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Vorwort

Das Tätigkeitsfeld des Fraunhofer Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüberhinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation darüber, wie aktuelle Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte transferiert werden, und wie umgekehrt Probleme der Praxis neue interessante mathematische Fragestellungen generieren.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Design of Zone Tariff Systems in Public Transportation

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Abstract

Given a public transportation system represented by its stops and direct connections between stops, we consider two problems dealing with the prices for the customers: The *fare problem* in which subsets of stops are already aggregated to zones and "good" tariffs have to be found in the existing zone system. Closed form solutions for the fare problem are presented for three objective functions. In the *zone problem* the design of the zones is part of the problem. This problem is NP hard and we therefore propose three heuristics which prove to be very succesful in the redesign of one of Germany's transportations systems.

1 Tariff systems in public transportation

In this paper we deal with the design of tariff systems in public transportation, a complex real-world problem, that was brought to our attention by a regional public transportation company several years ago. While working on the design of a fair tariff system we both developed a mathematical theory and a visualization tool to evaluate the effects of tariff changes. In this paper we present our studies and experience over the last years in this area.
When using a bus or a train, a passenger usually has to pay for his trip. There are several possiblities for defining ticket prices in public transportation.

- In a *distance tariff* system, the price for a trip is dependent on the length of the trip. The longer the trip is, the higher is the fare. This system is mostly considered as fair. To determine the ticket prices one needs the distance between each pair of stations. This makes a distance tariff inconvenient for the public transportation company and for the customers.

Figure 1: Zone tariff system with arbitrary prices

- The simplest tariff system is the *unit tariff*. In this case all trips cost the same, independent of their length. A unit tariff is very easy to handle. But it often is not accepted that a short trip between two neighbouring stations leads to the same ticket price as a long trip through the whole system.

- A model in between these two tariff systems is a *zone tariff system*. To establish a zone tariff, the whole area has to be divided into subregions (the *tariff zones*). The price for a trip in a zone tariff system is only dependent on the starting and the ending zone of the trip. If the price can be chosen arbitrarily for each pair of zones, we call the tariff system a *zone tariff with arbitrary prices*. An example for such a tariff system can, for instance, be found north of San Francisco, see Figure 1. The prices are given in form of a matrix, see Table 1.

  The most popular variant of a zone tariff system is the *counting zone tariff system*. To know his fare in this system, a customer has to count how

Table 1: Fares (in US-Dollar) for a one-way trip

zone	1	2	3	4	5	6	7	8	9	10
1	2.15									
2	2.35	1.50								
3	2.95	1.50	1.50							
4	3.55	1.50	1.50	1.50						
5	5.05	3.55	2.95	2.35	2.15					
6	5.70	4.10	3.55	2.95	2.15	2.15				
7	4.10	3.00	3.00	3.00	5.05	5.70	1.50			
8	4.75	3.00	3.00	3.00	4.45	5.85	3.00	1.50		
9	2.95	1.50	1.50	1.50	4.45	5.05	3.00	3.00	1.50	
10	4.75	2.95	2.35	2.95	4.45	5.05	4.75	4.10	3.55	1.45

Table 2: Fares by number of zones in journey for one-way trips

number of zones	1	2	3	4	5	6	7	8	9
fare in US-Dollar	1.25	2.00	2.75	3.25	4.00	4.75	5.25	6.00	6.75

many zones his trip will pass and read off the price assigned to the number of crossed zones. The prices in this system are dependent on the starting and the ending zone of the trip, but trips passing the same number of zones must have the same price. Figure 2 shows a counting zone tariff system south of San Francisco; the corresponding price for a single one-way trip can be read off Table 2.

Because of their simplicity, zone tariff systems are very popular. In Germany, nearly all tariff associations already have zone tariff systems or are currently introducing them. When a public transportation company wants to change its tariff system to a zone tariff, it has to design the zones and to fix the new fares, such that the resulting tariff system both is accepted by the customers and does not decrease the income of the company. The goal often is to design the zones in such a way, that the new and the old price for most of the trips are as close as possible. This means that neither the public transportation company nor the customers will have major disadvantages when changing the current tariff system to a zone tariff.

Another goal can be to design *fair zones*. In this case we do not consider the deviation to some old prices, but the deviation from a *reference price*, for instance one which is considered to be fair, like the distance tariff. In this approach, the public transportation company needs to estimate its new income.

Figure 2: Counting zone tariff system

In this paper we present a new optimization model for the zone tariff design problem. We focus on the case of counting zones. For the zone design problem with arbitrary prices we refer to [6, 12, 14, 1]. The remainder of the paper is organized as follows. Next, we present our model for the zone design problem with counting zones. Then we show how the fares for each number of crossed zones can be calculated easily by closed term formulas in Section 3. In Section 4 we present some important properties of the counting zone design problem and develop algorithmic approaches for designing good zones. We discuss their numerical behaviour in a real-world example in Section 5. In the end, some conclusions are given.

2 A model for the counting zone tariff

Let the *station graph* $G = (V, E)$ of the public transportation company be given, where V refers to the set of stops and $E \subseteq V \times V$ represents the available direct connections between pairs of stops. Furthermore, let d_{ij} be a reference price for traveling from station $i \in V$ to station $j \in V$. d_{ij} might be the current ticket price of the public transportation company, or it can be a fair price like a distance tariff.

If L denotes the number of planned zones, the *zone (planning) problem* identifies a partition

$$P = \{V_1, V_1, \dots, V_L\}$$

of V (i.e., $V_i \subseteq V, i = 1, 2, \dots, L$, pairwise disjoint and $\cup_{i=1}^{L} = V$). In the *fare (planning) problem* ticket prices

$$c(p), p = 0, 1, 2, \dots$$

are determined which are only dependent on the number of zones p in journey. Here $c(p)$ is the price for crossing p zone borders. In particular, $c(0)$ gives the fare for traveling within any zone, $c(1)$ is the price for crossing one zone border, i.e., for going from one zone to an adjacent one, and so on.

To evaluate some partition P with a price vector c we define for each pair of stations $i, j \in V$ n_{ij} as the number of passed zone borders when traveling from station i to station j. (In order to add to the confusion most public transportation companies count the number n'_{ij} of passed zones on the trip from station i to station j, including both the starting and the ending zone, i.e., $n'_{ij} = n_{ij} + 1$. We prefer our denotation for simplicity of our model.) The new ticket price for traveling from i to j is then given by

$$z_{ij} = \begin{cases} c(n_{ij}) & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases} .$$

Given the reference prices d_{ij} for a trip between stations i and j, the absolute deviation in ticket price is calculated by

$$|d_{ij} - z_{ij}| = |d_{ij} - c(n_{ij})| \text{ for all } i, j \in V.$$

Let w_{ij} be the number of customers traveling from station i to station j and let $W = \sum_{i,j \in V} w_{ij}$ be the sum of all customers of the public transportation company. The minimization of the following three objective functions is of interest.

maximum absolute deviation $b_{\max} = \max_{i,j \in V} w_{ij}|d_{ij} - z_{ij}|$

average absolute deviation $b_1 = \frac{1}{W} \sum_{i,j \in V} w_{ij}|d_{ij} - z_{ij}|$

average squared deviation $b_2 = \frac{1}{W} \sum_{i,j \in V} w_{ij}(d_{ij} - z_{ij})^2$

All three objectives lead to good results in practice. The first objective function, $b_{\max}$ with identical weights models the fact, that the greatest deviation of ticket prices in the two different tariffs should be as small as possible. It gives a bound for the largest change in the ticket price for any customer. In the weighted case, $b_{\max}$ minimizes the maximum deviation in income over all possible trips. b_1 gives the average of all absolute deviations, and b_2 the average of all squared deviations in ticket prices. The objective function b_2 leads to a smaller percentage of strongly affected customers than b_1. Nevertheless, from our experience, b_1 is slightly better accepted by the practitioners than b_2. It also should be mentioned that deviations in price increases and decreases are treated equally, such that the model reflects both the interests of customers and transportation companies.

