaling absolute permeability. *Discrete Contin. Dyn. Syst., Ser. B*, 2(2):185–204, 2002.

[WZ06] A. Wiegmann and A. Zemitis. Ej-heat: A fast explicit jump harmonic averaging solver for the effective heat conductivity of composite materials. Technical Report 94, Institut für Techno- und Wirtschaftsmathematik, 2006.

Fraunhofer Institut für Techno- und Wirtschaftsmathematik, Fraunhofer-Platz 1, 67663 Kaiserslautern, Germany,, and Institute of Mathematics, Bulg.Acad.Sci., Acad. G.Bonchev str., bl.8, 1113, Sofia, Bulgaria,, iliev@itwm.fhg.de

Institute of Mathematics, National Academy of Sciences of Belarus, Surganov Str. 11, 220072 Minsk, Belarus rybak@im.bas-net.by

Fraunhofer Institut für Techno- und Wirtschaftsmathematik, Fraunhofer-Platz 1, 67663 Kaiserslautern, Germany willems@itwm.fhg.de

Published reports of the Fraunhofer ITWM

The PDF-files of the following reports are available under:
www.itwm.fraunhofer.de/de/ zentral__berichte/berichte

1. D. Hietel, K. Steiner, J. Struckmeier
A Finite - Volume Particle Method for Compressible Flows
(19 pages, 1998)

2. M. Feldmann, S. Seibold
Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 pages, 1998)

3. Y. Ben-Haim, S. Seibold
Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 pages, 1998)

4. F.-Th. Lentes, N. Siedow
Three-dimensional Radiative Heat Transfer in Glass Cooling Processes
(23 pages, 1998)

5. A. Klar, R. Wegener
A hierarchy of models for multilane vehicular traffic
Part I: Modeling
(23 pages, 1998)

Part II: Numerical and stochastic investigations
(17 pages, 1998)

6. A. Klar, N. Siedow
Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes
(24 pages, 1998)

7. I. Choquet
Heterogeneous catalysis modelling and numerical simulation in rarified gas flows
Part I: Coverage locally at equilibrium
(24 pages, 1998)

8. J. Ohser, B. Steinbach, C. Lang
Efficient Texture Analysis of Binary Images
(17 pages, 1998)

9. J. Orlik
Homogenization for viscoelasticity of the integral type with aging and shrinkage
(20 pages, 1998)

10. J. Mohring
Helmholtz Resonators with Large Aperture
(21 pages, 1998)

11. H. W. Hamacher, A. Schöbel
On Center Cycles in Grid Graphs
(15 pages, 1998)

12. H. W. Hamacher, K.-H. Küfer
Inverse radiation therapy planning - a multiple objective optimisation approach
(14 pages, 1999)

13. C. Lang, J. Ohser, R. Hilfer
On the Analysis of Spatial Binary Images
(20 pages, 1999)

14. M. Junk
On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes
(24 pages, 1999)

15. M. Junk, S. V. Raghurame Rao
A new discrete velocity method for Navier-Stokes equations
(20 pages, 1999)

16. H. Neunzert
Mathematics as a Key to Key Technologies
(39 pages (4 PDF-Files), 1999)

17. J. Ohser, K. Sandau
Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem
(18 pages, 1999)

18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel
Solving nonconvex planar location problems by finite dominating sets
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 pages, 2000)

19. A. Becker
A Review on Image Distortion Measures
Keywords: Distortion measure, human visual system
(26 pages, 2000)

20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn
Polyhedral Properties of the Uncapacitated Multiple Allocation Hub Location Problem
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 pages, 2000)

21. H. W. Hamacher, A. Schöbel
Design of Zone Tariff Systems in Public Transportation
(30 pages, 2001)

22. D. Hietel, M. Junk, R. Keck, D. Teleaga
The Finite-Volume-Particle Method for Conservation Laws
(16 pages, 2001)

23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel
Location Software and Interface with GIS and Supply Chain Management
Keywords: facility location, software development, geographical information systems, supply chain management
(48 pages, 2001)

