Fraunhofer Institut Techno- und Wirtschaftsmathematik

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Mathematical Modelling of Evacuation Problems: A State of Art

Berichte des Fraunhofer ITWM, Nr. 24 (2001)

ISSN 1434-9973

Bericht 24 (2001)

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Vorwort

Das Tätigkeitsfeld des Fraunhofer Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüberhinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation darüber, wie aktuelle Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte transferiert werden, und wie umgekehrt Probleme der Praxis neue interessante mathematische Fragestellungen generieren.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

MATHEMATICAL MODELLING OF EVACUATION PROBLEMS : A STATE OF THE ART

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Abstract

This paper details models and algorithms which can be applied to evacuation problems. While it concentrates on building evacuation many of the results are applicable also to regional evacuation. All models consider the time as main parameter, where the travel time between components of the building is part of the input and the overall evacuation time is the output. The paper distinguishes between macroscopic and microscopic evacuation models both of which are able to capture the evacuees' movement over time.

Macroscopic models are mainly used to produce good lower bounds for the evacuation time and do not consider any individual behavior during the emergency situation. These bounds can be used to analyze existing buildings or help in the design phase of planning a building. Macroscopic approaches which are based on dynamic network flow models (minimum cost dynamic flow, maximum dynamic flow, universal maximum flow, quickest path and quickest flow) are described. A special feature of the presented approach is the fact, that travel times of evacuees are not restricted to be constant, but may be density dependent. Using multicriteria optimization priority regions and blockage due to fire or smoke may be considered. It is shown how the modelling can be done using time parameter either as discrete or continuous parameter.

Microscopic models are able to model the individual evacuee's characteristics and the interaction among evacuees which influence their movement. Due to the corresponding huge amount of data one uses simulation approaches. Some probabilistic laws for individual evacuee's movement are presented. Moreover ideas to model the evacuee's movement using cellular automata (CA) and resulting software are presented.

In this paper we will focuss on macroscopic models and only summarize some of the results of the microscopic approach. While most of the results are applicable to general evacuation situations, we concentrate on building evacuation.

1 Assumptions and Goals of Evacuation Models

Evacuation as one aspect of emergency processes can be simply defined as the removal of residents from a given area that has been considered as a danger zone to safety as quickly as possible and with utmost reliability. Two different evacuation scenarios can be considered.

(i) Precautionary
In this type of evacuation the estimation of the evacuation time compared to the hazard propagation time and the estimation of the risk can be done a priori. Hence, time and potential risks are the key components of this type of evacuation.

(ii) Life-saving operations
This type of evacuation occurs when insufficient warning has prevented the organizer from conducting a pre-emergency evacuation planning. Here it is more likely that problems as the rescue of injured evacuees in and around the damaged area, route clearance, etc. have to be dealt with.

Evacuation problems may arise in different types of systems, such as buildings, cities or regions, or transportation carriers (e.g. train, ship and airplane). The system structures (e.g. population and the behavior of people at risk, hazard propagation speed and charateristics) essentially influence the optimal planning in the corresponding system. In general, the following is a list of informations which would be ideal to have in evacuation planning (and needs to be estimated somehow, if not available) :

- Type of system defined by layout/geographic information and familiarity, for example : office building, shopping mal and airport
- Behaviour estimation of the occupants under panic situation.
- Occupants distribution (includes age, gender and disability).
- Source and location of hazard, hazard propagation speed/charateristics and factors affecting the hazard propagation.
- Safe destinations (refuge places).
- Availability of emergency service facilities and personel.

These informations are used to define the *dynamic severity matrix* $[\mathbf{r_{it}}]$ where vector $\mathbf{r_{it}}$ states the estimated severity or hazard level of area i at time t. The number of components of vector $\mathbf{r_{it}}$ is determined by the number of informations which are considered. Equation (1) shows an example of dynamic severity matrix.

$$[\mathbf{r_{it}}] = \begin{pmatrix} & \vdots & \\ \cdots \quad \mathbf{r_{it}} = & \begin{pmatrix} \text{fire level of area } i \text{ at time } t \\ \text{temperature level of area } i \text{ at time } t \\ \text{smoke density level of area } i \text{ at time } t \\ \text{toxicity level of area } i \text{ at time } t \end{pmatrix} & \cdots \\ & \vdots & \end{pmatrix} \tag{1}$$

By giving appropriate weights to vector components, vector $\mathbf{r_{it}}$ can be converted to a scalar r_{it}. If h is the number of vector components and w_l is the weight of component l, then r_{it} can be scalarized as follows.

$$r_{it} = \sum_{l=1}^{h} w_l r_{itl}$$

This matrix will be used to determine the evacuation priorities that will be discussed in more detail in Section 8.

The *evacuation time*, that is the time needed to complete an evacuation process, basically consists of three main time components ([30], [47]) namely :

- The time evacuees need to recognize a dangerous situation. This time is influenced mainly by the reliability of the alarm system and the familiarity of evacuees with emergency signals.

- The time evacuees need to decide which course of action to take. This time is influenced by the experience of evacuees in facing the emergency situation. This can, for instance, be generated through emergency practice and training.

Behavioural and organizational factors are the main contributors to the duration of these times. The subsequent decisions are made during the movement to the safety, especially when evacuees encounter a "hazard" route, i.e. one which is affected by fire, smoke, etc.

- The time evacuees need to move towards the safety area, which is known as *egress time*. The latter is influenced by the availability of emergency exit signs, well planned evacuation procedures, constructional factors (effective width of walkway, slope of stairs), and human behaviour during panic situations.

Since the behavioural and organizational factors are the main contributors to the first two time components, it is hard to predict analytically the duration of those time components. Therefore, most evacuation models emphasize the calculation of egress time and treat the result as the lower bound of the real evacuation time. This will also be the approach in this paper.

The next section describes the idea of macro- and microscopic models and give a list of classes of existing models which can be used in the evacuation problems. Informations on various specific mathematical models are given in Section 3 - 6, like dynamic flows (Section 3), maximum dynamic flows (Section 4), universal maximum flows (Section 5), quickest path and flows (Section 6). In Section 7 we show the interrelation between these models by establishing a triple optimization result. Section 8 deals with multiple criteria. The fact that the travel time may be dependend on the evacuation density is considered in Section 9. A stronger model, but sounds more difficult to solve is the continuous time dynamic network flow model which is the subject of Section 10. Before concluding the paper we discuss microscopic models, in particular simulation approaches, in Section 11.

2 Macro- and Microscopic Models

Approaches which are used to model evacuation problems may come from different problem fields, such as, network flow problems, traffic assignment problems, and simulation. Table 2 lists classes of models which can be used in the evacuation problem modeling although some of them may have been originally developed for different purposes. In general, there are two approaches used to model evacuation problems which emphasize on the estimation of the egress time, namely *macro-* and *microscopic models* .

Macroscopic models are mainly based on optimization approaches and do not consider individual differences and decisions for selecting egress routes, i.e. occupants are treated as a homogeneous group where only common characteristics are taken into account. Since the time is a decisive parameter in the evacuation process, most macroscopic approaches are based on dynamic network flow models (see e.g. [14], [18], [19], [41], [22], [12], [43], [50]). The common idea of these models is to represent a building and the attributes of the building's components in a static network G. The modeling of evacuation over time is then done in a dynamic network G_T which is the time expanded version of G and the network flows correspond to evacuation processes (see, for instance, [1] for an overview on network flow theory).

The static network G is used to model supply and demand points, and routes which are used to transfer supplies to demands. These routes may have some intermediate transshipment points. In the static network flow models, supply, demand and transshipment points are modeled by nodes while routes are modeled by paths of the graph. A path of the graph is composed by nodes and arcs, where an arc connects two adjacent nodes. The interrelation between nodes and arcs can, for instance, be described by the *node-arc incidence matrix.* In the representation of a building using a static network, nodes may represent rooms, lobbies or intersection points, while arcs can be used to model corridors, hallways, stairways or a connection between two intersection nodes. Some locations in the building that house a significant number of evacuees are considered to be source nodes in the network. The supply of a source node is given by an estimate of the number of evacuees in the location that the node represents. The building exits or safety locations that might be considered as the final destination of evacuees' movement, are considered as sink nodes. In the evacuation problem we have only one sink node by connecting all the exit nodes to one artificial node and assign the total number of evacuees as the demand value of this node. Hence, evacuation problems can be modeled as multi-source/single sink network flow problem. Each node has a capacity which is the upper bound of the number of evacuees simultaneously allowed to stay in the node. This node capacity can be determined, for instance, by

$$\text{node capacity} := \min \Big\{ \frac{\text{floor space area}}{\text{minimum required area per person}}, \frac{\text{maximum allowable weight}}{\text{average weight per person}} \Big\}$$

Arcs have other attributes, such as flow capacity and travel time. The arc flow capacity is the upper bound of the number of evacuees per unit time that can traverse the arc. The travel time is the time needed to travel from one node to another. This travel time is one of the important components that must be considered in the modeling of evacuation problems. During the evacuation process, the connection between two positions may only be temporary due to, for instance, blocking by fire or smoke. In this case, the arc that represents the connection must also be temporary, i.e. the arc capacity can be set to zero after some times. These time constraints can not be properly modeled by the static network flow models but the dynamic ones. Moreover, we can formulate the dynamic network flow problem in two ways depending on whether we use a discrete or continuous representation of time.

In the area of dynamic network flow problems, some of the existing models assume constant attributes, e.g constant travel time from one node to another and constant arc flow capacity. The constant travel time might be determined according to some predetermined queuing levels such that the model can be solved efficiently but still able to give quite realistic results. In this paper it will be shown that the models may be extended, thus providing a better estimate of the final evacuation time.

Microscopic models, in which the individual evacuees' movement is emphasized, are based on simulation. These models consider individual parameters (e.g. walking speed, reaction time, physical ability) and interaction of each evacuee with other evacuees during the movement. In recent years there is a growing interest to use cellular automata as the base of microscopic simulation in the field of pedestrians and traffic movement (see for example [8], [42], [51]) which have close interrelations with evacuation problems.

Model Class	Evacuation Model	References
Static Network	Shortest path	[22], [68]
	Minimum cost network flow	[68]
	Quickest path	[15], [16], [38], [61]
Discrete Time Dynamic network	Minimum turnstile cost	[14], [18], [41], [50]
	Quickest Flow	[12], [23]
	Universally maximum flow	[34], [48], [67]
	Minimum weight path (multi objectives)	[43]
	Lexicographically minimal cost	[33]
	Flow dependent exit capacity	[18], [19]
Continuous Time Dynamic Network	Constant capacity and travel time	[24]
	Time dependent capacity (maximal flow)	[2], [55]
	Universally maximum flow with zero travel time	[25], [52]
Traffic assignment	Transportation network	[62], [69]
	Density dependent travel time (single objective)	[13], [17], [35], [37], [39], [60]
Simulation	Probabilistic models	[21], [46], [47]
	Cellular Automata	[8], [10], [20], [42], [51]

Table 1: Summary of Existing Evacuation Approaches.

3 Discrete Time Dynamic Network Flow Model

A discrete time dynamic network flow problem is a discrete time expansion of a static network flow problem. In this case we distribute the flow over a set of predetermined time periods $t = 1, 2, \ldots, T$.

Definition 3.1 *Let $G = (N, A)$ be a directed network with N the set of nodes and A the set of arcs (the static network). On each arc $(i, j) \in A$ travel times λ_{ij} are given which are assumed to be constant. The time expansion of G over a time horizon T defines the dynamic network $G_T = (N_T, A_T)$ associated with G where*

$$N_T := \{i(t) \ \mid i \in N \ ; \ t = 0, 1, \ldots, T\}$$

and A_T consists of the set of movement arcs A_M

$$A_M := \{(i(t), j(t') \ \mid (i, j) \in A \ ; \ t' = t + \lambda_{ij} \leq T \ ; \ t = 0, 1, \ldots, T\}$$

and the set of holdover arcs A_H

$$A_H := \{(i(t), i(t+1)) \ \mid i \in N \ ; \ t = 0, 1, \ldots, T-1\}$$

i.e.

$$A_T := A_M \cup A_H$$

Figure 2 shows a T-time expansion of the static network of Figure 1, with $T = 4$.

The time period t is dependent on the basic unit θ in which travel times are measured. Thus, if we choose 5 seconds as the length of the basic unit (i.e. $\theta = 5$), then specifying three time periods (i.e. $t = 3$) for traversing an arc means we need fifteen seconds to do so. The number of time periods T is obtained by dividing the evacuation planning horizon of interest by the length θ of the basic unit. The smaller θ the more acurately the model represents the actual flow's evolution. Choosing θ too small, however, will result in undesirable size of the network and may have fractional arc capacities which make the problem difficult to solve. Hence, the choice of θ is a compromise between model realism and model complexity.