To obtain the numbers n_{ij} a shortest path algorithm, e.g., [3, 16] can be used according to one of the following models.

Station Graph Model: We use the station graph $G = (V, E)$, but introduce new weights u_{ij} for all $(i, j) \in E$, defined by

$$u_{ij} = \begin{cases} 0 & \text{if } i \text{ and } j \text{ are in the same zone} \\ 1 & \text{if } i \text{ and } j \text{ are in adjacent zones.} \end{cases}$$

The length of a shortest path between two stops equals the minimum number of crossed zone borders. This approach will be needed later to update the zone distances in the greedy heuristic in Section 4.

Zone Graph Model: To reduce the size of the network we define the *zone graph* $G' = (P, E')$ whose node set P is given by the zones and $(V_k, V_l) \in E'$ iff there exist stops $i \in V_k$, $j \in V_l$ such that $(i, j) \in E$, i.e., with a direct connection in the station graph G. All edges have weight 1. For $i \in V_k$ and $j \in V_l$ we get the minimum number of crossed zone borders n_{ij} on a trip from i to j as the length of a shortest path from V_k to V_l in G'.

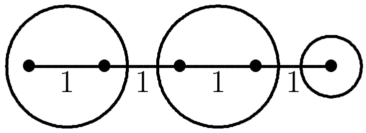

Figure 3: A station network with 5 stations and 3 zones.

The following example demonstrates the calculation of $b_{\max}$, b_1, and b_2.
Let a station graph G with a partition into three zones $V_1 = \{1, 2\}$, $V_2 = \{3, 4\}$, and $V_3 = \{5\}$ be given (see Figure 3).
Suppose that $w_{ij} = 1$ for all $i, j \in V, i \neq j$, i.e., $W = 20$. If we assume that the distance between any adjacent pair of nodes is 1, the matrix d_{ij} according to the distance tariff system may be

$$D = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 2 & 3 \\ 2 & 1 & 0 & 1 & 2 \\ 3 & 2 & 1 & 0 & 1 \\ 4 & 3 & 2 & 1 & 0 \end{pmatrix}.$$

The corresponding zone graph G' consists of three nodes (see Figure 4).

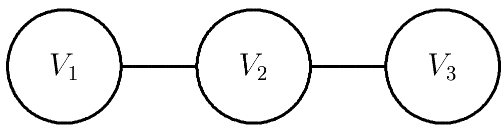

Figure 4: Zone graph with 3 zones.

The number of crossed zone borders between stations i and j is then given by

$$N = \begin{pmatrix} 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 2 & 2 & 1 & 1 & 0 \end{pmatrix}.$$

Suppose the new fares for crossing $p = 0, 1$, or 2 zone borders are given by

$$\begin{aligned} c(0) &= 0.5 \\ c(1) &= 1 \\ c(2) &= 1.5. \end{aligned}$$

Then new ticket prices can be calculated as

$$Z = \begin{pmatrix} 0 & 0.5 & 1 & 1 & 1.5 \\ 0.5 & 0 & 1 & 1 & 1.5 \\ 1 & 1 & 0 & 0.5 & 1 \\ 1 & 1 & 0.5 & 0 & 1 \\ 1.5 & 1.5 & 1 & 1 & 0 \end{pmatrix}.$$

The deviations between the reference prices d_{ij} and the new ticket prices z_{ij} are

$$D - Z = \begin{pmatrix} 0 & 0.5 & 1 & 2 & 2.5 \\ 0.5 & 0 & 0 & 1 & 1.5 \\ 1 & 0 & 0 & 0.5 & 1 \\ 2 & 1 & 0.5 & 0 & 0 \\ 2.5 & 1.5 & 1 & 0 & 0 \end{pmatrix}$$

and the objective values can be calculated as

$$\begin{aligned} b_{\max} &= 2.5 \\ b_1 &= \frac{20}{20} = 1 \\ b_2 &= \frac{32}{20} = 1.6 \end{aligned}$$

3 Solution for the Fare Problem with Fixed Zones

In this section we solve the fare problem with respect to a given zone partition. Our first result shows that a closed form solution is possible for each of the three objectives $b_{\max}, b_1$, and b_2 introduced in Section 2.

Theorem 1 *Let $P = \{V_1, V_2, \ldots, V_L\}$ be a given zone partition and let d_{ij} be given reference prices. In order to minimize $b_{\max}$, b_1, and b_2 we choose for all $p = 0, 1, \ldots, L$*

a)

$$c^*_{\max}(p) := \max_{\substack{i,j \in V: i \neq j, \\ n_{ij} = p}} d_{ij} - \frac{z^*_p}{w_{ij}}$$

where z^*_p is defined as

$$z^*_p = \max_{\substack{i_1, j_1, i_2, j_2 \in V: \\ n_{i_1 j_1} = n_{i_2 j_2} = p}} \frac{w_{i_1 j_1} w_{i_2 j_2}}{w_{i_1 j_1} + w_{i_2 j_2}} (d_{i_1 j_1} - d_{i_2 j_2}).$$

b)

$$c_1^*(p) := \text{median}\{\underbrace{d_{ij}, \ldots, d_{ij}}_{w_{ij} times} : i,j \in V : i \neq j, n_{ij} = p\}$$

c)

$$c_2^*(p) := \frac{1}{W} \sum_{i,j \in V: n_{ij} = p} w_{ij} d_{ij}$$

Proof: Given the zone partition P we have to find fares $c(p) \in \mathbb{R}$ for all $p = 0, 1, \ldots,$ minimizing $b_{\max}$, b_1, and b_2, respectively. Define

$$M_p = \{(i,j) : i,j \in V \text{ and } n_{ij} = p\}$$

and $W_p = \sum_{m \in M_p} w_m$ as the sum of all weights belonging to pairs of stations in the set M_p. First we note that each of the three objective functions can be separated into at most $L+1$ independent subproblems, $K_{\max}(p)$, $K_1(p)$, and $K_2(p)$, respectively (for $p = 0, 1, \ldots, L$).

$$\begin{aligned}
b_{\max} &= \max_{i,j \in V} w_{ij} |d_{ij} - z_{ij}| \\
&= \max_{p=0,1,\ldots,L} \max_{m \in M_p} w_m |d_m - c(p)| =: \max_{p=0,1,\ldots,L} K_{\max}(p) \\
b_1 &= \sum_{i,j \in V} w_{ij} |d_{ij} - z_{ij}| \\
&= \sum_{p=0}^{L} \sum_{m \in M_p} w_m |d_m - c(p)| =: \sum_{p=0}^{L} K_1(p) \\
b_2 &= \sum_{i,j \in V} w_{ij} (d_{ij} - z_{ij})^2 \\
&= \sum_{p=0}^{L} \sum_{m \in M_p} w_m (d_m - c(p))^2 =: \sum_{p=0}^{L} K_2(p)
\end{aligned}$$

Consequently, to minimize $b_{\max}$, b_1, and b_2 we determine the optimal fare $c(p)$ for $p = 0, 1, \ldots, L$ separately, in each of the three objective functions.

For $b_{\max}$: For all $p = 0, 1, \ldots, L$ the problem of minimizing

$$K_{\max}(p) = \max_{m \in M_p} w_m |d_m - c(p)|$$

is well-known from location theory when locating a point on a line such that the maximum distance to a given set of existing facilities on the same line is minimized. The proof for the formula given in part [a] of the theorem can therefore be found in the location literature, see e.g. [8, 5]. Note that $K_{\max}(p) = z_p^*$.