24. H. W. Hamacher, S. A. Tjandra
Mathematical Modelling of Evacuation Problems: A State of Art
(44 pages, 2001)

25. J. Kuhnert, S. Tiwari
Grid free method for solving the Poisson equation
Keywords: Poisson equation, Least squares method, Grid free method
(19 pages, 2001)

26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier
Simulation of the fiber spinning process
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD
(19 pages, 2001)

27. A. Zemitis
On interaction of a liquid film with an obstacle
Keywords: impinging jets, liquid film, models, numerical solution, shape
(22 pages, 2001)

28. I. Ginzburg, K. Steiner
Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models
(22 pages, 2001)

29. H. Neunzert
»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«
Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrskalenanalyse, Strömungsmechanik
(18 pages, 2001)

30. J. Kuhnert, S. Tiwari
Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation
AMS subject classification: 76D05, 76M28
(25 pages, 2001)

31. R. Korn, M. Krekel
Optimal Portfolios with Fixed Consumption or Income Streams
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems.
(23 pages, 2002)

32. M. Krekel
Optimal portfolios with a loan dependent credit spread
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics
(25 pages, 2002)

33. J. Ohser, W. Nagel, K. Schladitz
The Euler number of discretized sets – on the choice of adjacency in homogeneous lattices
Keywords: image analysis, Euler number, neighborhod relationships, cuboidal lattice
(32 pages, 2002)

34. I. Ginzburg, K. Steiner

Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting

Keywords: Lattice Boltzmann models; free-surface phenomena; interface boundary conditions; filling processes; injection molding; volume of fluid method; interface boundary conditions; advection-schemes; upwind-schemes
(54 pages, 2002)

35. M. Günther, A. Klar, T. Materne, R. Wegener

Multivalued fundamental diagrams and stop and go waves for continuum traffic equations

Keywords: traffic flow, macroscopic equations, kinetic derivation, multivalued fundamental diagram, stop and go waves, phase transitions
(25 pages, 2002)

36. S. Feldmann, P. Lang, D. Prätzel-Wolters

Parameter influence on the zeros of network determinants

Keywords: Networks, Equicofactor matrix polynomials, Realization theory, Matrix perturbation theory
(30 pages, 2002)

37. K. Koch, J. Ohser, K. Schladitz

Spectral theory for random closed sets and estimating the covariance via frequency space

Keywords: Random set, Bartlett spectrum, fast Fourier transform, power spectrum
(28 pages, 2002)

38. D. d'Humières, I. Ginzburg

Multi-reflection boundary conditions for lattice Boltzmann models

Keywords: lattice Boltzmann equation, boudary condistions, bounce-back rule, Navier-Stokes equation
(72 pages, 2002)

39. R. Korn

Elementare Finanzmathematik

Keywords: Finanzmathematik, Aktien, Optionen, Portfolio-Optimierung, Börse, Lehrerweiterbildung, Mathematikunterricht
(98 pages, 2002)

40. J. Kallrath, M. C. Müller, S. Nickel

Batch Presorting Problems: Models and Complexity Results

Keywords: Complexity theory, Integer programming, Assigment, Logistics
(19 pages, 2002)

41. J. Linn

On the frame-invariant description of the phase space of the Folgar-Tucker equation

Key words: fiber orientation, Folgar-Tucker equation, injection molding
(5 pages, 2003)

42. T. Hanne, S. Nickel

A Multi-Objective Evolutionary Algorithm for Scheduling and Inspection Planning in Software Development Projects

Key words: multiple objective programming, project management and scheduling, software development, evolutionary algorithms, efficient set
(29 pages, 2003)

43. T. Bortfeld , K.-H. Küfer, M. Monz, A. Scherrer, C. Thieke, H. Trinkaus

Intensity-Modulated Radiotherapy - A Large Scale Multi-Criteria Programming Problem

Keywords: multiple criteria optimization, representative systems of Pareto solutions, adaptive triangulation, clustering and disaggregation techniques, visualization of Pareto solutions, medical physics, external beam radiotherapy planning, intensity modulated radiotherapy
(31 pages, 2003)