Since the dynamic network has $(T + 1)$ copies of each source node and each sink node, the dynamic network will have multiple sources and multiple sinks. Therefore in order to handle many sources and sinks, one introduces a *super source* s and a *super sink* d to create a single source/single sink network (see Figures 2 and 3). In evacuation problems, the super sink can be interpreted as a common safety area. How the super source is connected to the source is actually problem-dependent. In the network clearing problem (clearing the network from initial occupancies), the super source is connected only to the time zero copy of the source nodes (see Figure 2). In this case, we may have holdover arcs for source nodes. Arcs from the super source have zero travel time and capacities are equal to initial occupancies. In the maximum dynamic flow problem (see Section 4), the super source is connected to all time-copies of the source nodes. In this case, we do not have holdover arcs for source nodes which do not have predecessors (e.g. node 1 in Figure 1) as shown in Figure 3. Arcs from the super source to other nodes have zero travel time and infinite capacities. On the other hand, generally all copies of every sink node are connected to the super sink and there is no holdover arc for sink nodes. All conections to the super sink have zero travel time and infinite flow capacities.

By constructing the dynamic network as defined above, dynamic network flow problems can always be solved as static flow problems in the expanded network. Also, it may be noted that the equivalent static problem does not require keeping arc capacities and travel times fixed over time, as assumed in Definition 3.1. But these assumptions are essential for building efficient algorithms to solve the problem. The upper bound for the number of nodes and arcs in discrete time dynamic network is described by Proposition 3.1.

Proposition 3.1 *If $n :=\mid N \mid$ and $m :=\mid A \mid$ then $n(T+1)$ and $(n+m)T + m - \sum_{(i,j)\in A} \lambda_{ij}$ are the upper bound for the number of nodes and arcs in G_T without considering super source and super sink, respectively.*

Since we do not use any arc in the path from the super source to any sink node at time greater than T, we can reduce the size of the time-expanded network by eliminating inessential arcs including the corresponding nodes (see Figure 3).

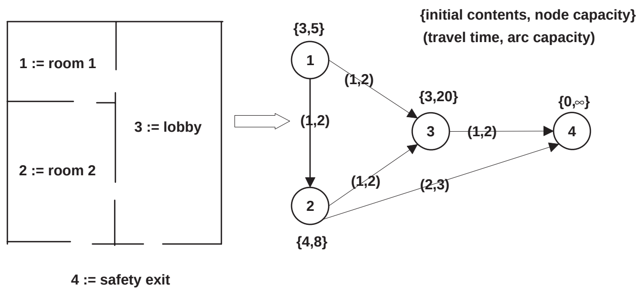

Figure 1: Static Network G of a Simple Building Layout.

In the dynamic network flow models, we denote by $x_{ij}(t)$ the flow (e.g. the number of evacuees moving at time t) that leave node i at time t and reach node j at time $t+\lambda_{ij}$. Flows from node i at time t to the same node with travel time $\lambda_{ii} = 1$ represent the number of evacuees who prefer to stay in the building component represented by node i at time t for at least one unit time. This flow is denoted by $y_i(t+1)$, i.e.

$$y_i(t+1) := x_{i(t),i(t+1)}$$

The capacity of movement arcs $(i(t), j(t+\lambda_{ij})) \in A_M$ is denoted by $b_{ij}(t)$ where we assume without loss of generality that

$$b_{ij}(t) := \min\{b_{ij}(t') : t' = t, t+1, \ldots, t+\lambda_{ij}\}$$

The capacity of a holdover arc $(i(t), i(t+1)) \in A_H$ is determined by the node capacity $a_i(t)$, and represents how many evacuees can stay in the node at a given time. With $\phi(X, Y)$ as the general objective and with q_i as the initial number of evacuees in any node $i \in N$, the discrete-time dynamic network flow

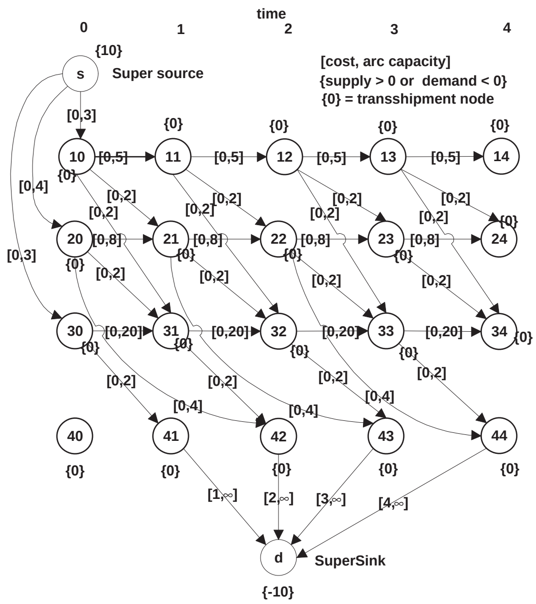

Figure 2: Dynamic Network G_T of the Static Network G of Figure 1, with $T = 4$.

model for evacuation processes can be formulated as follows.

$$\min/\max \qquad \sum_{t=0}^{T} \phi(X,Y) \tag{2}$$

$$y_i(t+1) - y_i(t) = \sum_{k \in pred(i)} x_{ki}(t - \lambda_{ki}) - \sum_{j \in succ(i)} x_{ij}(t), \quad t = 0, \ldots, T;\ \forall i \in N; \tag{3}$$

$$y_i(0) = q_i, \forall i \in N; \tag{4}$$

$$0 \leq y_i(t) \leq a_i(t), t = 1, \ldots, T-1; \forall i \in N; \tag{5}$$

$$0 \leq x_{ij}(t) \leq b_{ij}(t), t = 0, \ldots, T - \lambda_{ij}; \forall (i,j) \in A \tag{6}$$

where

$$pred(i) := \{j \mid \ (j,i) \in A\} \ \ ; \ succ(i) := \{j \mid \ (i,j) \in A\}$$

are the nodes which are predecessors and successors of node i, respectively.

In order to measure the time when evacuees reach their final destinations, so-called *turnstile cost* ([14], [33]) is defined on each arc as follows.

Definition 3.2 *If D is the set of sink nodes of the static network G and d is the super sink node of the associated dynamic network G_T, the (turnstile) cost of any arc $(i(t), j(t' = t + \lambda_{ij})) \in A_T$ is defined different from 0 if and only if $i \in D$ and $j(t') = d$. In this case $c(i(t), d) = t$.*

Let us denote $S \subset N$ as the set of source nodes of the static network G. Using the turnstile cost we can define the objective function $\phi(X,Y)$ to model the

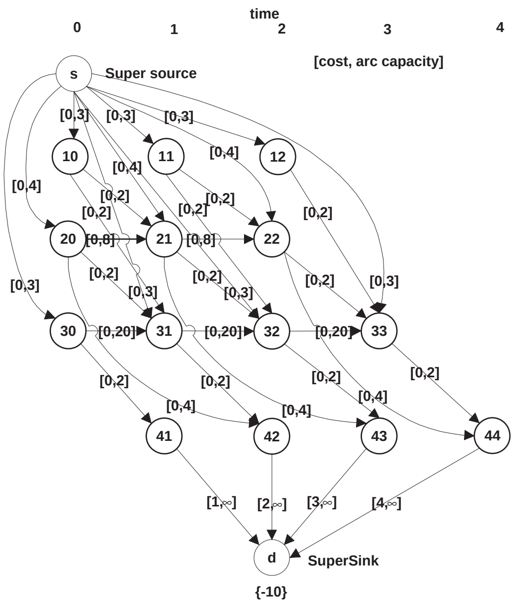

Figure 3: Dynamic Network G_T of the Static Network G of Figure 1, with $T = 4$, without Initial Contents, and by Deleting Inessential Arcs.

average evacuation time required by an evacuee to leave the network, i.e.

$$\phi(X, Y) := \frac{\sum_{t=0}^{T} \sum_{i \in D} t x_{id}(t)}{\sum_{i \in S} q_i}$$

Since the denominator is constant and ϕ depends only on the flow variables, one just needs to define the objective function ϕ as

$$\phi(X, Y) := \phi(X) = \sum_{t=0}^{T} \sum_{i \in D} t x_{id}(t)$$

The initial occupancies are modeled by using flow from the super-source s to each source node. Under assumptions of constant capacity (i.e., $b_{ij}(t) = b_{ij}, \forall (i,j) \in A$ and $a_i(t) = a_i, \forall i \in N; \forall t$) and constant travel time, the evacuation model that minimizes the average evacuation time can thus be formulated as follows.

$$\min \quad \sum_{t=0}^{T} \sum_{i \in D} t x_{id}(t) \tag{7}$$

$$x_{si}(0) = q_i, \forall i \in S, \tag{8}$$

$$\sum_{t=0}^{T} \sum_{i \in D} x_{id}(t) = \sum_{j \in S} q_j, \tag{9}$$

$$y_i(t+1) - y_i(t) = \sum_{k \in pred(i)} x_{ki}(t - \lambda_{ki}) - \sum_{j \in succ(i)} x_{ij}(t), \quad t = 0, \ldots, T; \ \forall i \in N \tag{10}$$

$$y_i(0) = 0, \forall i \in N, \tag{11}$$

$$y_i(t) = 0, \forall i \in D; t = 0, \ldots, T \tag{12}$$

$$0 \leq y_i(t) \leq a_i, t = 1, \ldots, T; i \in N - D \tag{13}$$

$$0 \leq x_{ij}(t) \leq b_{ij}, t = 0, \ldots, T - \lambda_{ij}; \forall (ij) \in A \tag{14}$$

We can treat the time-expanded network as defined in the Definition 3.1 as a static network and then apply any minimum cost static network flow algorithm (see, e.g., [1]) to obtain the solution.

Based on this minimum cost dynamic network optimization, Kisko and Francis [41] developed EVACNET+, an evacuation software which can be used to determine the egress time and possible bottleneck locations. The minimum cost dynamic network optimization problem is solved as a static network by using the NETFLO code [40].

In public buildings where the number of evacuees is difficult to estimate, one can model the evacuation problem as a maximum dynamic network flow problem. In the next section we will give a general description of the maximum dynamic network flow problems and the algorithms to solve them.

4 Maximum Dynamic Flows Problem

Given the time horizon T, maximum dynamic flow problems (MDF) maximize the dynamic flows reaching the sink. These problems can be used to model evacuation processes which have no reliable information about the number of evacuees. As already mentioned in Section 3, the super source node in G_T is connected to every time-copy of every source node and there is no holdover arc for source nodes and sink nodes. Arcs from super-source have zero travel time and infinite capacities.

The objective function ϕ of MDF is defined as follows.

$$\phi(X) := \sum_{t=0}^{t=T} \sum_{i \in D} x_{id}(t)$$

Using this definition, MDF with constant arc capacity and non-negative integral travel time can be formulated as follows.

$$
\begin{aligned}
\max \quad & \sum_{t=0}^{t=T} \sum_{i \in D} x_{id}(t) && (15)\\
y_i(t+1) - y_i(t) &= \sum_{k \in pred(i)} x_{ki}(t - \lambda_{ki}) - \sum_{j \in succ(i)} x_{ij}(t), \\
& \quad t = 0, \ldots, T; \forall i \in N && (16)\\
y_i(0) &= 0,\ \forall i \in N, && (17)\\
y_i(t) &= 0,\ \forall i \in S \cup D; t = 1, \ldots, T && (18)\\
0 \le y_i(t) &\le a_i, \\
& \quad t = 1, \ldots, T; \forall i \in N - S \cup D; && (19)\\
0 \le x_{ij}(t) &\le b_{ij},\ t = 0, \ldots, T - \lambda_{ij}; \forall (i,j) \in A && (20)
\end{aligned}
$$

The solution of MDF defined by Eq. (15) - (20) can be obtained by repeating the feasible flows along some chains (see Definition 4.1) of the static network from the source to the sink. The flows on these static chains are repeated in the dynamic network for every time period within the time horizon T. This approach is called *temporally repeated flow technique* (see Definition 4.2).

Definition 4.1 (Chain, chain flow and chain decomposition)

- *A chain is a sequence of nodes* $P = \{i_1, i_2, \ldots, i_k\}$, $k \geq 2$, *such that* $(i_j, i_{j+1}) \in A$ *and* $i_j \neq i_{j'}$ *when* $j \neq j'$, *for* $j, j' = 1, \ldots, k-1$, *i.e. a chain has no repeated nodes.*

- *A chain flow* $\gamma = <\mid P \mid, P>$ *is a static flow of value* $\mid P \mid$ *along the chain* P.