For b_1**:** Since

$$K_1(p) = \min \sum_{m \in M} w_m |d_m - c(p)|$$

is a one-dimensional, piecewise linear and convex fuction, its minimization is known in statistics (see e.g., [7]) and in location theory as the one-dimensional median problem (see, e.g. [5, 10]). It is shown that the above problem is solved by the so-called *weighted median* of the set $\{d_m : m \in M_p\}$, i.e., by any real number $c = c_1^*(p)$ which satisfies

$$\sum_{m: d_m < c} w_m \leq \frac{W_p}{2} \text{ and}$$
$$\sum_{m: d_m > c} w_m \leq \frac{W_p}{2}.$$

For b_2**:** Here we have to minimize $K_2(p)$, i.e.,

$$\min \sum_{d \in M} w_d (d - c(p))^2.$$

Using the theorem of Steiner (see e.g. [11]) of statistics, we note that the weighted mean of the values in $\{d_m : m \in M_p\}$ is the unique optimal solution for $c(p)$.

To demonstrate the result of Theorem 1 we continue the example of Section 2 The optimal values for the zone prices and the resulting values for the objective functions $b_{\max}$, b_1, and b_2 are listed in the following table.

zones	$c_{\max}$	c_1	c_2	example
0	1	1	1	0.5
1	2	2	$\frac{11}{6}$	1
2	3.5	3	3.5	1.5
$b_{\max}$	**1**	1	1.167	2.5
b_1	8	**8**	8.667	20
b_2	7	8	**6.722**	32

Calculating the objective function by using the optimal fares according to Theorem 1 yields the following Corollary.

Corollary 1 *Given a zone partition* $P = \{V_1, V_2, \ldots, V_L\}$ *and reference prices* d_{ij} *the optimal values of the objective functions are given as follows.*

a)

$$b^*_{\max} = \max_p z^*_p$$

b)

$$b^*_1 = \sum_p \left(\sum_{(i,j)\in V_p^+ : n_{ij}=p} d_{ij} - \sum_{(i,j)\in V_p^- : n_{ij}=p} d_{ij} \right)$$

where:
$V_p^+ := \{(i,j) : n_{ij} = p, d_{ij} > c^*_1(p)\}$
$V_p^- := \{(i,j) : n_{ij} = p, d_{ij} < c^*_1(p)\}$

c)

$$b^*_2 = \sum_k \mathrm{Var}\{d_{ij} : i,j \in V, n_{ij} = p\}$$

where Var denotes the variance of the set.

In practice, often restrictions on the new fares are given; sometimes there even exist "politically" desired fares for the number of zones in journey that have to be realized. With the help of Corollary 1 one can easily calculate the increase of the objective functions when using such given fares instead of the optimal ones. In particular, Corollary 1 shows that for the objective function $b_{\max}$ the optimal fares $c^*_{max}(p)$ are not needed to calculate the optimal objective value for a given zone partition. This will be needed in the next section when we are going to optimize the zone partition with respect to $b_{\max}$. If, additionally, $b_{\max}$ is used in the unweighted case, i.e. with $w_{ij} = 1$ for all $i,j \in V$, we can further simplify Theorem 1 and Corollary 1.

Corollary 2 *In the case of equal weights, the optimal fares $c^*_{\max}(p)$ and the corresponding objective value $b_{\max}$ are given by*

$$c^*_{\max}(p) = \frac{1}{2}\left(\max_{i,j\in V : n_{ij}=p} d_{ij} + \min_{i,j\in V : n_{ij}=p} d_{ij} \right)$$

$$K_{\max}(p) = \frac{1}{2}\left(\max_{i,j\in V : n_{ij}=p} d_{ij} - \min_{i,j\in V : n_{ij}=p} d_{ij} \right)$$

$$b^*_{\max} = \frac{1}{2} \max_{p=1,\dots,L} \left(\max_{i,j\in V : n_{ij}=p} d_{ij} - \min_{i,j\in V : n_{ij}=p} d_{ij} \right)$$

Proof: We calculate z_p^* as

$$\begin{aligned} z_p^* &= \max_{m_1,m_2\in M_p} \frac{w_{m_1}w_{m_2}}{w_{m_1}+w_{m_2}}(d_{m_1}-d_{m_2}) \\ &= \frac{1}{2}\left(\max_{m\in M_p} d_m - \min_{m\in M_p} d_m\right) \end{aligned}$$

and consequently,

$$\begin{aligned} c_p^* &= \max_{m\in M_p} d_m - \frac{z_p^*}{w_m} \\ &= \max_{m\in M_p}\left(d_m - \frac{1}{2}\max_{\tilde{m}\in M_p} d_{\tilde{m}} + \frac{1}{2}\min_{\tilde{m}\in M_p} d_{\tilde{m}}\right) \\ &= \frac{1}{2}\left(\max_{m\in M_p} d_m + \min_{m\in M_p} d_m\right) \end{aligned}$$

Using Corollary 1 and $z_p^* = K_{\max}(p)$, the remaining parts follow immediately.

QED

We remark that for the zone design problem with arbitrary prices, similar results can be derived (see [6, 13]).

4 Finding zone partitions for maximum deviation problems

The consequence of the results of Section 3 is that we can concentrate on finding the zones, since the zone pricing follows easily from the choice of the objective function. We now focus our attention to the maximum deviation problem. Unfortunately, this problem is NP-hard and therefore difficult to solve.
A first observation deals with the monotonicity of the objective function dependent on the number of planned zones L. While it is easy to see that for the zone design problem with arbitrary prices all three objectives are monotone in L, this is not true for the zone design problem with counting zones, as Figure 5 shows. The station network consists of 8 nodes, and we assume that $w_{ij} = 1$ for all pairs of nodes i, j. The reference prices are given as weights between any two adjacent nodes, as shown in the figure. Between any other pair of nodes the reference prices are given as the sum of the weights along a shortest path connecting the nodes. For the (unweighted) max absolute deviation problem, Corollary 2 shows that any solution with $L = 5$ zones leads to a strictly higher objective value than the graphed solution with $L = 4$ and $b_{\max} = 1$. We will therefore fix L in the following.

Theorem 2 *The zone design problem with counting zones and objective function $b_{\max}$ is NP-hard for all fixed $L \geq 3$.*

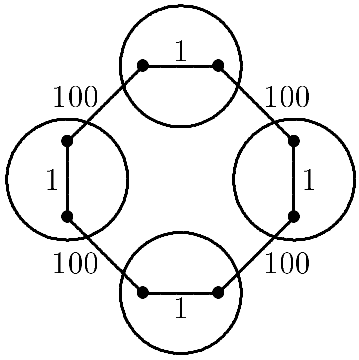

Figure 5: A station network where the objective value for $L = 4$ is better than the objective value for $L = 5$.

The proof of Theorem 2 is given in the appendix. Note that also the zone design problem with arbitrary prices is NP-hard, see [1].
To motivate the heuristics of this section, we first present the following two observations for getting upper and lower bounds on the objective value $b_{\max}$.