44. T. Halfmann, T. Wichmann

Overview of Symbolic Methods in Industrial Analog Circuit Design

Keywords: CAD, automated analog circuit design, symbolic analysis, computer algebra, behavioral modeling, system simulation, circuit sizing, macro modeling, differential-algebraic equations, index
(17 pages, 2003)

45. S. E. Mikhailov, J. Orlik

Asymptotic Homogenisation in Strength and Fatigue Durability Analysis of Composites

Keywords: multiscale structures, asymptotic homogenization, strength, fatigue, singularity, non-local conditions
(14 pages, 2003)

46. P. Domínguez-Marín, P. Hansen, N. Mladenovi ́c , S. Nickel

Heuristic Procedures for Solving the Discrete Ordered Median Problem

Keywords: genetic algorithms, variable neighborhood search, discrete facility location
(31 pages, 2003)

47. N. Boland, P. Domínguez-Marín, S. Nickel, J. Puerto

Exact Procedures for Solving the Discrete Ordered Median Problem

Keywords: discrete location, Integer programming
(41 pages, 2003)

48. S. Feldmann, P. Lang

Padé-like reduction of stable discrete linear systems preserving their stability

Keywords: Discrete linear systems, model reduction, stability, Hankel matrix, Stein equation
(16 pages, 2003)

49. J. Kallrath, S. Nickel

A Polynomial Case of the Batch Presorting Problem

Keywords: batch presorting problem, online optimization, competetive analysis, polynomial algorithms, logistics
(17 pages, 2003)

50. T. Hanne, H. L. Trinkaus

knowCube for MCDM – Visual and Interactive Support for Multicriteria Decision Making

Key words: Multicriteria decision making, knowledge management, decision support systems, visual interfaces, interactive navigation, real-life applications.
(26 pages, 2003)

51. O. Iliev, V. Laptev

On Numerical Simulation of Flow Through Oil Filters

Keywords: oil filters, coupled flow in plain and porous media, Navier-Stokes, Brinkman, numerical simulation
(8 pages, 2003)

52. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva

On a Multigrid Adaptive Refinement Solver for Saturated Non-Newtonian Flow in Porous Media

Keywords: Nonlinear multigrid, adaptive refinement, non-Newtonian flow in porous media
(17 pages, 2003)

53. S. Kruse

On the Pricing of Forward Starting Options under Stochastic Volatility

Keywords: Option pricing, forward starting options, Heston model, stochastic volatility, cliquet options
(11 pages, 2003)

54. O. Iliev, D. Stoyanov

Multigrid – adaptive local refinement solver for incompressible flows

Keywords: Navier-Stokes equations, incompressible flow, projection-type splitting, SIMPLE, multigrid methods, adaptive local refinement, lid-driven flow in a cavity
(37 pages, 2003)

55. V. Starikovicius

The multiphase flow and heat transfer in porous media

Keywords: Two-phase flow in porous media, various formulations, global pressure, multiphase mixture model, numerical simulation
(30 pages, 2003)

56. P. Lang, A. Sarishvili, A. Wirsen

Blocked neural networks for knowledge extraction in the software development process

Keywords: Blocked Neural Networks, Nonlinear Regression, Knowledge Extraction, Code Inspection
(21 pages, 2003)

57. H. Knaf, P. Lang, S. Zeiser

Diagnosis aiding in Regulation Thermography using Fuzzy Logic

Keywords: fuzzy logic,knowledge representation, expert system
(22 pages, 2003)

58. M. T. Melo, S. Nickel, F. Saldanha da Gama

Largescale models for dynamic multicommodity capacitated facility location

Keywords: supply chain management, strategic planning, dynamic location, modeling
(40 pages, 2003)

59. J. Orlik

Homogenization for contact problems with periodically rough surfaces

Keywords: asymptotic homogenization, contact problems
(28 pages, 2004)

60. A. Scherrer, K.-H. Küfer, M. Monz, F. Alonso, T. Bortfeld

IMRT planning on adaptive volume structures – a significant advance of computational complexity

Keywords: Intensity-modulated radiation therapy (IMRT), inverse treatment planning, adaptive volume structures, hierarchical clustering, local refinement, adaptive clustering, convex programming, mesh generation, multi-grid methods
(24 pages, 2004)