- *Let* $\Gamma = \{P_1, P_2, \ldots, P_l\}$ *be a set of chain flows and let* $\mid P_i \mid$ *be the chain flow along path* P_i. Γ *is a chain decomposition of the static flow* f *if* $\sum_{i=1}^{i=l} \mid P_i \mid = f$.

It is well-known that any network flow can be decomposed into chain flows (plus possibly some flows on cycles).

Definition 4.2 (Temporally repeated flows) *Let* $\gamma = <\mid P \mid, P>$ *be a chain flow. The temporally repeated flow* γ^T *is a dynamic flow obtained by repeating* $(T + 1 - \lambda(P))$ *times the chain flow* γ *, i.e., by sending* $\mid P \mid$ *units of flow every time period from time zero to time* $T + 1 - \lambda(P)$ *along the same path (static)* P.

The next theorem shows that the maximum dynamic flow problem can be solved as a minimum cost flow problem (MCFP) in the static network. The reader may refer books on network flow theory for more details on MCFP (see for instance [1]).

Proposition 4.1 ([26]) *Finding a maximum dynamic flow is equivalent to solving a MCFP. In particular, the temporally repeated flow obtained from the chain decomposition of any min cost flow is a maximum dynamic flow.*

In order to solve the maximum dynamic flow problem, and thus find the maximal number of persons, which can be evacuated within T time periods from a given building, we only have to solve a min cost flow problem in the small, static network G.

Algorithm 4.1

step 1 *Apply a minimum cost flow algorithm to the original static network G. Let x^* be an optimal solution.*

step 2 *Decompose x^* into k chain flows on $P_1, P_2, \ldots, P_l$ such that*

$$x^* = \sum_{i=1}^{i=l} \mid P_i \mid$$

step 3 *Repeat each chain flow P_i from time 0 till time $T - \lambda(P_i)$.*

Example 4.1
Figure 4 shows a static network of a simple building layout with travel time and capacity parameters attached on every arc. Node 1 and 6 are source and sink nodes, respectively. It is desired to calculate the maximum number of evacuees who can reach the safety during $T = 7$ time units. The optimal solution of MCFP is obtained as shown in table 4.1 (only the positive flows).

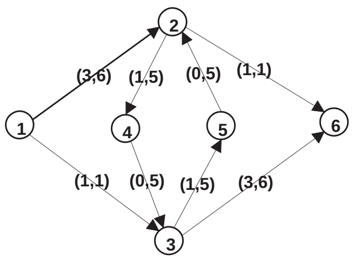

Figure 4: Static Network G for Example 4.1.

Table 4.1 : Optimal Maximum Flows for Static Network in Example 4.1.

Arc	(1,2)	(1,3)	(2,4)	(2,6)	(3,6)	(4,3)
Flow (f)	6	1	5	1	6	5

Step 2 of the algorithm gives the following chain flows :

- $P_1 = (1, 2, 6), \lambda(P_1) = 4, \mid P_1 \mid = 1$; P_1 must be repeated four times for $t = 0, 1, 2, 3$.
- $P_2 = (1, 2, 4, 3, 6), \lambda(P_2) = 7, \mid P_2 \mid = 5$; P_2 must be repeated only once at time $t = 0$.
- $P_3 = (1, 3, 6), \lambda(P_3) = 4, \mid P_3 \mid = 1$; P_3 must be repeated four times for time $t = 0, 1, 2, 3$.

The dynamic flows are shown in Figure 5. The total dynamic flow is equal to 13 unit of flow in which 2 units arrive at the sink at time $t = 4, 5, 6$ and 7 units at time $t = 7$. It means that one needs at least 7 time units to evacuate 13 evacuees who are at the location represented by node 1 in the beginning of the evacuation process.

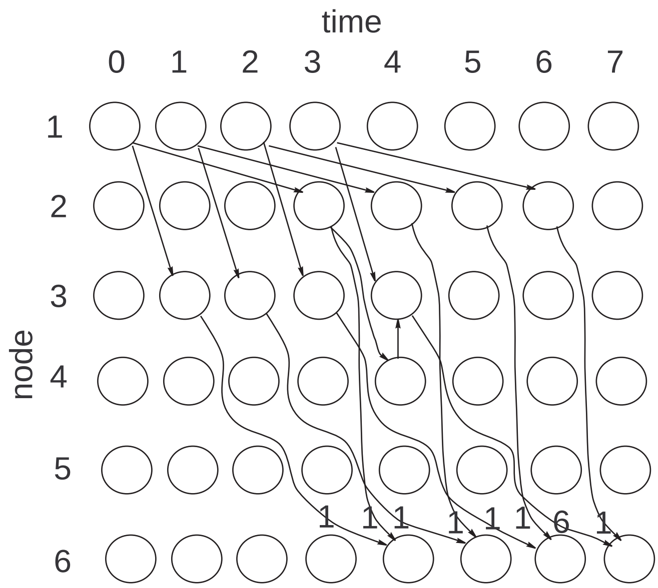

Figure 5: A maximum Dynamic Flow for Example 4.1.

The solution of the maximum dynamic flow in this example does not use any holdover arcs, i.e. $x_{ii}(t) = y_i(t+1) = 0, \forall i \in N; \forall t = 0, \ldots, T-1$. In fact, it can be shown, that the maximum dynamic flow poblem with constant capacities and travel times never requires hold-over at any nodes [34]. Therefore, variables $y_i(t)$ can be eliminated from the problem formulation.

5 Universal Maximum Flow Problem

The *(discrete) Universal maximum flow problem* (UMF) was introduced by Gale [28] as a variant of the maximum dynamic flow problem. UMF is the problem of finding maximum dynamic flows reaching the sink at every time period $t = 1, \ldots, T$. Hence, the optimal solution of UMF is the solution of the maximum dynamic flow problem, not only for the alloted time horizon T, but also for any smaller time horizons. Such a flow is also known in the literature as *earliest arrival dynamic flow* (see Hoppe and Tardos [34]). It can be formulated as

$$\begin{aligned} \max \quad & \sum_{t=0}^{t=T'} \sum_{i \in D} x_{id}(t), \forall T' = 1, \ldots, T \qquad (21) \\ \text{subject to :} \quad & (16) - (20) \end{aligned}$$

Table 2: Maximum Dynamic Flows (1-st row) vs Universal Maximum Flows (2-nd row).

Time Arc	0	1	2	3	4	5	6	7
(1,2)	6	1	1	1				
	6	1	1					
(1,3)	1	1	1	1				
	1	1	1	1	1			
(2,4)				5				
				5				
(2,6)				1	1	1	1	
			1	1	1	1	1	
(3,6)		1	1	1	6			
			1	1	6			
(3,5)								
		1				1		
(4,3)					5			
					5			
(5,2)								
			1				1	
arrival at 6					2	2	2	7
				1	1	2	2	7

The relevance of UMF for the evacuation problem is obvious. In every time period the maximal number of evacuees is brought to safety, such that an evacuation modeled by universal maximum flows is a very safe one.

By definition, every universal maximum flow is a maximum dynamic flow, but the reverse is not true as is illustrated by the flow distribution in Table 2 using the data of Example 4.1. In Table 2 the first line of each arc flow shows the optimal solution of maximum dynamic flow problem and the second line shows the one of UMF.

The following algorithm finds a universal maximum flow in the case of static networks with single source and single sink.

Algorithm 5.1 ([36])

step 0 : *Identify K, the value of the maximum dynamic flow with respect to the time horizon T, and set K as the total capacity of arcs from super source s to time-copies of source node, i.e.*

$$K = \sum_{t=0}^{T-\lambda} b_{si(t)}$$

with $i(t)$ the t-th time-copy of source node i of the static network and λ the shortest possible time to reach the sink node from the source node. Set all flows to zero and set time counter t to 1.

step 1 : *Maximize the flow from s to d via arc* $(i(t), d) \in G_T$ *by seting temporarily the capacity of arcs* $(i(t'), d), i \in D; t' > t$ *to zero (i.e., close temporarily those arcs) using the Ford-Fulkerson labeling algorithm([26]).*

step 2 : *If* $t = T$ *stop. Otherwise, increase* t *by 1, open arc* $(i(t), d)$ *and go to step 1.*

The validity of the algorithm has been proved in Minieka [48]. Note that this algorithm as well as alternative algorithms proposed by Minieka [48] and Wilkinson [67] are non-polynomial (both authors use shortest augmenting path algorithm). Moreover, they all work with the time expanded network, the size of which is dependend on the time horizon T.

Hoppe and Tardos [34] proposed a polynomial approximation algorithm for the discrete UMF by introducing a generalization of the chain decomposition introduced in the previous section, the *non-standard chain decomposition*. It is a chain decomposition of the static flow which can use arcs in the opposite direction of the static flow, i.e. it is allowed to send flows in the direction of arc (j, i) where $(i, j) \in A$ and $(j, i) \notin A$. In this case the travel time along (j, i) is the negative value of the one on $(i, j) \in A$. By using the non-standard chain decomposition, one can again use the idea of temporally repeated flows to produce UMF. Figure 6 shows dynamic flows induced by the non standard chain decomposition. Consider the static network of Example 4.1. Let $\Gamma =$

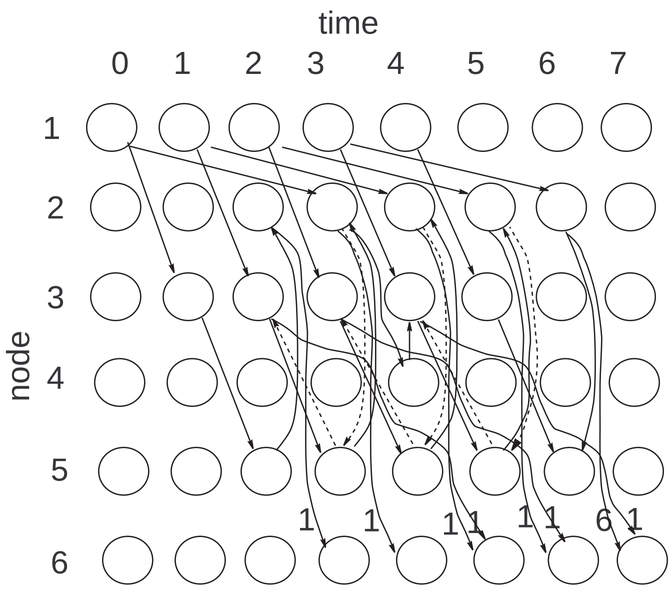

Figure 6: Universal Maximal Flow Induced by Non-standard Chain Decomposition

$\{P_1, P_2, P_3\}$ be the set of chains, with
$P_1 = \{1, 3, 5, 2, 6\}; P_2 = \{1, 2, 5, 3, 6\}; P_3 = \{1, 2, 4, 3, 6\}$
Γ is a non-standard chain decomposition since $P_2 \in \Gamma$ uses arcs $(5, 2)$ and $(3, 5)$ in the opposite direction (shown in Figure 6 as dot lines), i.e. uses arc (2,5) and (5,3) that both are not in A.

In general, suppose P is a chain flow that sends flow along arc (i,j) but in the opposite direction of the static flow. In order to keep the feasibility due to capacity constraints, there must be another chain flow P^* that also uses arc (i,j) but sends flow in the opposite direction of P to cancel chain flow P on (i,j). Since the travel time of the opposite arc is nonpositive, if chain flow P arrives at node i at time t, then the chain flow P^* must arrive at i at time $t^* \leq t$. Similarly, if chain flow P stops using arc (i,j) at time t, then P^* must continue sending flow from j until time $t^* \geq t$. In Figure 6, chain flow P_2 starts using arc $(3,5)$ in the opposite direction $(5,3)$ at time $t=3$ and stops using it at time $t=4$. On the other hand, chain flow P_1 starts using arc $(3,5)$ at time $t=1$ and stops using it at time $t=6$.

By using this non-standard chain decomposition and applying the capacity scaling shortest augmenting path algorithm, Hoppe and Tardos [34] developed the first polynomial approximation algorithm, with time complexity $O(\frac{m}{\epsilon}(m + n\text{log}n)\text{log}U)$, where U is the maximum arc capacity. It is proved to be within $(1+\epsilon)$ of optimality.