Lemma 1

$$b_{\max} \leq \frac{1}{2}\left(\max_{i,j\in V} d_{ij} - \min_{i,j\in V} d_{ij}\right)$$

Proof: For any zone partition P and any integer p we have that

$$\begin{aligned} K_{\max}(p) &= \frac{1}{2}\left(\max_{i,j\in V, n_{ij}=p} d_{ij} - \min_{i,j\in V, n_{ij}=p} d_{ij}\right) \\ &\leq \frac{1}{2}\left(\max_{i,j\in V} d_{ij} - \min_{i,j\in V} d_{ij}\right), \end{aligned}$$

yielding the result. QED

Lemma 2 *Given a zone partition P let $\mathrm{INT} \subseteq E$ be the set of edges with both end nodes within the same zone and $\mathrm{BET} = E \setminus \mathrm{INT}$. Then we have:*

$$1.\ b_{\max} \geq \frac{1}{2}\left(\max_{(i,j)\in \mathrm{INT}} d_{ij} - \min_{(i,j)\in \mathrm{INT}} d_{ij}\right)$$

$$2.\ b_{\max} \geq \frac{1}{2}\left(\max_{(i,j)\in \mathrm{BET}} d_{ij} - \min_{(i,j)\in \mathrm{BET}} d_{ij}\right)$$

Proof:

1. Since $\min_{i,j \in V, n_{ij}=0} d_{ij} = \min_{(i,j)\in \text{INT}} d_{ij}$ we get

$$\begin{aligned} b_{\max} &\geq K_{\max}(0) \\ &\geq \frac{1}{2}\left(\max_{i,j\in V, n_{ij}=0} d_{ij} - \min_{i,j\in V, n_{ij}=0} d_{ij} \right) \\ &\geq \frac{1}{2}\left(\max_{(i,j)\in \text{INT}} d_{ij} - \min_{(i,j)\in \text{INT}} d_{ij} \right) \end{aligned}$$

2. Analogously, $\min_{i,j\in V, n_{ij}=1} d_{ij} = \min_{(i,j)\in \text{BET}} d_{ij}$ and we get the result by using $b_{\max} \geq K_{\max}(1)$. QED

Lemma 2 suggests a zone design in which edges with high weights are collected in BET and edges with small weights in INT or vice versa. To be more specific, let D be the maximal diameter over all zones. Assuming that edge weights along a path are additive, we get

$$\begin{aligned} K_{max}(p) &= \frac{1}{2}\left(\max_{i,j\in V, n_{ij}=p} d_{ij} - \min_{i,j\in V, n_{ij}=p} d_{ij} \right) \\ &\leq \frac{1}{2}\left((p+1)D + p\left(\max_{(i,j)\in \text{BET}} d_{ij} - \min_{(i,j)\in \text{BET}} d_{ij} \right)\right) \end{aligned}$$

yielding that the maximal diameter D should be small, and consequently edges with large weights should be in BET while edges with small weights should be in INT.

Following these considerations, we present three heuristics for solving the zone design problem with counting zones. As **Input Data** we need — for any of the following algorithms — a set of n stations with reference prices d_{ij} and a number L of planned zones. The **Output** is then given by a zone partition with L zones.

Algorithms based on Clustering Theory

The first algorithm is based on ideas from clustering theory and here in particular on the SAHN (sequential agglomerative hierarchical non-overlapping) algorithms, see e.g., [2]). The idea is to start with n zones, each of them containing one single station and to combine in each step the two closest zones to a new one. Depending on the particular definition of the distance between two zones, different algorithms can be obtained. Two of them have been applied to the zone design problem: *Single Linkage* and *Complete Linkage*.

ALGORITHM 1 (Zone design using SAHN-algorithms)

1. Start with a partition P consisting of n zones each of which containing a single station. Let $d(C_i, C_j) := d_{ij}$ for all zones $C_i, C_j \in P$.

2. Determine two zones $C_i \neq C_j \in P$ with minimum distance $d(C_i, C_j)$.

3. Join C_i and C_j to a new zone C_k and get a new partition P.

4. Calculate the new distances for all $C \in P$:
$d(C_k, C) := \frac{1}{2}\left(d(C_i, C) + d(C_j, C) + \mathsf{c}|d(C_i, C) - d(C_j, C)|\right)$

5. If the number of planned zones is attained, then Stop, Output P,
else goto Step 2.

The parameter c in Step 4 determines the formula for calculating the distance between two zones. In the context of the zone design problem, we have used

- $\mathsf{c} = -1$ for the Single Linkage algorithm and

- $\mathsf{c} = 1$ for the Complete Linkage algorithm.

The interpretation for Single Linkage is the following: The distance between two zones is defined as the smallest distance between elements of them, and consequently in each step we join along a shortest edge. Note that, in the Complete Linkage the distance between two zones is defined as the maximum distance between their elements, and in each step complete linkage tries to minimize the maximum diameter of the zones.

Greedy Approach

This approach is a variant of the SAHN algorithms discussed above, but with more emphasis on the specific structure of the zone design problem. Using the basics of Algorithm 1, we calculate for all edges (i, j) the objective value $b_{\max}^{ij}$ when contracting (i, j) of the current zone graph. Finally, we contract the edge with smallest increase in the objective function. This is rather time consuming, but as we will show in the next section, leads to very good results in practice. The formulation of the greedy approach is the following:

ALGORITHM 2 (Zone design by Greedy approach)

1. For all edges $(i, j) \in E$ with $n_{ij} = 1$:
Contract i and j temporally and calculate $b_{\max}^{ij}$.

2. Contract the edge (i^0, j^0) permanently, where

$$b_{\max}^{i^0,j^0} = \min\{(i,j) : b_{\max}^{ij}\},$$

and let $n_{i^0 j^0} = 0$. If the graph has L nodes, Stop.

3. For all $i, j \in V$ recalculate n_{ij} as shortest distance, and goto Step 1.

Spanning Tree Approach

The idea of the following heuristic is to determine a set of edges BET which contains mostly edges with high weights.

ALGORITHM 3 (Zone design by spanning tree approach)

1. Find a maximum spanning tree T in the complete graph with edge weights d_{ij}.

2. Omit the $L - 1$ largest edges of T and get a forest with L components.

3. Output: Zones are the connected components.

Note that in trees, the spanning tree approach is equivalent to the single linkage algorithm of clustering theory. In general graphs, it is always possible to find a spanning tree such that omitting its $L - 1$ largest edges leads to the same result as Single Linkage. However, if we start with a spanning tree with maximal weight (which performed best in practice) the spanning tree approach differs significantly from Single Linkage.

5 Theory Put to Work in Saarland, Germany

As an example for the practical value of our approach, we consider the situation in the State of Saarland, Germany. Currently, there are six public transportation companies operating in the Saarland, each of them with its own tariff system.

- Four public transportation companies already use a counting zone tariff system, but their fares for crossing p zones and the structure of their zones are completely different, although they are partly operating in the same geographical region.

- The Deutsche Bahn (German Rail) still applies its distance tariff.

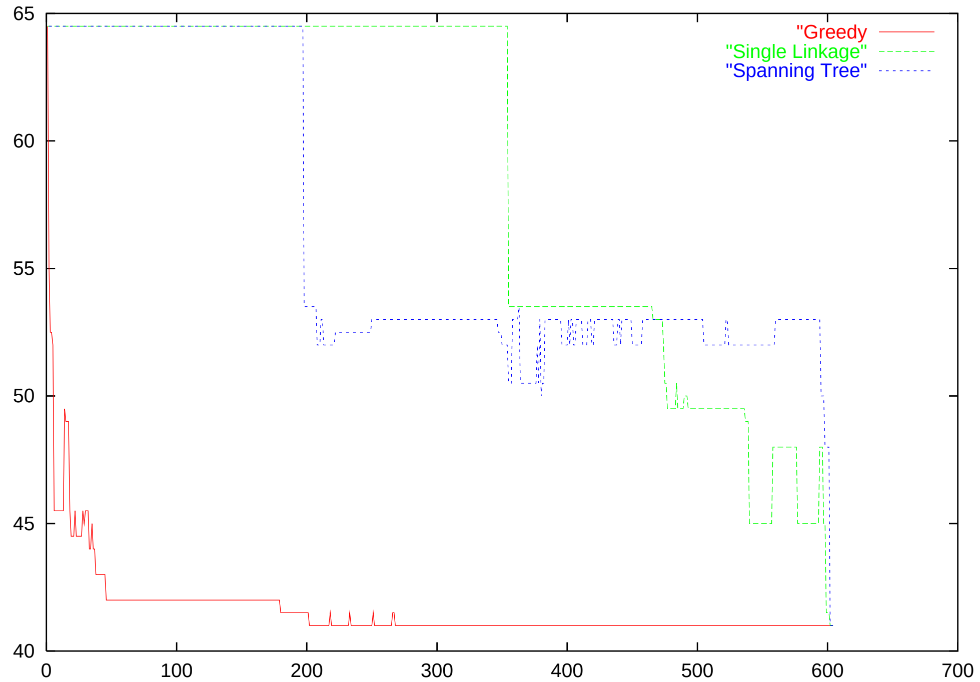

Figure 6: Comparison of the heuristics: $b_{\max}$ graphed for any number L of planned zones.