61. D. Kehrwald
Parallel lattice Boltzmann simulation of complex flows
Keywords: Lattice Boltzmann methods, parallel computing, microstructure simulation, virtual material design, pseudo-plastic fluids, liquid composite moulding
(12 pages, 2004)

62. O. Iliev, J. Linn, M. Moog, D. Niedziela, V. Starikovicius
On the Performance of Certain Iterative Solvers for Coupled Systems Arising in Discretization of Non-Newtonian Flow Equations
Keywords: Performance of iterative solvers, Preconditioners, Non-Newtonian flow
(17 pages, 2004)

63. R. Ciegis, O. Iliev, S. Rief, K. Steiner
On Modelling and Simulation of Different Regimes for Liquid Polymer Moulding
Keywords: Liquid Polymer Moulding, Modelling, Simulation, Infiltration, Front Propagation, non-Newtonian flow in porous media
(43 pages, 2004)

64. T. Hanne, H. Neu
Simulating Human Resources in Software Development Processes
Keywords: Human resource modeling, software process, productivity, human factors, learning curve
(14 pages, 2004)

65. O. Iliev, A. Mikelic, P. Popov
Fluid structure interaction problems in deformable porous media: Toward permeability of deformable porous media
Keywords: fluid-structure interaction, deformable porous media, upscaling, linear elasticity, stokes, finite elements
(28 pages, 2004)

66. F. Gaspar, O. Iliev, F. Lisbona, A. Naumovich, P. Vabishchevich
On numerical solution of 1-D poroelasticity equations in a multilayered domain
Keywords: poroelasticity, multilayered material, finite volume discretization, MAC type grid
(41 pages, 2004)

67. J. Ohser, K. Schladitz, K. Koch, M. Nöthe
Diffraction by image processing and its application in materials science
Keywords: porous microstructure, image analysis, random set, fast Fourier transform, power spectrum, Bartlett spectrum
(13 pages, 2004)

68. H. Neunzert
Mathematics as a Technology: Challenges for the next 10 Years
Keywords: applied mathematics, technology, modelling, simulation, visualization, optimization, glass processing, spinning processes, fiber-fluid interaction, trubulence effects, topological optimization, multicriteria optimization, Uncertainty and Risk, financial mathematics, Malliavin calculus, Monte-Carlo methods, virtual material design, filtration, bio-informatics, system biology
(29 pages, 2004)

69. R. Ewing, O. Iliev, R. Lazarov, A. Naumovich
On convergence of certain finite difference discretizations for 1D poroelasticity interface problems
Keywords: poroelasticity, multilayered material, finite volume discretizations, MAC type grid, error estimates
(26 pages,2004)

70. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva
On Efficient Simulation of Non-Newtonian Flow in Saturated Porous Media with a Multigrid Adaptive Refinement Solver
Keywords: Nonlinear multigrid, adaptive renement, non-Newtonian in porous media
(25 pages, 2004)

71. J. Kalcsics, S. Nickel, M. Schröder
Towards a Unified Territory Design Approach – Applications, Algorithms and GIS Integration
Keywords: territory desgin, political districting, sales territory alignment, optimization algorithms, Geographical Information Systems
(40 pages, 2005)

72. K. Schladitz, S. Peters, D. Reinel-Bitzer, A. Wiegmann, J. Ohser
Design of acoustic trim based on geometric modeling and flow simulation for non-woven
Keywords: random system of fibers, Poisson line process, flow resistivity, acoustic absorption, Lattice-Boltzmann method, non-woven
(21 pages, 2005)

73. V. Rutka, A. Wiegmann
Explicit Jump Immersed Interface Method for virtual material design of the effective elastic moduli of composite materials
Keywords: virtual material design, explicit jump immersed interface method, effective elastic moduli, composite materials
(22 pages, 2005)

74. T. Hanne
Eine Übersicht zum Scheduling von Baustellen
Keywords: Projektplanung, Scheduling, Bauplanung, Bauindustrie
(32 pages, 2005)