6 Quickest Path and Quickest Flow

The quickest path problem as introduced by Chen and Chin [15] is another variant of the shortest path problem. The objective is to send a predetermined number of units from their initial position (i.e. the source node) to the destination (i.e. the sink node) as quickly as possible along a single path. But the notion of "shortest" depends not only on the travel time but also on the number of units that have to be delivered along the path. Flows are sent continuously over the time. The quickest path problem is relevant to a special evacuation problem where evacuees may use only a single path or tunnel from their initial position, that will not be interfered by evacuees from other places. An example of this problem is the evacuation of spectators from a sports stadium. In this case, the quickest path model is applied independently to each network that models the evacuation of each stand in the stadium. Another problem known as quickest flow [12] is similar to the quickest path problem. Here, it is allowed to send flows along multiple paths. The latter problem is known as the minimum time network clearing problem with its obvious relevance for evacuation problems. In this section we first discuss the quickest path and then the quickest flow problem.

6.1 Quickest Path Problems

Definition 6.1 *Given a path $P := (i_1, i_2, \ldots, i_k)$ in the static network G.*

- *The capacity of path P, $b(P)$ is defined as*

$$b(P) = \min_{1 \leq i \leq k-1} b(i_i, i_{i+1})$$

- *The length of path P (in time unit) $\lambda(P)$ is defined as*

$$\lambda(P) := \sum_{i=1}^{i=k-1} \lambda(i_i, i_{i+1})$$

- *The egress time to send out σ units of flow from i_1 to i_k along P is*

$$T(\sigma, P) = \lambda(P) + \frac{\sigma}{b(P)}$$

Different from the classical shortest path problem the concatenation property (i.e. the property that every subpath of a shortest path is also a shortest path) is no longer true for the quickest path problem, as is shown by the following example.

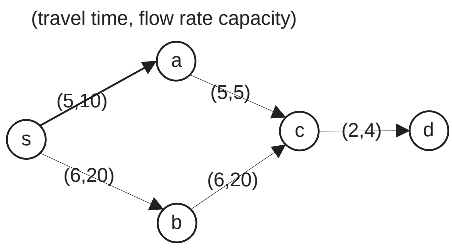

Figure 7: Example for Quickest Path Problem.

Example 6.1 Refering to the example of Figure 7, we want to send out 20 evacuees from source node s to sink node c. Then the quickest path is $P_1 = (s, b, c)$ with egress time $T(20, P_1) = 12 + \frac{20}{20} = 13$. Suppose instead of c we take d as the final destination, then the quickest path is $P_2 = (s, a, c, d)$ with $T(20, P_2) = 17$. But $P_3 = (s, a, c) \subset P_2$ is not the quickest path from s to c, violating the concatenation property. We see also that P_1 is not the (classical) shortest path (with respect to travel time) from s to c. The number of evacuees reaching the destination d over time t is shown as function $I_d(t)$ in Figure 8 with

$$I_d(t) = \begin{cases} 4(t-12) & , t \geq 12 \\ 0 & , \text{otherwise} \end{cases}$$

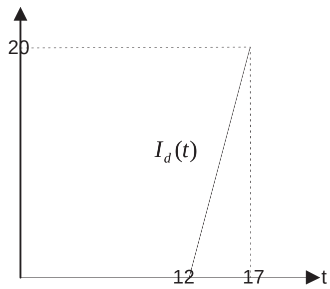

Figure 8: The number of Evacuees Reaching the Destination d.

The following two theorems show the interrelation between quickest and shortest path in G which turn out to be useful in developing an algorithm for solving the quickest path problem. For this purpose we define for given network G and value z the subgraph $G(z) = (N, A(z))$ of G defined by $A(z) := \{(i, j) : (i, j) \in A \text{ and } b_{ij} \geq z\}$

Theorem 6.1 (Rosen [61]) *If P is the quickest s-d path in the static network G sending σ units of flow then*

- *P is a shortest s-d path in $G(b(P))$.*
- *Any subpath of P itself must be a shortest path in $G(b(P))$.*

Theorem 6.2 (Rosen [61]) *Let r be the number of distinct capacity values and let P_j be a shortest $s-d$ path in $G(b_j), j = 1, \dots, r$. If*

$$\lambda(P_l) + \frac{\sigma}{b(P_l)} = \min_{1 \leq j \leq r} \{\lambda(P_j) + \frac{\sigma}{b(P_j)}\},$$

then P_l is the quickest $s-d$ path in G sending σ unit of flows.

Based on these two theorems, Rosen, et al. [61] developed a simple algorithm as follows. In the initial step we compute for each $j = 1, \dots, r$ a shortest $s-d$ path in $G(b_j)$ and then apply Theorem 6.2. Using Fredman and Tarjan [27], each computation of P_j requires $O(m+n \log n)$ time such that the overal complexity of the algorithm is $O(rm + rn \log n)$.

This result can be extended by considering the more realistic assumption that the travel time is density dependent (i.e. flow dependent). Here we assume that the travel time is a step function of the flow, which is nondecreasing, and constant in each unit of flow. Let k_{ij} be the number of distinct travel times of arc (i, j) and $k^* := \max_{(i,j)\in A}\{k_{ij}\}$. The travel time of arc (i, j) then can be defined as follows.

$$\lambda_{ij}(x_{ij}) = \begin{cases} \lambda_{ij}^1 & ,0 \leq x_{ij} \leq b_{ij}^1 \\ \lambda_{ij}^2 & ,0 \leq x_{ij} \leq b_{ij}^2 \\ \dots & ,\dots \\ \lambda_{ij}^{k_{ij}} & ,0 \leq x_{ij} \leq b_{ij}^{k_{ij}} \end{cases}$$

For each arc (i, j) we create k_{ij} artificial nodes denoted $ij^1, \dots, ij^{k_{ij}}$. Then, we connect node i to node ij^l and node ij^l to node j for each $l = 1, \dots, k_{ij}$ as shown in Figure 9. The capacity of arc (i, ij^l) is b_{ij}^l with travel time λ_{ij}^l. The capacity and travel time for arc (ij^l, j) are ∞ and 0 respectively. The modified network will have maximum $(n + mk^*)$ nodes, $2mk^*$ arcs and mk^* number of different capacities. The quickest path then can be obtained by applying the previous algorithm to the modified network.

Proposition 6.1 ([64]) *Let P is the quickest path with arc $(i, j) \in P$. Then arc (i, j) will use the closest capacity to the $b(P)$, i.e. if $b_{ij}^{l-1} < b(P) \leq b_{ij}^l$ then $(i, ij^r) \in P$ for $r = l$ and $(i, ij^r) \notin P$ for $r \neq l$.*

6.2 Quickest Flow Problems

Unlike the quickest path problem, the quickest flow problem (QFP) relaxes the limitation of a single path to multiple paths. QFP is a dynamic network flow problem with single source and single sink that clear the network in the minimum possible time ([12], [23]). The objective function of QFP can be formulated as minimizing the time horizon $T =: T(v)$ where v is the number of initial occupants of the building. The set of constraints is equal to the one used

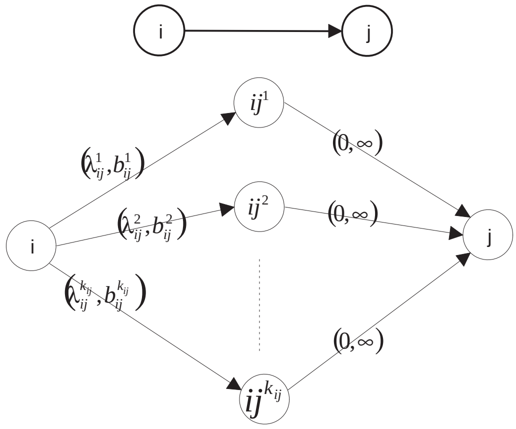

Figure 9: Network Modification for Density Dependent Travel Time.

in the minimum turnstile cost dynamic network flow model discussed in Section 3. The following theorem states properties of the value $v(T)$ of a maximum dynamic flow over time period T, that can be used to derive an algorithm to solve the quickest flow problem.

Theorem 6.3 ([12])

- *Let T_0 be the length of the shortest path from the source to the sink with respect to the travel time. $v(T)$ is a monotone increasing function and for $T \geq T_0$ it increases strictly.*
- *$\triangle(T) := v(T) - v(T-1)$ is for $T > 0$ monotonously increasing, i.e.*

$$\triangle(T+1) \geq \triangle(T), \forall T > 0$$

- *$\triangle(T)$ attains its value only from the set $\{0, 1, \ldots, \mid x_{max} \mid\}$, where $\mid x_{max} \mid$ is the value of maximum static flow in the network G.*

The interrelations between maximum dynamic flow and quickest flow are described by the following lemma.

Lemma 6.1 ([12])

- *$T(v) = min\{T \mid v(T) \geq v\}$ where $v(T)$ is a maximum flow over time period T.*
- *Let x be a maximum dynamic flow of value v in the time interval $[0, T], T \geq 0$. If $v(T-1) < v$ then x is a quickest flow of value v, and for the minimum egress time $T(v)$, we get $T(v) = T$.*
- *$v(T(v) - 1) < v$*

The solution is obtained by applying an iterative process with two main steps [12]. The first step estimates the time period T by utilizing a binary search, Newton or interpolation techniques. The second one solves the minimum cost flow problem with parameter T. The process is repeated until $v(T) \approx v$. Burkard, et. al. [12] developed several polynomial and strongly polynomial algorithms for the quickest flow problem using a continuous version of $v(T)$ in case of single source and single sink.

7 Tripple Optimization Result

Using the turnstile cost definition introduced in Section 3, very interesting properties can be derived interrelating the evacuation time and the maximum number of evacuees that can be sent out to safety in every time period. This interrelation is described in the following theorem known as *tripple optimization theorem.* It says that the flow pattern which minimizes the average evacuation time (see Section 3) also maximizes the number of evacuees reaching the safety at each time period (see Section 5) and vice versa. Moreover, the solution of the second problem also minimizes the total time needed to evacuate all evacuees (see Section 6.2) but the converse in general is not true.

Theorem 7.1 ([36]) *Let F_t be the flow vector of arcs connected to the super sink at time t and let c_t is the associated weight (or cost) vector where the weight c is increasing over the time t. Consider three different problems under the assumption that there exists a feasible flow of K units, i.e. the value of the maximum dynamic flow within a time T is not less than K.*

(a) *Universal maximum flow problem :*

$$max \sum_{t=0}^{T'} F_t, \forall T' \leq T$$

(b) *Minimum weighted sum flow problem :*

$$\sum_{t=0}^{T} c_t F_t$$

(c) *Quickest flow problem of initial occupancies K :*

$$min\{T \mid F_{T'} = 0, \forall T' > T\}$$

Then the solution for either problem (a) *or* (b) *is also the solution of the other two problems, i.e.*

$$(a) \Leftrightarrow (b) \Rightarrow (c)$$

As a consequence it suffices to solve either of the two problems using the turnstile cost approach or the UMF approach to obtain a best evacuation plan according to the three goals listed at the beginning of this section!

8 Multiple Objectives

Using the values contained in the dynamic severity matrix (see Section 1), one can divide the evacuation regions into priority levels. For example, locations where hazards initiated, can be classified as regions with the highest priority level. Obviously, evacuees should not move from lower priority regions to regions with higher priority. In the modeling one can enforce this by charging high costs to corresponding arcs. With this priority system, the evacuation process may have several objectives that must be satisfied simultaneously. For example, let $P_1, \ldots, P_k$ be a partition of the underlying system into k different parts such that the evacuation of P_1 has the highest, the evacuation of P_2 has the next highest, and finally, the evacuation of P_k has the lowest priority. The objective of the evacuation process is to minimize the evacuation time such that ([33])

(1) The evacuation time of P_1 is as small as possible.

(2) Among all plans optimizing (1), the evacuation time of P_2 is as small as possible.

$\vdots$

(k) Among all plans optimizing (1) until $(k-1)$, the evacuation time of P_k is as small as possible.

Hamacher and Tufekci ([33]) consider this multiple priority level problem as lexicographical minimum cost dynamic flow problem.

The lexicographical ordering is defined as follows.

Definition 8.1 *Suppose we have two vectors with k-components,* $\mathbf{c}$ *and* $\overline{\mathbf{c}}$.

$$\mathbf{c} = \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix} \quad , \quad \overline{\mathbf{c}} = \begin{pmatrix} \overline{c}_1 \\ \vdots \\ \overline{c}_k \end{pmatrix}$$

Vector $\mathbf{c}$ *is lexicographically smaller than* $\overline{\mathbf{c}}$ *if and only if c_i is strictly smaller than $\overline{c}_i$ for the first component i, where c_i and $\overline{c}_i$ are different.*

Given a set K of vectors with k-components its lexicographic minimum is denoted as *lex min* K.