- There is also a public transportation company (serving the city of Saarland's capitol, Saarbrücken) which uses a zone tariff with variable prices.

The traffic association of the Saarland considers to introduce one common counting zone tariff system which should be applied by all the public transportation companies. The public transportation network in the Saarland consists of roughly 4000 stations, where a pre-clustering into 600 mini-zones is given. The goal is to design about 100 zones and install a counting zone tariff systems in such a way that the difference between the current fares and the new ones is as small as possible. It is also important, that the new income of each of the public transportation companies should not differ too much from the current income. Consequently, the reference prices in this application are the current prices for travelling. While the current fare structure is known and therefore relatively easy to get it is usually hard to get realistic data about the customers' behaviour. In our project in the Saarland this was solved by using the income data of each of the transportaion companies and to divide the income with the help of available statistics over the origin-destination pairs used by the customers.

We tested our algorithms on the data described above. The results of Algorithms 1,2, and 3 are shown in Figure 6. This figure shows the objective value $b_{\max}$

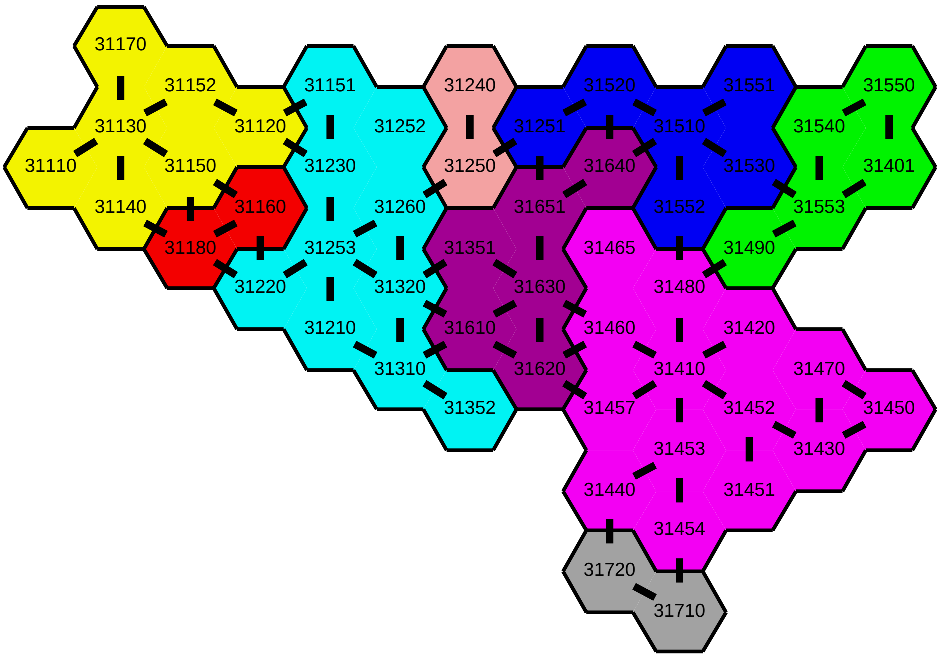

Figure 7: Political suggestion; $b_{\max} = 5.15$ DM

for any number of possible zones from 1 to 600. All objective values refer to a single trip ticket for an adult, given in German Marks (DM). It turns out that in this practical application the greedy heuristic (Algorithm 2) is the clear winner in terms of the objective value: it generated the best results for any number of desired zones. On the other hand, the running time for Algorithm 2 for all possible number of zones, i.e. from $L = 1, \ldots, 600$ was nearly two weeks alltogether in our first implementation (on a AixJ90). The Spanning Tree Approach (Algorithm 3) and the Single Linkage Algorithm (Algorithm 1) both needed only a few hours, but the results are much less convincing regarding the objective value $b_{\max}$ again. For a small number L of desired zones, Single Linkage did better than the Spanning Tree approach, while for a higher number of planned zones it was the other way round. This is due to the fact that the Spanning Tree Approach starts with only one zone, while Single Linkage starts with 600 zones.

On a subset consisting of only 400 stations (or 54 mini-zones) the heuristics have also been tested. In this smaller setting the running times of Algorithms 1 and 3 were within seconds, and also Algorithm 2 needed only 2 minutes to obtain again the clearly best results. The results for 9 zones are shown graphically in Figures 7,8, and 9. Figure 7 shows a suggestion for a zone partition which is due to the political districts in this area of the Saarland. The objective value for this zone partition is $b_{\max} = 5.15$ DM, i.e. there exists a customer which will have a

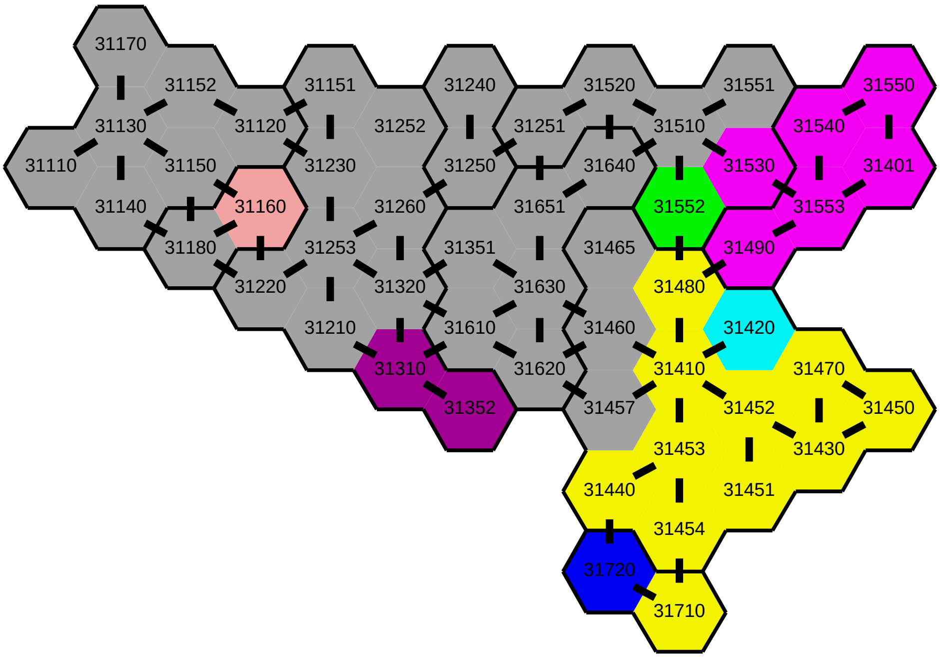

Figure 8: Solution of Single Linkage; $b_{\max} = 5.00$ DM

difference of 5.15 DM between his current fare and the new one. The result of the Single Linkage Algorithm for 9 zones is shown in Figure 8. As it is reported also in literature (see, e.g., [2]), Single Linkage tends to form one large zone and a lot of smaller zones surrounding it. This behavior is also shown in Figure 8. The objective value of the
graphed zone partition is 5.00 DM. The objective value in the Spanning Tree approach also was 5.00 DM for 9 zones, but without these big differences in the sizes of the zones. The best results, however, were obtained by the Greedy Approach with an objective value of only 3.75 DM. The corresponding zone partition is shown in Figure 9.