75. J. Linn
The Folgar-Tucker Model as a Differetial Algebraic System for Fiber Orientation Calculation
Keywords: fiber orientation, Folgar–Tucker model, invariants, algebraic constraints, phase space, trace stability
(15 pages, 2005)

76. M. Speckert, K. Dreßler, H. Mauch, A. Lion, G. J. Wierda
Simulation eines neuartigen Prüfsystems für Achserprobungen durch MKS-Modellierung einschließlich Regelung
Keywords: virtual test rig, suspension testing, multibody simulation, modeling hexapod test rig, optimization of test rig configuration
(20 pages, 2005)

77. K.-H. Küfer, M. Monz, A. Scherrer, P. Süss, F. Alonso, A.S.A. Sultan, Th. Bortfeld, D. Craft, Chr. Thieke
Multicriteria optimization in intensity modulated radiotherapy planning
Keywords: multicriteria optimization, extreme solutions, real-time decision making, adaptive approximation schemes, clustering methods, IMRT planning, reverse engineering
(51 pages, 2005)

78. S. Amstutz, H. Andrä
A new algorithm for topology optimization using a level-set method
Keywords: shape optimization, topology optimization, topological sensitivity, level-set
(22 pages, 2005)

79. N. Ettrich
Generation of surface elevation models for urban drainage simulation
Keywords: Flooding, simulation, urban elevation models, laser scanning
(22 pages, 2005)

80. H. Andrä, J. Linn, I. Matei, I. Shklyar, K. Steiner, E. Teichmann
OPTCAST – Entwicklung adäquater Strukturoptimierungsverfahren für Gießereien Technischer Bericht (KURZFASSUNG)
Keywords: Topologieoptimierung, Level-Set-Methode, Gießprozesssimulation, Gießtechnische Restriktionen, CAE-Kette zur Strukturoptimierung
(77 pages, 2005)

81. N. Marheineke, R. Wegener
Fiber Dynamics in Turbulent Flows Part I: General Modeling Framework
Keywords: fiber-fluid interaction; Cosserat rod; turbulence modeling; Kolmogorov's energy spectrum; double-velocity correlations; differentiable Gaussian fields
(20 pages, 2005)
Part II: Specific Taylor Drag
Keywords: flexible fibers; k-ε turbulence model; fiber-turbulence interaction scales; air drag; random Gaussian aerodynamic force; white noise; stochastic differential equations; ARMA process
(18 pages, 2005)

82. C. H. Lampert, O. Wirjadi
An Optimal Non-Orthogonal Separation of the Anisotropic Gaussian Convolution Filter
Keywords: Anisotropic Gaussian filter, linear filtering, orientation space, nD image processing, separable filters
(25 pages, 2005)

83. H. Andrä, D. Stoyanov
Error indicators in the parallel finite element solver for linear elasticity DDFEM
Keywords: linear elasticity, finite element method, hierarchical shape functions, domain decom-position, parallel implementation, a posteriori error estimates
(21 pages, 2006)

84. M. Schröder, I. Solchenbach
Optimization of Transfer Quality in Regional Public Transit
Keywords: public transit, transfer quality, quadratic assignment problem
(16 pages, 2006)

85. A. Naumovich, F. J. Gaspar
On a multigrid solver for the three-dimensional Biot poroelasticity system in multilayered domains
Keywords: poroelasticity, interface problem, multigrid, operator-dependent prolongation
(11 pages, 2006)

86. S. Panda, R. Wegener, N. Marheineke
Slender Body Theory for the Dynamics of Curved Viscous Fibers
Keywords: curved viscous fibers; fluid dynamics; Navier-Stokes equations; free boundary value problem; asymptotic expansions; slender body theory
(14 pages, 2006)

87. E. Ivanov, H. Andrä, A. Kudryavtsev
Domain Decomposition Approach for Automatic Parallel Generation of Tetrahedral Grids
Key words: Grid Generation, Unstructured Grid, Delaunay Triangulation, Parallel Programming, Domain Decomposition, Load Balancing
(18 pages, 2006)

88. S. Tiwari, S. Antonov, D. Hietel, J. Kuhnert, R. Wegener

A Meshfree Method for Simulations of Interactions between Fluids and Flexible Structures