In the multiple objective evacuation problem, we replace the real valued cost $c_{ij}(t)$ of the single objective problem by the vector

$$\mathbf{c}_{ij}(t) = \begin{pmatrix} c_{1ij}(t) \\ \vdots \\ c_{kij}(t) \end{pmatrix}$$

for each arc $(i,j) \in A$ and for $t = 0, \ldots, T$. The cost value for each priority level $l = 1, \ldots, k$ can be defined by

$$c_{lij}(t) = \begin{cases} t' = t + \lambda_{ij} & , \text{ if } i(t) \in P_l \text{ and } j(t') \in P_{l'},\ l' > l \\ M & , \text{ if } i(t) \in P_l \text{ and } j(t') \in P_{l'},\ l' < l \\ 0 & , \text{ otherwise.} \end{cases}$$

The objective function ϕ can be defined as

$$\phi(X) = \sum_{t=0}^{T} \sum_{l=1}^{k} \sum_{(i,j)\in A} c_{lij}(t) x_{ij}(t)$$

The constraints are (8) - (14). This problem can be solved by using the concept of *lexicographically shortest augmenting path* .

Definition 8.2 *Let $P = \{s = i_0, i_1, \ldots, i_k = d\}$ be a path from super source s to super sink d in the time-expanded network G_T and let*

$$P^+ = \{e \in P \mid e = (i_j, i_{j+1}),\ j = 0, \ldots, k-1\}$$

$$P^- = \{e \in P \mid e = (i_{j+1}, i_j),\ j = 0, \ldots, k-1\}$$

If x is a flow in G_T define

$$\epsilon_+(P) = \ min\ \{b_e - x_e \mid e \in P^+\}$$

$$\epsilon_-(P) = \ min\ \{x_e \mid e \in P^-\}$$

$$\epsilon(P) = \ min\ \{\epsilon_+(P), \epsilon_-(P)\}$$

and

$$\mathbf{c}(P) = \mathbf{c}(P^+) - \mathbf{c}(P^-) = \sum_{e \in P^+} \mathbf{c}(e) - \sum_{e \in P^-} \mathbf{c}(e)$$

P is called a lexicographically (lex) shortest augmenting path if $\epsilon(P) > 0$ and

$$\mathbf{c}(P) = lex\ min\{\mathbf{c}(P') \mid \epsilon(P') > 0,\ \forall path\ P' in\ G_T\}$$

In the definition, the time-expanded network G_T is considered as a static network. The proposed algorithm ([33]) to solve the lexicographically minimum cost flow problem needs the notion of lex extreme flow as explained in Definition 8.3.

Definition 8.3 *A flow x is a lex extreme flow in G_T if x is a feasible flow with value v and if the cost*

$$\mathbf{c}(x) = \sum_{e \in A_T} x_e \mathbf{c}(e)$$

is minimal among all flows with the same flow value v.

The algorithm starts with finding a lexicographically shortest augmenting path and then augments the current flow along this path. Theorem 8.1 shows that the updated flow after the augmentation is again a lex extreme flow.

Theorem 8.1 *Let x be a lex extreme flow with flow value v and let P be a lex shortest augmenting path. Then the new flow x' defined by*

$$x'_e = \begin{cases} x_e + \epsilon(P) & , \ if\ e \in P^+ \\ x_e - \epsilon(P) & , \ if\ e \in P^- \\ x_e & , \ if\ e \notin P \end{cases}$$

is a lex extreme flow with respect to flow value $v + \epsilon(P)$.

The search for lex shortest augmenting paths is repeated until no more augmenting path is available. If this situation is reached, the current flow is a lex min cost flow.

Another area, where multi criteria network flows may be used, is when arcs may have several attributes which are time dependent. Travel cost, travel distance and risk are examples of attributes which may be attached to each arc. We may want to consider, for instance, the effect of fire, smoke or toxical chemicals on the availability of arcs over time. If the development is such that evacuees can not walk as fast as usual or even may not be able to pass this arc then the cost change over time in all or some components of the cost vector. If, for instance, the arc (i, j) becomes impassable at time t', then the travel cost of going from node i to node j can be modified to be

$$c_{ij}(t) = \begin{cases} c_{ij}(t) & , \text{ if } t < t' \\ M & , \text{ if } t \geq t' \end{cases}$$

with M a large positive number.

Let $\{\mathbf{F}_i\}$ be the set of non-dominated evacuation routes from node i to safety d, and let $\mathbf{c}_{ij}$ be the vector of attributes associated with the arc from node i to node j. Defines also *vmin* as the vector minimization to find the non-dominated evacuation routes. The dynamic programming formulation to find the non-dominated evacuation routes given by ([43], [44]).

$$\{\mathbf{F}_i\} = \text{ vmin }_{j \neq i, (i,j) \in A} \{\{\mathbf{F}_j\} + \mathbf{c}_{ij}\} \quad , \forall i \in N - \{d\}$$

$$\{\mathbf{F}_d\} = \{\mathbf{0}\}$$

Here, $\mathbf{0}$ is the null vector. Kostreva and Wiecek ([43]) proposed backward and forward dynamic programming algorithms to solve this problem. They allow discontinuity in the cost functions, an asumption which is certainly relevant for evacuation planning, as seen above.

9 Dynamic Network with Density Dependent Travel Time

So far, we have mainly considered travel times which are either constant or dependent on time. In reality, they are however dependent on the density of the flow. During any kind of movement, and this is in particular true for evacuation movement, the speed (i.e. travel time) will grow with higher density, until we encounter slow down and queuing phenomena at certain degrees of densitiy. In this section we consider therefore travel times $\lambda_e(t) = g_e(x_e(t))$ where g is an appropriately chosen function depending both on time and the flow at this time.

This model is more realistic, but also more difficult to handle from a mathematical point of view. If we consider, for instance, the flow augmentation process, the amount of time a specific arc in a source-sink path is available is depending on the amount of flow sent through this arc. Moreover, since the density dependent travel time is in general nonlinear, the dynamic conservation flow contraints (see constraint (3)) are also nonlinear, which makes the problem much more difficult to solve.

Several references on the dynamic network flow problem with density dependent travel times can be found in the field of traffic assignment (see Table 2). Since many concepts and models of traffic assignment problems can be adapted to tackle the evacuation problems, we review some of these methods in the evacuation context. There are two main objectives in traffic assignment problem, system optimum and user optimum. The former objective tries to minimize the total travel time. It considers the willingnes of the community to share the lateness. The latter tries to optimize every individual's travel time where it is assumed that each individual behaves egotistical, If we assume single destination and allow exogenous flows only at the beginning (i.e. at time zero), the dynamic traffic assignment problem becomes an evacuation problem with known initial occupancy distribution.

In traffic assignment, the nonlinearity of the travel time function is handled by the following approaches.

- Use linear approximation of the travel time by introducing 0-1 decision variables for selecting the appropriate travel time (Kaufmann et al. [39]).
- Use piecewise linear approximation (Carey and Subrahmanian [13]).
- Apply an iterative process where in each iteration the travel time is fixed temporarily according to the current flow (see [37], [60]).

The first approach uses the decision variable δ_{ij}^{ts} with value equal to 1 if the flow entering arc (i, j) at time t needs arc travel time s and zero otherwise. The free-flow travel time gives the lower bound to the parameter s whereas the upper bound is given by the value $(T - t)$. The flow variable x_{ij}^{ts} represents the flows enter arc (i, j) at time t and exits at time $t + s$. The flow value is bounded by the arc capacity corresponds to the selected travel time determined by δ_{ij}^{ts}. The problem is thus formulated as a mixed integer programming problem.

The second approach uses a piecewise linear approximation of the travel times. A time expanded arc is obtained by joining node i at time t with node j at time $(t + k)$ with $k = 0, 1, \ldots, K$ integer breakpoints of the travel time function. The arc flow must lie between at most two neighbouring breakpoints, i.e. the arc flow is represented as a convex combination of two flow values at two neigbouring breakpoints. Using this approach, the problem is formulated as a linear programming problem.

In the last approach, the value of the travel time is approximated using a 2-level iterative process. In the first level, the travel time is temporarily fixed in each iteration. In the second level, a nonlinear optimization problem is solved iteratively as a convex programming problem. This approach is equivalent to the problem of finding the fixed-point of two interdependent algorithmic maps. One algorithmic map is for adjusting the travel time and the other one is for finding the optimal flow under a temporarily fixed travel time.

An even more realistic modelling can be achieved by considering a dependence of the travel time not only on the existing flow, but on three flow components on each arc $(i, j) \in A$ at time t, namely: incoming flow $u_e(t)$, existing flows $x_e(t)$ and outgoing flow $v_e(t)$. Hence, the travel time of any arc e at any time t is given by

$$\lambda : A \times T \rightarrow \mathbb{R}$$
$$\lambda_e(t) = g_e(u_e(t), x_e(t), v_e(t)),$$

where g is assumed to be a nondecreasing function, which is convex, continuous and differentiable. This model has been considered by Ran and Boyce [60].

In the evacuation context, the monotonicity assumption mimics properly the situation under congestion. Instead of the node-arc flow formulation used so far, their approach uses an *arc-path flow formulation* in the constraints. Let us define $O(j)$ as the set of arcs leaving node j and $I(j)$ as the set of arcs entering node j. The objective of the model is to minimize the average evacuation time, i.e. we minimize

$$\phi(U) := \sum_{t=0}^{T} \sum_{e \in I(d)} (t + \lambda_e(t)) u_e(t)$$

Constraints in the model include flow conservation, flow propagation, nonnegativity and boundary constraints as follows.

- Let $A(p)$ be the set of all feasible paths from source s to sink d. If ep is an arc e on path $P \in A(p)$, then the interrelation among incoming, existing and outgoing flows of arc e along path p is

$$x_{ep}(t+1) = u_{ep}(t) + x_{ep}(t) - v_{ep}(t), \forall ep \ ; \ \forall t \tag{22}$$

- Evacuees who are in the source s at the beginning can directly move to arcs leaving s or wait in the node if they think those arcs are too crowded. Assuming a single source s and a single sink d, the total movement out of the the source must be equal to the initial contents q. Therefore the supply-demand constraints can be formulated as

$$\sum_{t=0}^{T} \sum_{e \in O(s)} u_e(t) - \sum_{t=0}^{T} \sum_{e \in I(s)} v_e(t) = q \tag{23}$$

$$\sum_{t=0}^{T} \sum_{e \in I(d)} v_e(t) - \sum_{t=0}^{T} \sum_{e \in O(d)} u_e(t) = q \tag{24}$$

- Flows conservation constraints :

$$\sum_{e \in O(i)} u_e(t) - \sum_{e \in I(i)} v_e(t) = 0, \forall i \neq s, d \ ; \ \forall t \tag{25}$$

- Flow propagation constraints :
  If platoon dispersion is not allowed then the incoming flow at time t will cause exactly outgoing flow after $\lambda_e(t)$ unit time, i.e.

$$u_{ep}(t) = v_{ep}(t + \lambda_e(t)), \forall e \in A; \forall p \in A(p); \forall t \tag{26}$$

- Boundary conditions :

$$x_e(0) = 0, \forall e \in A \tag{27}$$

- Nonnegativity constraints :

$$u_e(t), x_e(t), v_e(t) \geq 0, \forall e \in A \ ; \ \forall t \tag{28}$$

- Interrelation between node-arc and arc-path formulation:

$$\sum_{p \in A(p)} u_{ep}(t) = u_e(t) \ ; \ \sum_{p \in A(p)} x_{ep}(t) = x_e(t) \ ;$$
$$\sum_{p \in A(p)} x_{ep}(t) = x_e(t), \ \forall e \in A \ ; \ \forall t \qquad (29)$$

To handle nonlinearity of the travel time we use again a two level iterative algorithm. Initially the first level (outer level) estimates the travel times using free-flow travel times. The travel times are fixed temporally in order to be used in the second level (inner level). Under fixed travel time, the model has convex objective function but linear constraints. The inner level uses the Frank-Wolfe algorithm (see [40], [60]) to obtain the optimal flows. The resulting optimal flows are used to recalculate travel times and then compare the newest travel times to the previous ones. If the result is not significantly different, then the algorithm stops. Otherwise, the second level is repeated using the new travel times as input. The reader is refered to Tjandra ([64]) for more details on numerical procedures and results.

10 Continuous Time Dynamic Network Flow Model

We have seen in the prevous sections, that discretization plays a vital role in the modeling of evacuation using dynamic network flows. To increase the accuracy of the model one can set the basic time unit θ (see Section 3) very small, but this will enlarge the size of the network and thus the computational complexity of the solutions algorithm. Obviously, a tradeoff is necessary between good accuracy and computational tracktabiliy. Independent of this, the fact that the choice of discretization by choosing a specific basic time unit θ predetermines the possible set of evacuation plans is somewhat unsatisfying.