For evaluating tariff zones in more detail, we use the software package *WabPlan* [15]. A graphical front-end provides a detailed analysis of all trips for which the fare will increase or decrease dramatically (see Figure 10). Furthermore, the expected income for each of the transportation companies in each ticket category is compared with its current income.

For practical purposes a lot of special rules for using fare zones are common. A lot of them have also been implemented in our algorithms and tested on the data of the Saarland.

Empty zones : First of all, in most zone tariff systems, *empty zones* are used to increase the fare on some special trips without affecting all other relations.

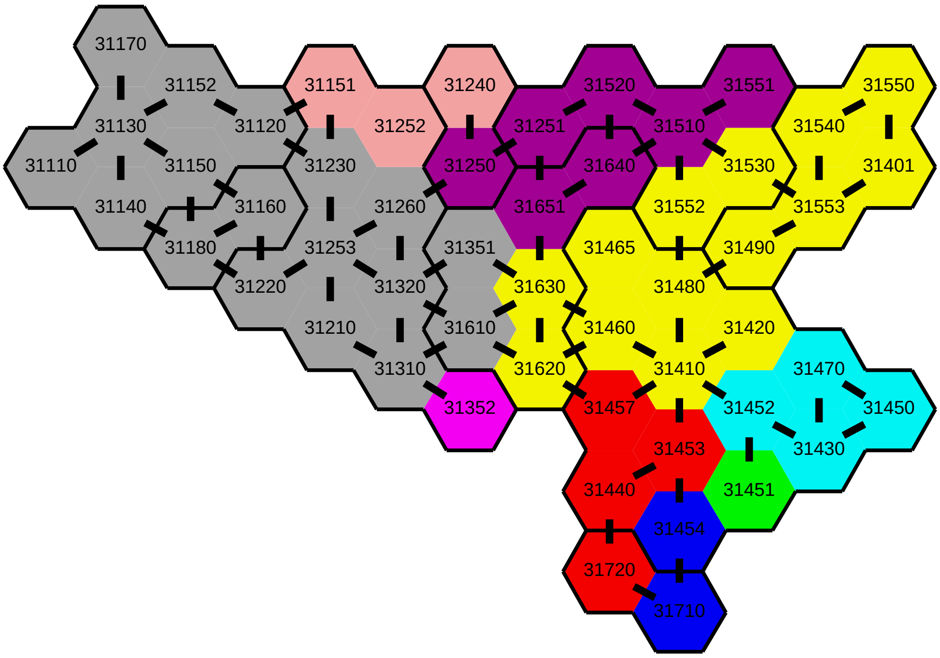

Figure 9: Solution of Greedy algorithm; $b_{\max} = 3.75$ DM

This seems to make sense in practice and can easily be incorporated in the algorithms presented in Section 4. In this way, given reference prices can be approximated arbitrarily close if the number L of zones is large enough. In our model this means that the optimal objective value goes to zero for $b_{\max}, b_1$, and b_2 in this case.

Border stations : To avoid injustice, stations can be located on zone borders, meaning that they belong to more than one zone, and apply the cheapest choice for determining the fare. Since the zone tariff system should be clear and understandable, we tried to avoid this in the Saarland. In most cases it turned out that border stations can be avoided without loosing anything in the objective values only by changig the zone design.

Special rules for large zones : Also, some zones might be so large that they have to be counted twice when crossing them, and that a special fare structure has to be implemented within such a large zone.

In the Saarland, the tariff system proposed by using our methods is now in the implementation process.

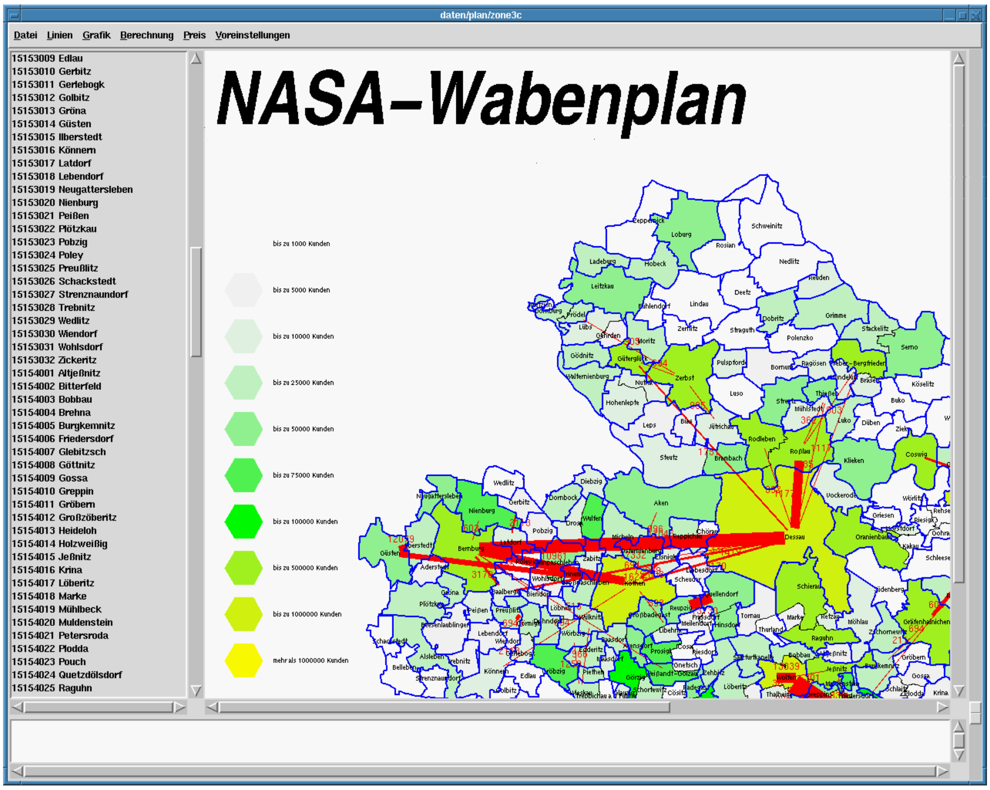

Figure 10: Zone planning with the software *WabPlan* in the state Sachsen-Anhalt. The red lines show the (fictional) customers which will have a change in their ticket price which is more than 5 Percent.

6 Conclusion

We have shown that closed form solutions can be provided for the fare problem (with fixed zones). In contrast, the zone problem, i.e. the design of zones, is NP-hard. Three heuristics have been proposed and compared with respect to their numerical behaviour. The practical usefulness of the approach is shown by its actual implementation in the state Saarland, Germany. Other states of Germany are currently using our system to evaluate their tariff structure.

Appendix

Proof of Theorem 2

We use a reduction to the problem "partition into cliques" for $L = 3$ Cliques, which is NP-hard, see problem GT15 in [4]. Let a graph $G = (V, E)$ be given. We want to answer the question, if there exists a partition of V into 3 node sets

V_1, V_2, and V_3 such that the induced subgraphs G_1, G_2, and G_3 are complete. To reduce this problem to a zone design problem, we consider the complete graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, defined by

$$\mathcal{V} = V \cup \{a_1, a_2, a_3, b_1, b_2, b_3\} \text{ and}$$

$$\mathcal{E} = \{e = (k,l) : k, l \in \mathcal{V}\}, k \neq l.$$

Furthermore, let $w_{ij} = 1$ for all $i, j \in \mathcal{V}$ and define the reference prices as the following edge weights.

$$d_{kl} = \begin{cases} 1 & \text{if } (k,l) \in E \\ 1 & \text{if } k \in V, l \notin V \\ \frac{1}{2} & \text{if there exists } i = 1,2,3 \text{ such that } (k,l) = (a_i, b_i) \\ 2 & \text{if } k, l \in V, (k,l) \notin E \\ 2 & \text{if } k, l \notin V, (k,l) \neq (a_i, b_i) \text{ for all } i = 1,2,3. \end{cases}$$

Now we show that G can be partitioned into 3 cliques if and only if the zone design problem in G has a solution with 3 zones and with $b_{\max} < \frac{3}{4}$.