Key words: Meshfree Method, FPM, Fluid Structure Interaction, Sheet of Paper, Dynamical Coupling
(16 pages, 2006)

89. R. Ciegis , O. Iliev, V. Starikovicius, K. Steiner

Numerical Algorithms for Solving Problems of Multiphase Flows in Porous Media

Keywords: nonlinear algorithms, finite-volume method, software tools, porous media, flows
(16 pages, 2006)

90. D. Niedziela, O. Iliev, A. Latz

On 3D Numerical Simulations of Viscoelastic Fluids

Keywords: non-Newtonian fluids, anisotropic viscosity, integral constitutive equation
(18 pages, 2006)

91. A. Winterfeld

Application of general semi-infinite Programming to Lapidary Cutting Problems

Keywords: large scale optimization, nonlinear programming, general semi-infinite optimization, design centering, clustering
(26 pages, 2006)

92. J. Orlik, A. Ostrovska

Space-Time Finite Element Approximation and Numerical Solution of Hereditary Linear Viscoelasticity Problems

Keywords: hereditary viscoelasticity; kern approximation by interpolation; space-time finite element approximation, stability and a priori estimate
(24 pages, 2006)

93. V. Rutka, A. Wiegmann, H. Andrä

EJIIM for Calculation of effective Elastic Moduli in 3D Linear Elasticity

Keywords: Elliptic PDE, linear elasticity, irregular domain, finite differences, fast solvers, effective elastic moduli
(24 pages, 2006)

94. A. Wiegmann, A. Zemitis

EJ-HEAT: A Fast Explicit Jump Harmonic Averaging Solver for the Effective Heat Conductivity of Composite Materials

Keywords: Stationary heat equation, effective thermal conductivity, explicit jump, discontinuous coefficients, virtual material design, microstructure simulation, EJ-HEAT
(21 pages, 2006)

95. A. Naumovich

On a finite volume discretization of the three-dimensional Biot poroelasticity system in multilayered domains

Keywords: Biot poroelasticity system, interface problems, finite volume discretization, finite difference method.
(21 pages, 2006)

96. M. Krekel, J. Wenzel

A unified approach to Credit Default Swaption and Constant Maturity Credit Default Swap valuation

Keywords: LIBOR market model, credit risk, Credit Default Swaption, Constant Maturity Credit Default Swap-method.
(43 pages, 2006)

97. A. Dreyer

Interval Methods for Analog Circiuts

Keywords: interval arithmetic, analog circuits, tolerance analysis, parametric linear systems, frequency response, symbolic analysis, CAD, computer algebra
(36 pages, 2006)

98. N. Weigel, S. Weihe, G. Bitsch, K. Dreßler

Usage of Simulation for Design and Optimization of Testing

Keywords: Vehicle test rigs, MBS, control, hydraulics, testing philosophy
(14 pages, 2006)

99. H. Lang, G. Bitsch, K. Dreßler, M. Speckert

Comparison of the solutions of the elastic and elastoplastic boundary value problems

Keywords: Elastic BVP, elastoplastic BVP, variational inequalities, rate-independency, hysteresis, linear kinematic hardening, stop- and play-operator
(21 pages, 2006)

100. M. Speckert, K. Dreßler, H. Mauch

MBS Simulation of a hexapod based suspension test rig

Keywords: Test rig, MBS simulation, suspension, hydraulics, controlling, design optimization
(12 pages, 2006)

101. S. Azizi Sultan, K.-H. Küfer

A dynamic algorithm for beam orientations in multicriteria IMRT planning

Keywords: radiotherapy planning, beam orientation optimization, dynamic approach, evolutionary algorithm, global optimization
(14 pages, 2006)

102. T. Götz, A. Klar, N. Marheineke, R. Wegener

A Stochastic Model for the Fiber Lay-down Process in the Nonwoven Production

Keywords: fiber dynamics, stochastic Hamiltonian system, stochastic averaging
(17 pages, 2006)

103. Ph. Süss, K.-H. Küfer

Balancing control and simplicity: a variable aggregation method in intensity modulated radiation therapy planning

Keywords: IMRT planning, variable aggregation, clustering methods
(22 pages, 2006)