We therefore discuss in this section a continuous-time approach to evacuation modelling. Continous time dynamic network flow problems have been considered by various authors including Tyndall [65] [66], Grinold [31], Perold [54], Anderson, et al. [3], Philpott [55], Pullan [57], Philpott and Craddock [56], Pullan [59]. Most of existing works emphasize on the analysis of primal-dual relationships and the existence of the optimal solution.

In the continuous model we consider bounded measurable functions $c(t)$ and $b(t)$ where each of the m components assigns the cost of flow and upper bound on the rate of flow in one of the arcs at time t, respectively. Variables $x(t)$ and $y(t)$ define rates of flow in each arc and levels of storage in each node at time t, respectively. The level of storage is bounded above by continuous function $a(t)$. The *continuous time dynamic (min cost) network flow problem* is formulated as follows (see [4], [55], [6], [59]).

$$\min\ Z = \int_{t=0}^{T} \sum_{(i,j)\in A} c_{ij}(t) x_{ij}(t) dt \tag{30}$$

$$\text{s.t.} \quad y_j(t) = y_j(0) + \int_0^t [\sum_{(i,j)\in A} x_{ij}(\tau - \lambda_{ij}) - \sum_{(j,i)\in A} x_{ji}(\tau)] d\tau,\ t \in [0,T] \tag{31}$$

$$y_j(t) \le a_j(t), j \in N\ ;\ t \in [0,T], \tag{32}$$

$$x_{ij}(t) \le b_{ij}(t), (i,j) \in A, t \in [0,T] \tag{33}$$

$$x_{ij}(t), y_j(t) \ge 0 \tag{34}$$

Constraint (31) is called *integral constraint* and constraints (32)-(33) are called *instantaneous constraints* .

Since the integral and instantaneous constraints are separated, this network flow problem is included into a specific class of continuous linear programs, namely *separated continuous linear programs* (SCLP) proposed by Anderson, Nash and Perold ([3]). The following theorem gives the format of the optimal flow function of SCLP under specific assumptions on the capacity and cost functions.

Theorem 10.1 ([58]) *Suppose that $a(t), b(t)$ and $c(t)$ are piecewise analytic on $[0,T]$ with $a(t)$ continuous. If the feasible region of SCLP is bounded and nonempty, then there exists an optimal solution for SCLP in which $x(t)$ is piecewise analytic on $[0,T]$.*

Analogous to the three problems in Theorem 7.1 derived in the discrete dynamic network model context, Philpott [55] formulated three continuous models which can be used for evacuation planning.

(a) Maximize flows into the sink node in the interval $[0,T]$.
$\max \int_0^T [\sum_{(i,d)\in A} x_{id}(\tau - \lambda_{id}) - \sum_{(d,i)\in A} x_{di}(\tau)] d\tau$
subject to (31) - (34)

(b) Minimize the time to clear the network initial occupancies.
$\min T$
subject to (31) - (34)
$y(t) = 0, t \ge T; x_{ij}(t) = 0, t \ge T - \lambda_{ij}$

(c) Minimizing total egress time.
$\min \int_0^T \sum_{(i,j)\in A} c_{ij}(t) x_{ij}(t) dt + \int_0^T \sum_{j\in N-\{d\}} h_j(t) y_j(t) dt$
subject to (31) - (34)
with h_j is holding cost in node j.

Problem (b) is known as the continuous version of the quickest flow problem explained in Section 6. By defining $c_{ij}(t)$ as turnstile cost, problem (c) solves the problem of minimizing the average evacuation time as in Section 3. Relationships among those three problems are explained by the next result.

Theorem 10.2

- *Any flow which solves (a) for any time $T^{'} \le T$ and empty the network in time T will also solves (b) but not the converse*

- *Assuming that*
  - *holding cost for each node is equal to one unit,*
  - *arc cost at any time t is defined as*

$$c_{ij}(t) = \begin{cases} \lambda_{ij} & ,0 \leq t < T - \lambda_{ij} \\ T - t & ,T - \lambda_{ij} \leq t < T, \end{cases}$$

  Problems (a) and (c) are equivalent.

This Theorem guarantees the existence of a universal maximum flow in the continuous time domain.

From a model accuracy point of view, modeling evacuation problems using dynamic network problems with continuous time is preferable to discrete time models. But continuous models can to date not be solved satisfactorily for the large scale problems which need to be tackled in the evacuation context. Solution algorithms for continuous dynamic network flow algorithms are due to Anderson and Philpott [5] (continuous-time network simplex algorithm), Pullan [57], Philpott and Craddock [56] (discretization approaches). Anderson, et.al [2] and Philpott [55] (continuous- time version of Ford-Fulkerson's maximum flow labelling algorithm).

More work is certainly needed in this area. First additional results can be found in Tjandra ([64]). He proposes an algorithm to solve UMF problem with time dependent capacity and utilize the result to solve the quickest flow problem under the same assumption.

11 Microscopic Models

In microscopic models each evacuee is considered as a separate flow object. An evacuee will be exposed to accident effects depending on the route he/she follows and the length of time spent in different locations. An evacuee selects the route 'step by step', which means that the choice of the next piece of the route is decided at every node along this route. The initially selected route might be changed due to some reasons, for instance, blockage by fire or high congestion. Figure 10 shows an example of the evacuation process as it can be modeled for each person. Microscopic models emphasize the modeling of human behavior during an emergency situation. The human model can be provided with some personal attributes, for example, walking speed, personal memory and psychological condition. These attribute will be used to determine the movement decisions, for example [21], to select the nearest walkway, move on the walkway only when there is no blockage at the end, or change the destination target before reaching it. Løvås [47] proposed some different probability laws for personal movement relative to the route components (nodes and arcs) as follows.

- Random choice. It is applied when the evacuee is not familiar with the surrounding. Lets define X as the state of the underlying system, for instance, the number of evacuees on each arc/node or the hazard (smoke, fire, etc.) level of each arc/node. Moreover, let $p_k(i, j, X)$ be the probability that person k, with his personal attributes, will move from node i to

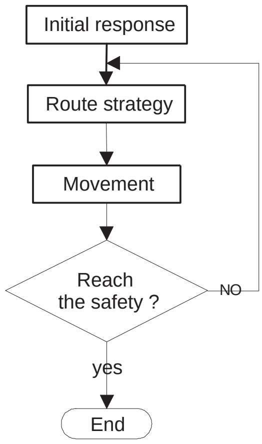

Figure 10: Individual Evacuation Process [46].

node j when the whole system is in state X. If the number of walkways connected to node i is denoted as δ_i, then

$$p_k(i,j,X) := \frac{1}{\delta_i}$$

- Modified random choice. It includes the possibility that evacuees will return to their previous walkway. Define n_k as the node-number of the last node visited by a person k and introduce a number ω, in which $0 \leq \omega \leq 1$. The parameter ω is a factor showing how a person is willing to return to his/her previous walkway. Using this parameter, the probability $p_k(i,j,X)$ can be redefined as follows.

$$p_k(i,j,X) := \begin{cases} \frac{\omega}{\delta_i} & , j = n_k \\ \frac{1-\frac{\omega}{\delta_i}}{\delta_i - 1} & , j \neq n_k \\ 0 & , otherwise \end{cases}$$

- Suppose there is an evacuation route P determined a priori by the evacuation planner (this is a typical situation, indicated by evacuation sign attached to the wall of rooms or corridors).

$$P := (s = \text{origin}, \ldots, i, j*, \ldots, \text{destination} = d)$$

Suppose that $\delta_i > 1$ for all nodes in this route, except possibly the origin and destination. When an emergency situation arises there is a high tendency that evacuees will use familiar routes that may not follow the route recommended in the evacuation plan. For all $i \in P - \{d\}$ the parameter

$\gamma_i \in [0,1]$ indicates the probability that an evacuee will leave the planned route at an intersection node. Then $p_k(i,j,X)$ can be written as

$$p_k(i,j,X) := \begin{cases} 1-\gamma_i & ,j = j* \\ \frac{\gamma_i}{\delta_i - 1} & ,j \neq n_k \end{cases}$$

Path choice rules like the ones presented above can be used together with attributes of evacuees and building components to design a simulation of a building evacuation (see [63]). Obviously, such simulation is a good tool to model individual behaviour in evacuation planning and can also be used to validate optimization models as presented in the preceding sections.

In recent years, there is an increasing trend to use simulation based on cellular automata (CA). Either deterministic or probabilistic rules can be applied to model the movement patterns between time periods relative to the movement of other persons and/or physical barriers. CA simulation offers the possibility to emulate the essential, diverse movements of evacuees as behavioural responses to varying and uncertain local conditions. A *cellular automata* is defined as a regular n-dimensional lattice partitioned into discrete elements called *cells* or *sites* which has a dicrete step time evolution. Using cellular automata, the space of the evacuation area is divided into accessible and non accessible cells, each of fixed and equal size. The state of each cell is one of finitely many values and has a dynamical behaviour. It is updated simultaneously based on the values of the states in its neighborhood at the preceding time step and according to a specific *local rules* . The set of local rules is defined to control the movement of evacuees, or the state transition of each cell. Since the rules are needed to govern only local relationships among the neighboring cells, CA is considered to be very effective for simulating physical phenomena, the relationships of which over the whole domain are unknown. In the 1-dimensional lattice, a cell has only two neighbourhood cells, the left and right cell adjacent to it. For 2-dimensional lattice, there are some neighbourhood classifications of a cell as shown in Figure 11. The von Neumann neighbourhood consists of 4 cells, the cell above, below, right and left of the reference cell. The radius r of the von Neumann neighborhood is 1, since it considers only the next layer of a cell. The Moore neighborhood is an extension of the von Neumann neighborhood in which the diagonal cells are added. Its radius is still $r = 1$. The expanded Moore neighbourhood further extends the Moore neighborhood by including two layers (i.e., $r = 2$). Another type of neigborhood is the Margolus neigborhood in which 2×2 cells of lattices are considerd at once. The transition from one state to another is arranged by a set of rules according to the neighboring state of the preceding time step. If $x(t)$ is a state of cell x at time t then the state of cell x at time $t+1$ can be written as

$$x(t+1) = f(x(t), N_x(t))$$

where $N_x(t)$ is state of the neighborhood cells of cell x at time t and where f is the rule. A simple example of a rule can be given as follows. Consider a narrow corridor (assumed wide enough only for one person) with a given length that can be modeled as a one dimensional lattice (see Figure 12). A cell represents a space in the corridor that may be occupied by one person or it may be empty. The walking speed of the i^{th} person $v(i)$ corresponds to the number of cells

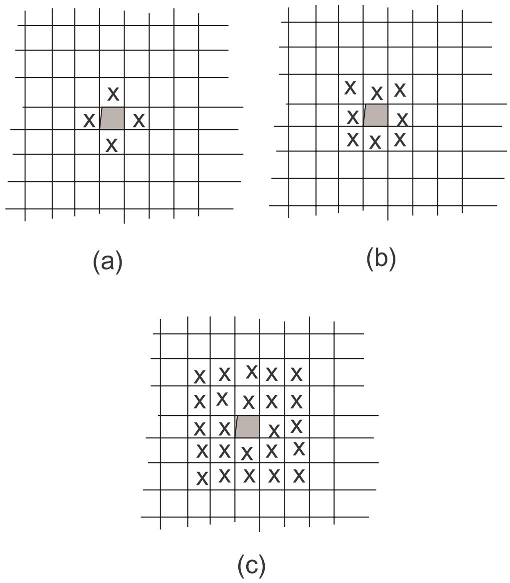

Figure 11: (a) von Neumann Neighborhood (b) Moore Neighborhood (c) Expanded Moore Neighborhood of a Cell (Shaded Cell).

that a person advances in one iteration (one time step). By defining $x(i)$ as the position of the i^{th} person in the corridor, the distance between the i^{th} person and the person immediately ahead is given by

$$g(i) = x(i+1) - x(i) - 1$$

Using these 3 variables the transition rule can be defined as follow :

- Acceleration of free person :
  If $v(i) < v_{max}$ and $g(i) \geq v(i) + 1$ then $v(i) = v(i) + 1$, where v_{max} is the maximum possible speed.
- Slowing down due to other person :
  If $v(i) > g(i) - 1$ then $v(i) = g(i)$
- Movement : person is advanced $v(i)$ cells.

As a method of discrete simulation, CA is used by some evacuation softwares including EGRESS and FlightSim.