$\Longrightarrow$: Let $V = V_1 \cup V_2 \cup V_3$ the partition of G into cliques. Define $C_i = V_i \cup \{a_i, b_i\}$ for $i = 1, 2, 3$. Using Corollary 2 and 1 we calculate

$$\begin{aligned} K_{\max}(0) &= \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4} \\ K_{\max}(1) &= \frac{1}{2}(2-1) = \frac{1}{2}, \end{aligned}$$

such that we get $b_{\max} < \frac{3}{4}$.

$\Longleftarrow$: Let C_1, C_2, C_3 be a partition of $\mathcal{V}$ with $b_{\max} < \frac{3}{4}$. Define $V_i = C_i \cap V$ for $i = 1, 2, 3$. First we note, that by renaming the C_i we can assume that $a_i, b_i \in C_i$ for $i = 1, 2, 3$. This can be proved by a simple case analysis (see [9]) which verifies the following.

1. If a_i and b_i do not belong to the same zone this yields $b_{\max} \geq \frac{3}{4}$.
2. If a_i, b_i and another a_j or b_j, $j \neq i$ belong to a single common zone, then $b_{\max} \geq \frac{3}{4}$.

Now let $k, l \in V_i$. We have to show that the edge $(k,l) \in E$. Assume the contrary, i.e., $d_{kl} = 2$. But this yields

$$K_{\max}(0) \geq \frac{1}{2}\left(2 - \frac{1}{2}\right) = \frac{3}{4},$$

(again using Corollary 2 and 1), a contradiction.

For more than 3 zones, the proof can be done analogously with a reduction to partition into $L > 3$ cliques. QED

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1. D. Hietel, K. Steiner, J. Struckmeier
A Finite - Volume Particle Method for Compressible Flows

We derive a new class of particle methods for conservation laws, which are based on numerical flux functions to model the interactions between moving particles. The derivation is similar to that of classical Finite-Volume methods; except that the fixed grid structure in the Finite-Volume method is substituted by so-called mass packets of particles. We give some numerical results on a shock wave solution for Burgers equation as well as the well-known one-dimensional shock tube problem.
(19 S., 1998)

2. M. Feldmann, S. Seibold
Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing

In this paper, a combined approach to damage diagnosis of rotors is proposed. The intention is to employ signal-based as well as model-based procedures for an improved detection of size and location of the damage. In a first step, Hilbert transform signal processing techniques allow for a computation of the signal envelope and the instantaneous frequency, so that various types of non-linearities due to a damage may be identified and classified based on measured response data. In a second step, a multi-hypothesis bank of Kalman Filters is employed for the detection of the size and location of the damage based on the information of the type of damage provided by the results of the Hilbert transform.
Keywords:
Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 S., 1998)

3. Y. Ben-Haim, S. Seibold
Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery

Damage diagnosis based on a bank of Kalman filters, each one conditioned on a specific hypothesized system condition, is a well recognized and powerful diagnostic tool. This multi-hypothesis approach can be applied to a wide range of damage conditions. In this paper, we will focus on the diagnosis of cracks in rotating machinery. The question we address is: how to optimize the multi-hypothesis algorithm with respect to the uncertainty of the spatial form and location of cracks and their resulting dynamic effects. First, we formulate a measure of the reliability of the diagnostic algorithm, and then we discuss modifications of the diagnostic algorithm for the maximization of the reliability. The reliability of a diagnostic algorithm is measured by the amount of uncertainty consistent with no-failure of the diagnosis. Uncertainty is quantitatively represented with convex models.
Keywords:
Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 S., 1998)

4. F.-Th. Lentes, N. Siedow
Three-dimensional Radiative Heat Transfer in Glass Cooling Processes

For the numerical simulation of 3D radiative heat transfer in glasses and glass melts, practically applicable mathematical methods are needed to handle such problems optimal using workstation class computers. Since the exact solution would require super-computer capabilities we concentrate on approximate solutions with a high degree of accuracy. The following approaches are studied: 3D diffusion approximations and 3D ray-tracing methods.
(23 S., 1998)

5. A. Klar, R. Wegener
A hierarchy of models for multilane vehicular traffic
Part I: Modeling

In the present paper multilane models for vehicular traffic are considered. A microscopic multilane model based on reaction thresholds is developed. Based on this model an Enskog like kinetic model is developed. In particular, care is taken to incorporate the correlations between the vehicles. From the kinetic model a fluid dynamic model is derived. The macroscopic coefficients are deduced from the underlying kinetic model. Numerical simulations are presented for all three levels of description in [10]. Moreover, a comparison of the results is given there.
(23 S., 1998)

Part II: Numerical and stochastic investigations

In this paper the work presented in [6] is continued. The present paper contains detailed numerical investigations of the models developed there. A numerical method to treat the kinetic equations obtained in [6] are presented and results of the simulations are shown. Moreover, the stochastic correlation model used in [6] is described and investigated in more detail.
(17 S., 1998)

6. A. Klar, N. Siedow
Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes

In this paper domain decomposition methods for radiative transfer problems including conductive heat transfer are treated. The paper focuses on semi-transparent materials, like glass, and the associated conditions at the interface between the materials. Using asymptotic analysis we derive conditions for the coupling of the radiative transfer equations and a diffusion approximation. Several test cases are treated and a problem appearing in glass manufacturing processes is computed. The results clearly show the advantages of a domain decomposition approach. Accuracy equivalent to the solution of the global radiative transfer solution is achieved, whereas computation time is strongly reduced.
(24 S., 1998)

7. I. Choquet
Heterogeneous catalysis modelling and numerical simulation in rarified gas flows
Part I: Coverage locally at equilibrium

A new approach is proposed to model and simulate numerically heterogeneous catalysis in rarefied gas flows. It is developed to satisfy all together the following points:
1) describe the gas phase at the microscopic scale, as required in rarefied flows,
2) describe the wall at the macroscopic scale, to avoid prohibitive computational costs and consider not only crystalline but also amorphous surfaces,
3) reproduce on average macroscopic laws correlated with experimental results and
4) derive analytic models in a systematic and exact way.
The problem is stated in the general framework of a non static flow in the vicinity of a catalytic and non porous surface (without aging). It is shown that the exact and systematic resolution method based on the Laplace transform, introduced previously by the author to model collisions in the gas phase, can be extended to the present problem. The proposed approach is applied to the modelling of the Eley-Rideal and Langmuir-Hinshelwood recombinations, assuming that the coverage is locally at equilibrium. The models are developed considering one atomic species and extended to the general case of several atomic species. Numerical calculations show that the models derived in this way reproduce with accuracy behaviors observed experimentally.
(24 S., 1998)

8. J. Ohser, B. Steinbach, C. Lang
Efficient Texture Analysis of Binary Images

A new method of determining some characteristics of binary images is proposed based on a special linear filtering. This technique enables the estimation of the area fraction, the specific line length, and the specific integral of curvature. Furthermore, the specific length of the total projection is obtained, which gives detailed information about the texture of the image. The influence of lateral and directional resolution depending on the size of the applied filter mask is discussed in detail. The technique includes a method of increasing directional resolution for texture analysis while keeping lateral resolution as high as possible.
(17 S., 1998)