104. A. Beaudry, G. Laporte, T. Melo, S. Nickel

Dynamic transportation of patients in hospitals

Keywords: in-house hospital transportation, dial-a-ride, dynamic mode, tabu search
(37 pages, 2006)

105. Th. Hanne

Applying multiobjective evolutionary algorithms in industrial projects

Keywords: multiobjective evolutionary algorithms, discrete optimization, continuous optimization, electronic circuit design, semi-infinite programming, scheduling
(18 pages, 2006)

106. J. Franke, S. Halim

Wild bootstrap tests for comparing signals and images

Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression
(13 pages, 2007

107. Z. Drezner, S. Nickel

Solving the ordered one-median problem in the plane

Keywords: planar location, global optimization, ordered median, big triangle small triangle method, bounds, numerical experiments
(21 pages, 2007)

108. Th. Götz, A. Klar, A. Unterreiter, R. Wegener

Numerical evidance for the non-existing of solutions of the equations desribing rotational fiber spinning

Keywords: rotational fiber spinning, viscous fibers, boundary value problem, existence of solutions
(11 pages, 2007)

109. Ph. Süss, K.-H. Küfer

Smooth intensity maps and the Bortfeld-Boyer sequencer

Keywords: probabilistic analysis, intensity modulated radiotherapy treatment (IMRT), IMRT plan application, step-and-shoot sequencing
(8 pages, 2007)

110. E. Ivanov, O. Gluchshenko, H. Andrä, A. Kudryavtsev

Parallel software tool for decomposing and meshing of 3d structures

Keywords: a-priori domain decomposition, unstructured grid, Delaunay mesh generation
(14 pages, 2007)

111. O. Iliev, R. Lazarov, J. Willems

Numerical study of two-grid preconditioners for 1d elliptic problems with highly oscillating discontinuous coefficients

Keywords: two-grid algorithm, oscillating coefficients, preconditioner
(20 pages, 2007)

112. L. Bonilla, T. Götz, A. Klar, N. Marheineke, R. Wegener

Hydrodynamic limit of the Fokker-Planck-equation describing fiber lay-down processes

Keywords: stochastic dierential equations, Fokker-Planck equation, asymptotic expansion, Ornstein-Uhlenbeck process
(17 pages, 2007)

113. S. Rief

Modeling and simulation of the pressing section of a paper machine

Keywords: paper machine, computational fluid dynamics, porous media
(41 pages, 2007)

114. R. Ciegis, O. Iliev, Z. Lakdawala

On parallel numerical algorithms for simulating industrial filtration problems

Keywords: Navier-Stokes-Brinkmann equations, finite volume discretization method, SIMPLE, parallel computing, data decomposition method
(24 pages, 2007)

115. N. Marheineke, R. Wegener

Dynamics of curved viscous fibers with surface tension

Keywords: Slender body theory, curved viscous bers with surface tension, free boundary value problem
(25 pages, 2007)

116. S. Feth, J. Franke, M. Speckert

Resampling-Methoden zur mse-Korrektur und Anwendungen in der Betriebsfestigkeit
Keywords: Weibull, Bootstrap, Maximum-Likelihood, Betriebsfestigkeit
(16 pages, 2007)

117. H. Knaf

Kernel Fisher discriminant functions – a concise and rigorous introduction
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression
(30 pages, 2007)

118. O. Iliev, I. Rybak

On numerical upscaling for flows in heterogeneous porous media
Keywords: numerical upscaling, heterogeneous porous media, single phase flow, Darcy's law, multiscale problem, effective permeability, multipoint flux approximation, anisotropy
(17 pages, 2007)

119. O. Iliev, I. Rybak

On approximation property of multipoint flux approximation method
Keywords: Multipoint flux approximation, finite volume method, elliptic equation, discontinuous tensor coefficients, anisotropy
(15 pages, 2007)

120. O. Iliev, I. Rybak, J. Willems

On upscaling heat conductivity for a class of industrial problems
Keywords: Multiscale problems, effective heat conductivity, numerical upscaling, domain decomposition
(21 pages, 2007)

Status quo: July 2007