EGRESS is a C++ program developed for modelling the behaviour of evacuees in emergency situations, especially in offshore environments. The physical structure of the offshore installation is represented by using hexagonal cellular grids (see, e.g., Doheny and Fraser [20]). It models evacuation using cellular automata where the movements and interactions of the automata on the cellular grid simulate the movements and physical interactions of evacuees on the platform. EGRESS integrates MOBEDIC (Modelling Behavioural Decisions In Computer) for evacuees's decision making and a movement model which models the subsequent movement of evacuees throughout the platform. MOBEDIC

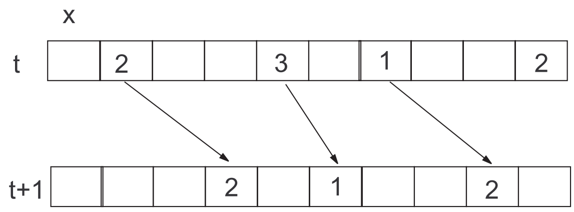

Figure 12: CA Model of Evacuees Moving in a Corridor with $v_{max} = 3$ cells/time-step.

represents the *brain* of the automata in the movement model. It is responsible for making decisions and for instructing an automaton to carry out some action. The Movement Model distributes informations such as the presence of smoke or fire, the occurrence of alarms and the current position of automaton. These informations are combined with some properties of evacuees, such as reactions to alarm and hazards, familiarity with the building structure, knowledge and experiences about the emergency situation. The combined informations determine the movement of evacuees. In the movement model evacuees are represented using cellular automata which can move about the plan from cell to cell. Movement algorithms determine which cell each automaton should occupy at any given time and move the automata accordingly.

FlightSim was developed at the University of Duisburg, Germany (see Klüpfel et al. [42]). It was originally developed for analyzing the evacuation process of a passenger ship. Some adjusted parameters are included in the model, for example the time that every evacuee needs to enter the saving boats, the distribution of the maximum speed among the evacuee, and the distribution of the number of cells a person looks forward for orientation. Instead of using *parallel updating* when updating the evacuees' positions, FlightSim uses *sequential updating* ([11]). In parallel updating, variables of all evacuees (e.g., position, speed) are changed at the same time. In contrast, the sequential updating selects the evacuees one after the other and updates his/her attributes. The selection on who will be the first is done randomly in order not to give advantage to any specific person. Figures 13 and 14 show the difference of these two updating systems ([11]). Looking at Figure 13, during the first step under parallel updating, evacuee a must wait until the next update, since he is blocked by evacuee b. The left column of Figure 14 describes the sequential updating when evacuee a is always chosen first. The movement pattern is the same as one with parallel update. If evacuee b is chosen first, the situation changes, as is shown in the right column of Figure 14.

Another simulation based evacuation software, EXODUS, was developed by the Fire Safety Engineering group at the university of Greenwich, United of Kingdom (see [29], [32]) and designed to simulate a large number of occupants in a closed environment, for instance building and airplane. It is an expert system-based software which has a set of heuristics or rules to determine the progressive motion and behaviour of each individual. It tracks each individual either making his/her way out of the evacuation area or is being overcome by fire hazards. A more complete survey on evacuation softwares using either macro approaches or micro approaches is discussed by Gwynne, et al. [32].

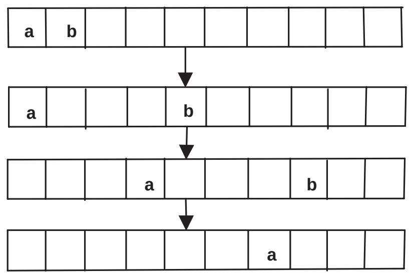

Figure 13: Parallel Update.

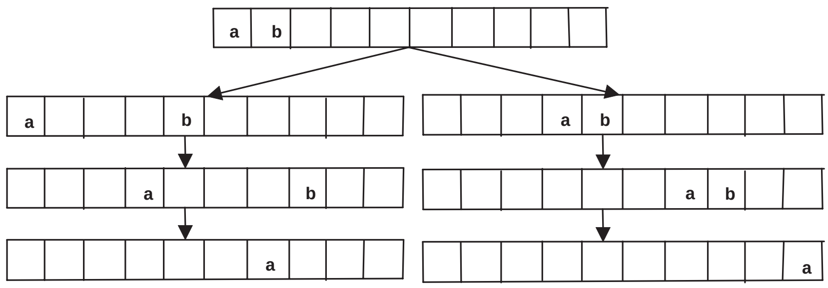

Figure 14: Sequential Update.

12 Summary and Conclusion

In this paper, a review of models and algorithms for evacuation planning has been presented. The review covered macroscopic models quite extensively and sketched microscopic models. Both approaches are able to mirror the flows of evacuations over time. The former has its strength in its possibility to optimize the system (while neglecting individuals' behaviour), while the latter is able to capture and utilize properties of each of the evacuees.

Under the macroscopic approach, minimum turnstile cost dynamic network flow models can be applied to estimate the average evacuation time per evacuee. Maximum dynamic flows and universal maximum flows can be used to estimate the maximum number of evacuees which can reach safety during any given time horizon for the evacuation. Quickest flow models allow the estimatation of the minimum time required to bring a given number of evacuees to safety. Considering the source and propagation of hazards, availability of emergency service units and better organization, the evacuation region can be divided into some regions with different priority levels. Therefore, multiple objective models are presented to cope with this problem. Constant travel time is mostly assumed in the literature. This time can be obtained by taking the travel time of the average flow or travel time of a specific queuing level. In order to reflect the congestion phenomenon, it was shown how the constant travel time assumption can be strengthened by considering density dependent travel time. This approach will, however, significantly increase the complexity of the model.

In contrast to macroscopic models, the microscopic approach, usually implemented as simulation, offers possibilities to include individual characteristics and interaction among evacuees. This makes up for the lack of optimization in this approach. Succesful implementations of the macro approach are based on cellular automata.

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Bisher erschienene Berichte des Fraunhofer ITWM

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1. D. Hietel, K. Steiner, J. Struckmeier

A Finite - Volume Particle Method for Compressible Flows

We derive a new class of particle methods for conservation laws, which are based on numerical flux functions to model the interactions between moving particles. The derivation is similar to that of classical Finite-Volume methods; except that the fixed grid structure in the Finite-Volume method is substituted by so-called mass packets of particles. We give some numerical results on a shock wave solution for Burgers equation as well as the well-known one-dimensional shock tube problem.
(19 S., 1998)

2. M. Feldmann, S. Seibold

Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing

In this paper, a combined approach to damage diagnosis of rotors is proposed. The intention is to employ signal-based as well as model-based procedures for an improved detection of size and location of the damage. In a first step, Hilbert transform signal processing techniques allow for a computation of the signal envelope and the instantaneous frequency, so that various types of non-linearities due to a damage may be identified and classified based on measured response data. In a second step, a multi-hypothesis bank of Kalman Filters is employed for the detection of the size and location of the damage based on the information of the type of damage provided by the results of the Hilbert transform.
Keywords:
Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 S., 1998)

3. Y. Ben-Haim, S. Seibold

Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery

Damage diagnosis based on a bank of Kalman filters, each one conditioned on a specific hypothesized system condition, is a well recognized and powerful diagnostic tool. This multi-hypothesis approach can be applied to a wide range of damage conditions. In this paper, we will focus on the diagnosis of cracks in rotating machinery. The question we address is: how to optimize the multi-hypothesis algorithm with respect to the uncertainty of the spatial form and location of cracks and their resulting dynamic effects. First, we formulate a measure of the reliability of the diagnostic algorithm, and then we discuss modifications of the diagnostic algorithm for the maximization of the reliability. The reliability of a diagnostic algorithm is measured by the amount of uncertainty consistent with no-failure of the diagnosis. Uncertainty is quantitatively represented with convex models.
Keywords:
Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 S., 1998)

4. F.-Th. Lentes, N. Siedow

Three-dimensional Radiative Heat Transfer in Glass Cooling Processes

For the numerical simulation of 3D radiative heat transfer in glasses and glass melts, practically applicable mathematical methods are needed to handle such problems optimal using workstation class computers. Since the exact solution would require super-computer capabilities we concentrate on approximate solutions with a high degree of accuracy. The following approaches are studied: 3D diffusion approximations and 3D ray-tracing methods.
(23 S., 1998)

5. A. Klar, R. Wegener

A hierarchy of models for multilane vehicular traffic
Part I: Modeling

In the present paper multilane models for vehicular traffic are considered. A microscopic multilane model based on reaction thresholds is developed. Based on this model an Enskog like kinetic model is developed. In particular, care is taken to incorporate the correlations between the vehicles. From the kinetic model a fluid dynamic model is derived. The macroscopic coefficients are deduced from the underlying kinetic model. Numerical simulations are presented for all three levels of description in [10]. Moreover, a comparison of the results is given there.
(23 S., 1998)

Part II: Numerical and stochastic investigations

In this paper the work presented in [6] is continued. The present paper contains detailed numerical investigations of the models developed there. A numerical method to treat the kinetic equations obtained in [6] are presented and results of the simulations are shown. Moreover, the stochastic correlation model used in [6] is described and investigated in more detail.
(17 S., 1998)

6. A. Klar, N. Siedow

Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes

In this paper domain decomposition methods for radiative transfer problems including conductive heat transfer are treated. The paper focuses on semi-transparent materials, like glass, and the associated conditions at the interface between the materials. Using asymptotic analysis we derive conditions for the coupling of the radiative transfer equations and a diffusion approximation. Several test cases are treated and a problem appearing in glass manufacturing processes is computed. The results clearly show the advantages of a domain decomposition approach. Accuracy equivalent to the solution of the global radiative transfer solution is achieved, whereas computation time is strongly reduced.
(24 S., 1998)

7. I. Choquet

Heterogeneous catalysis modelling and numerical simulation in rarified gas flows
Part I: Coverage locally at equilibrium

A new approach is proposed to model and simulate numerically heterogeneous catalysis in rarefied gas flows. It is developed to satisfy all together the following points:
1) describe the gas phase at the microscopic scale, as required in rarefied flows,
2) describe the wall at the macroscopic scale, to avoid prohibitive computational costs and consider not only crystalline but also amorphous surfaces,
3) reproduce on average macroscopic laws correlated with experimental results and
4) derive analytic models in a systematic and exact way.
The problem is stated in the general framework of a non static flow in the vicinity of a catalytic and non porous surface (without aging). It is shown that the exact and systematic resolution method based on the Laplace transform, introduced previously by the author to model collisions in the gas phase, can be extended to the present problem. The proposed approach is applied to the modelling of the Eley-Rideal and Langmuir-Hinshelwood recombinations, assuming that the coverage is locally at equilibrium. The models are developed considering one atomic species and extended to the general case of several atomic species. Numerical calculations show that the models derived in this way reproduce with accuracy behaviors observed experimentally.
(24 S., 1998)

8. J. Ohser, B. Steinbach, C. Lang

Efficient Texture Analysis of Binary Images

A new method of determining some characteristics of binary images is proposed based on a special linear filtering. This technique enables the estimation of the area fraction, the specific line length, and the specific integral of curvature. Furthermore, the specific length of the total projection is obtained, which gives detailed information about the texture of the image. The influence of lateral and directional resolution depending on the size of the applied filter mask is discussed in detail. The technique includes a method of increasing directional resolution for texture analysis while keeping lateral resolution as high as possible.
(17 S., 1998)

9. J. Orlik

Homogenization for viscoelasticity of the integral type with aging and shrinkage

A multi-phase composite with periodic distributed inclusions with a smooth boundary is considered in this contribution. The composite component materials are supposed to be linear viscoelastic and aging (of the non-convolution integral type, for which the Laplace transform with respect to time is not effectively applicable) and are subjected to isotropic shrinkage. The free shrinkage deformation can be considered as a fictitious temperature deformation in the behavior law. The procedure presented in this paper proposes a way to determine average (effective homogenized) viscoelastic and shrinkage (temperature) composite properties and the homogenized stress-field from known properties of the

components. This is done by the extension of the asymptotic homogenization technique known for pure elastic non-homogeneous bodies to the non-homogeneous thermo-viscoelasticity of the integral non-convolution type. Up to now, the homogenization theory has not covered viscoelasticity of the integral type.
Sanchez-Palencia (1980), Francfort & Suquet (1987) (see [2], [9]) have considered homogenization for viscoelasticity of the differential form and only up to the first derivative order. The integral-modeled viscoelasticity is more general then the differential one and includes almost all known differential models. The homogenization procedure is based on the construction of an asymptotic solution with respect to a period of the composite structure. This reduces the original problem to some auxiliary boundary value problems of elasticity and viscoelasticity on the unit periodic cell, of the same type as the original non-homogeneous problem. The existence and uniqueness results for such problems were obtained for kernels satisfying some constrain conditions. This is done by the extension of the Volterra integral operator theory to the Volterra operators with respect to the time, whose 1 kernels are space linear operators for any fixed time variables. Some ideas of such approach were proposed in [11] and [12], where the Volterra operators with kernels depending additionally on parameter were considered. This manuscript delivers results of the same nature for the case of the space-operator kernels.
(20 S., 1998)

10. J. Mohring

Helmholtz Resonators with Large Aperture

The lowest resonant frequency of a cavity resonator is usually approximated by the classical Helmholtz formula. However, if the opening is rather large and the front wall is narrow this formula is no longer valid. Here we present a correction which is of third order in the ratio of the diameters of aperture and cavity. In addition to the high accuracy it allows to estimate the damping due to radiation. The result is found by applying the method of matched asymptotic expansions. The correction contains form factors describing the shapes of opening and cavity. They are computed for a number of standard geometries. Results are compared with numerical computations.
(21 S., 1998)

11. H. W. Hamacher, A. Schöbel

On Center Cycles in Grid Graphs

Finding "good" cycles in graphs is a problem of great interest in graph theory as well as in locational analysis. We show that the center and median problems are NP hard in general graphs. This result holds both for the variable cardinality case (i.e. all cycles of the graph are considered) and the fixed cardinality case (i.e. only cycles with a given cardinality p are feasible). Hence it is of interest to investigate special cases where the problem is solvable in polynomial time.
In grid graphs, the variable cardinality case is, for instance, trivially solvable if the shape of the cycle can be chosen freely.
If the shape is fixed to be a rectangle one can analyze rectangles in grid graphs with, in sequence, fixed dimension, fixed cardinality, and variable cardinality. In all cases a complete characterization of the optimal cycles and closed form expressions of the optimal objective values are given, yielding polynomial time algorithms for all cases of center rectangle problems.
Finally, it is shown that center cycles can be chosen as rectangles for small cardinalities such that the center cycle problem in grid graphs is in these cases completely solved.
(15 S., 1998)

12. H. W. Hamacher, K.-H. Küfer

Inverse radiation therapy planning - a multiple objective optimisation approach

For some decades radiation therapy has been proved successful in cancer treatment. It is the major task of clinical radiation treatment planning to realize on the one hand a high level dose of radiation in the cancer tissue in order to obtain maximum tumor control. On the other hand it is obvious that it is absolutely necessary to keep in the tissue outside the tumor, particularly in organs at risk, the unavoidable radiation as low as possible.
No doubt, these two objectives of treatment planning - high level dose in the tumor, low radiation outside the tumor - have a basically contradictory nature. Therefore, it is no surprise that inverse mathematical models with dose distribution bounds tend to be infeasible in most cases. Thus, there is need for approximations compromising between overdosing the organs at risk and underdosing the target volume.
Differing from the currently used time consuming iterative approach, which measures deviation from an ideal (non-achievable) treatment plan using recursively trial-and-error weights for the organs of interest, we go a new way trying to avoid a priori weight choices and consider the treatment planning problem as a multiple objective linear programming problem: with each organ of interest, target tissue as well as organs at risk, we associate an objective function measuring the maximal deviation from the prescribed doses.
We build up a data base of relatively few efficient solutions representing and approximating the variety of Pareto solutions of the multiple objective linear programming problem. This data base can be easily scanned by physicians looking for an adequate treatment plan with the aid of an appropriate online tool.
(14 S., 1999)

13. C. Lang, J. Ohser, R. Hilfer

On the Analysis of Spatial Binary Images

This paper deals with the characterization of microscopically heterogeneous, but macroscopically homogeneous spatial structures. A new method is presented which is strictly based on integral-geometric formulae such as Crofton's intersection formulae and Hadwiger's recursive definition of the Euler number. The corresponding algorithms have clear advantages over other techniques. As an example of application we consider the analysis of spatial digital images produced by means of Computer Assisted Tomography.
(20 S., 1999)

14. M. Junk

On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes

A general approach to the construction of discrete equilibrium distributions is presented. Such distribution functions can be used to set up Kinetic Schemes as well as Lattice Boltzmann methods. The general principles are also applied to the construction of Chapman Enskog distributions which are used in Kinetic Schemes for compressible Navier-Stokes equations.
(24 S., 1999)

15. M. Junk, S. V. Raghurame Rao

A new discrete velocity method for Navier-Stokes equations

The relation between the Lattice Boltzmann Method, which has recently become popular, and the Kinetic Schemes, which are routinely used in Computational Fluid Dynamics, is explored. A new discrete velocity model for the numerical solution of Navier-Stokes equations for incompressible fluid flow is presented by combining both the approaches. The new scheme can be interpreted as a pseudo-compressibility method and, for a particular choice of parameters, this interpretation carries over to the Lattice Boltzmann Method.
(20 S., 1999)

16. H. Neunzert

Mathematics as a Key to Key Technologies

The main part of this paper will consist of examples, how mathematics really helps to solve industrial problems; these examples are taken from our Institute for Industrial Mathematics, from research in the Technomathematics group at my university, but also from ECMI groups and a company called TecMath, which originated 10 years ago from my university group and has already a very successful history.
(39 S. (vier PDF-Files), 1999)

17. J. Ohser, K. Sandau

Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem

Wicksell's corpuscle problem deals with the estimation of the size distribution of a population of particles, all having the same shape, using a lower dimensional sampling probe. This problem was originary formulated for particle systems occurring in life sciences but its solution is of actual and increasing interest in materials science. From a mathematical point of view, Wicksell's problem is an inverse problem where the interesting size distribution is the unknown part of a Volterra equation. The problem is often regarded ill-posed, because the structure of the integrand implies unstable numerical solutions. The accuracy of the numerical solutions is considered here using the condition number, which allows to compare different numerical methods with different (equidistant) class sizes and which indicates, as one result, that a finite section thickness of the probe reduces the numerical problems. Furthermore, the relative error of estimation is computed which can be split into two parts. One part consists of the relative discretization error that increases for increasing class size, and the second part is related to the relative statistical error which increases with decreasing class size. For both parts, upper bounds can be given and the sum of them indicates an optimal class width depending on some specific constants.
(18 S., 1999)

18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel

Solving nonconvex planar location problems by finite dominating sets

It is well-known that some of the classical location problems with polyhedral gauges can be solved in polynomial time by finding a finite dominating set, i. e. a finite set of candidates guaranteed to contain at least one optimal location.
In this paper it is first established that this result holds for a much larger class of problems than currently considered in the literature. The model for which this result can be proven includes, for instance, location problems with attraction and repulsion, and location-allocation problems.
Next, it is shown that the approximation of general gauges by polyhedral ones in the objective function of our general model can be analyzed with regard to the subsequent error in the optimal objective value. For the approximation problem two different approaches are described, the sandwich procedure and the greedy algorithm. Both of these approaches lead - for fixed epsilon - to polynomial approximation algorithms with accuracy epsilon for solving the general model considered in this paper.
Keywords:
Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 S., 2000)

19. A. Becker

A Review on Image Distortion Measures

Within this paper we review image distortion measures. A distortion measure is a criterion that assigns a "quality number" to an image. We distinguish between mathematical distortion measures and those distortion measures in-cooperating a priori knowledge about the imaging devices (e. g. satellite images), image processing algorithms or the human physiology. We will consider representative examples of different kinds of distortion measures and are going to discuss them.
Keywords:
Distortion measure, human visual system
(26 S., 2000)

20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn

Polyhedral Properties of the Uncapacita*ted Multiple Allocation Hub Location Problem*

We examine the feasibility polyhedron of the uncapacitated hub location problem (UHL) with multiple allocation, which has applications in the fields of air passenger and cargo transportation, telecommunication and postal delivery services. In particular we determine the dimension and derive some classes of facets of this polyhedron. We develop some general rules about lifting facets from the uncapacitated facility location (UFL) for UHL and projecting facets from UHL to UFL. By applying these rules we get a new class of facets for UHL which dominates the inequalities in the original formulation. Thus we get a new formulation of UHL whose constraints are all facet–defining. We show its superior computational performance by benchmarking it on a well known data set.
Keywords:
integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 S., 2000)

21. H. W. Hamacher, A. Schöbel

Design of Zone Tariff Systems in Public Transportation

Given a public transportation system represented by its stops and direct connections between stops, we consider two problems dealing with the prices for the customers: The fare problem in which subsets of stops are already aggregated to zones and "good" tariffs have to be found in the existing zone system. Closed form solutions for the fare problem are presented for three objective functions. In the zone problem the design of the zones is part of the problem. This problem is NP hard and we therefore propose three heuristics which prove to be very successful in the redesign of one of Germany's transportation systems.
(30 S., 2001)

22. D. Hietel, M. Junk, R. Keck, D. Teleaga:

The Finite-Volume-Particle Method for Conservation Laws

In the Finite-Volume-Particle Method (FVPM), the weak formulation of a hyperbolic conservation law is discretized by restricting it to a discrete set of test functions. In contrast to the usual Finite-Volume approach, the test functions are not taken as characteristic functions of the control volumes in a spatial grid, but are chosen from a partition of unity with smooth and overlapping partition functions (the particles), which can even move along prescribed velocity fields. The information exchange between particles is based on standard numerical flux functions. Geometrical information, similar to the surface area of the cell faces in the Finite-Volume Method and the corresponding normal directions are given as integral quantities of the partition functions.
After a brief derivation of the Finite-Volume-Particle Method, this work focuses on the role of the geometric coefficients in the scheme.
(16 S., 2001)

23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel

Location Software and Interface with GIS and Supply Chain Management

The objective of this paper is to bridge the gap between location theory and practice. To meet this objective focus is given to the development of software capable of addressing the different needs of a wide group of users. There is a very active community on location theory encompassing many research fields such as operations research, computer science, mathematics, engineering, geography, economics and marketing. As a result, people working on facility location problems have a very diverse background and also different needs regarding the software to solve these problems. For those interested in non-commercial applications (e. g. students and researchers), the library of location algorithms (LoLA can be of considerable assistance. LoLA contains a collection of efficient algorithms for solving planar, network and discrete facility location problems. In this paper, a detailed description of the functionality of LoLA is presented. In the fields of geography and marketing, for instance, solving facility location problems requires using large amounts of demographic data. Hence, members of these groups (e. g. urban planners and sales managers) often work with geographical information too s. To address the specific needs of these users, LoLA was inked to a geographical information system (GIS) and the details of the combined functionality are described in the paper. Finally, there is a wide group of practitioners who need to solve large problems and require special purpose software with a good data interface. Many of such users can be found, for example, in the area of supply chain management (SCM). Logistics activities involved in strategic SCM include, among others, facility location planning. In this paper, the development of a commercial location software tool is also described. The too is embedded in the Advanced Planner and Optimizer SCM software developed by SAP AG, Walldorf, Germany. The paper ends with some conclusions and an outlook to future activities.
Keywords:
facility location, software development, geographical information systems, supply chain management.
(48 S., 2001)

24. H. W. Hamacher, S. A. Tjandra

Mathematical Modelling of Evacuation Problems: A State of Art

This paper details models and algorithms which can be applied to evacuation problems. While it concentrates on building evacuation many of the results are applicable also to regional evacuation. All models consider the time as main parameter, where the travel time between components of the building is part of the input and the overall evacuation time is the output. The paper distinguishes between macroscopic and microscopic evacuation models both of which are able to capture the evacuees' movement over time.
Macroscopic models are mainly used to produce good lower bounds for the evacuation time and do not consider any individual behavior during the emergency situation. These bounds can be used to analyze existing buildings or help in the design phase of planning a building. Macroscopic approaches which are based on dynamic network flow models (minimum cost dynamic flow, maximum dynamic flow, universal maximum flow, quickest path and quickest flow) are described. A special feature of the presented approach is the fact, that travel times of evacuees are not restricted to be constant, but may be density dependent. Using multicriteria optimization priority regions and blockage due to fire or smoke may be considered. It is shown how the modelling can be done using time parameter either as discrete or continuous parameter.
Microscopic models are able to model the individual evacuee's characteristics and the interaction among evacuees which influence their movement. Due to the corresponding huge amount of data one uses simulation approaches. Some probabilistic laws for individual evacuee's movement are presented. Moreover ideas to model the evacuee's movement using cellular automata (CA) and resulting software are presented.
In this paper we will focus on macroscopic models and only summarize some of the results of the microscopic approach. While most of the results are applicable to general evacuation situations, we concentrate on building evacuation.
(44 S., 2001)

Stand: Juni 2001