9. J. Orlik
Homogenization for viscoelasticity of the integral type with aging and shrinkage

A multi-phase composite with periodic distributed inclusions with a smooth boundary is considered in this contribution. The composite component materials are supposed to be linear viscoelastic and aging (of the non-convolution integral type, for which the Laplace transform with respect to time is not effectively applicable) and are subjected to isotropic shrinkage. The free shrinkage deformation can be considered as a fictitious temperature deformation in the behavior law. The procedure presented in this paper proposes a way to determine average (effective homogenized) viscoelastic and shrinkage (temperature) composite properties and the homogenized stress-field from known properties of the

components. This is done by the extension of the asymptotic homogenization technique known for pure elastic non-homogeneous bodies to the non-homogeneous thermo-viscoelasticity of the integral non-convolution type. Up to now, the homogenization theory has not covered viscoelasticity of the integral type.
Sanchez-Palencia (1980), Francfort & Suquet (1987) (see [2], [9]) have considered homogenization for viscoelasticity of the differential form and only up to the first derivative order. The integral-modeled viscoelasticity is more general then the differential one and includes almost all known differential models. The homogenization procedure is based on the construction of an asymptotic solution with respect to a period of the composite structure. This reduces the original problem to some auxiliary boundary value problems of elasticity and viscoelasticity on the unit periodic cell, of the same type as the original non-homogeneous problem. The existence and uniqueness results for such problems were obtained for kernels satisfying some constrain conditions. This is done by the extension of the Volterra integral operator theory to the Volterra operators with respect to the time, whose 1 kernels are space linear operators for any fixed time variables. Some ideas of such approach were proposed in [11] and [12], where the Volterra operators with kernels depending additionally on parameter were considered. This manuscript delivers results of the same nature for the case of the space-operator kernels.
(20 S., 1998)

10. J. Mohring

Helmholtz Resonators with Large Aperture

The lowest resonant frequency of a cavity resonator is usually approximated by the classical Helmholtz formula. However, if the opening is rather large and the front wall is narrow this formula is no longer valid. Here we present a correction which is of third order in the ratio of the diameters of aperture and cavity. In addition to the high accuracy it allows to estimate the damping due to radiation. The result is found by applying the method of matched asymptotic expansions. The correction contains form factors describing the shapes of opening and cavity. They are computed for a number of standard geometries. Results are compared with numerical computations.
(21 S., 1998)

11. H. W. Hamacher, A. Schöbel

On Center Cycles in Grid Graphs

Finding "good" cycles in graphs is a problem of great interest in graph theory as well as in locational analysis. We show that the center and median problems are NP hard in general graphs. This result holds both for the variable cardinality case (i.e. all cycles of the graph are considered) and the fixed cardinality case (i.e. only cycles with a given cardinality p are feasible). Hence it is of interest to investigate special cases where the problem is solvable in polynomial time.
In grid graphs, the variable cardinality case is, for instance, trivially solvable if the shape of the cycle can be chosen freely.
If the shape is fixed to be a rectangle one can analyze rectangles in grid graphs with, in sequence, fixed dimension, fixed cardinality, and variable cardinality. In all cases a complete characterization of the optimal cycles and closed form expressions of the optimal objective values are given, yielding polynomial time algorithms for all cases of center rectangle problems.
Finally, it is shown that center cycles can be chosen as rectangles for small cardinalities such that the center cycle problem in grid graphs is in these cases completely solved.
(15 S., 1998)

12. H. W. Hamacher, K.-H. Küfer

Inverse radiation therapy planning - a multiple objective optimisation approach

For some decades radiation therapy has been proved successful in cancer treatment. It is the major task of clinical radiation treatment planning to realize on the one hand a high level dose of radiation in the cancer tissue in order to obtain maximum tumor control. On the other hand it is obvious that it is absolutely necessary to keep in the tissue outside the tumor, particularly in organs at risk, the unavoidable radiation as low as possible.
No doubt, these two objectives of treatment planning - high level dose in the tumor, low radiation outside the tumor - have a basically contradictory nature. Therefore, it is no surprise that inverse mathematical models with dose distribution bounds tend to be infeasible in most cases. Thus, there is need for approximations compromising between overdosing the organs at risk and underdosing the target volume.
Differing from the currently used time consuming iterative approach, which measures deviation from an ideal (non-achievable) treatment plan using recursively trial-and-error weights for the organs of interest, we go a new way trying to avoid a priori weight choices and consider the treatment planning problem as a multiple objective linear programming problem: with each organ of interest, target tissue as well as organs at risk, we associate an objective function measuring the maximal deviation from the prescribed doses.
We build up a data base of relatively few efficient solutions representing and approximating the variety of Pareto solutions of the multiple objective linear programming problem. This data base can be easily scanned by physicians looking for an adequate treatment plan with the aid of an appropriate online tool.
(14 S., 1999)

13. C. Lang, J. Ohser, R. Hilfer

On the Analysis of Spatial Binary Images

This paper deals with the characterization of microscopically heterogeneous, but macroscopically homogeneous spatial structures. A new method is presented which is strictly based on integral-geometric formulae such as Crofton's intersection formulae and Hadwiger's recursive definition of the Euler number. The corresponding algorithms have clear advantages over other techniques. As an example of application we consider the analysis of spatial digital images produced by means of Computer Assisted Tomography.
(20 S., 1999)

14. M. Junk

On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes

A general approach to the construction of discrete equilibrium distributions is presented. Such distribution functions can be used to set up Kinetic Schemes as well as Lattice Boltzmann methods. The general principles are also applied to the construction of Chapman Enskog distributions which are used in Kinetic Schemes for compressible Navier-Stokes equations.
(24 S., 1999)

15. M. Junk, S. V. Raghurame Rao

A new discrete velocity method for Navier-Stokes equations

The relation between the Lattice Boltzmann Method, which has recently become popular, and the Kinetic Schemes, which are routinely used in Computational Fluid Dynamics, is explored. A new discrete velocity model for the numerical solution of Navier-Stokes equations for incompressible fluid flow is presented by combining both the approaches. The new scheme can be interpreted as a pseudo-compressibility method and, for a particular choice of parameters, this interpretation carries over to the Lattice Boltzmann Method.
(20 S., 1999)

16. H. Neunzert

Mathematics as a Key to Key Technologies

The main part of this paper will consist of examples, how mathematics really helps to solve industrial problems; these examples are taken from our Institute for Industrial Mathematics, from research in the Technomathematics group at my university, but also from ECMI groups and a company called TecMath, which originated 10 years ago from my university group and has already a very successful history.
(39 S. (vier PDF-Files), 1999)

17. J. Ohser, K. Sandau

Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem

Wicksell's corpuscle problem deals with the estimation of the size distribution of a population of particles, all having the same shape, using a lower dimensional sampling probe. This problem was originary formulated for particle systems occurring in life sciences but its solution is of actual and increasing interest in materials science. From a mathematical point of view, Wicksell's problem is an inverse problem where the interesting size distribution is the unknown part of a Volterra equation. The problem is often regarded ill-posed, because the structure of the integrand implies unstable numerical solutions. The accuracy of the numerical solutions is considered here using the condition number, which allows to compare different numerical methods with different (equidistant) class sizes and which indicates, as one result, that a finite section thickness of the probe reduces the numerical problems. Furthermore, the relative error of estimation is computed which can be split into two parts. One part consists of the relative discretization error that increases for increasing class size, and the second part is related to the relative statistical error which increases with decreasing class size. For both parts, upper bounds can be given and the sum of them indicates an optimal class width depending on some specific constants.
(18 S., 1999)

18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel

Solving nonconvex planar location problems by finite dominating sets

It is well-known that some of the classical location problems with polyhedral gauges can be solved in polynomial time by finding a finite dominating set, i. e. a finite set of candidates guaranteed to contain at least one optimal location.
In this paper it is first established that this result holds for a much larger class of problems than currently considered in the literature. The model for which this result can be proven includes, for instance, location problems with attraction and repulsion, and location-allocation problems.
Next, it is shown that the approximation of general gauges by polyhedral ones in the objective function of our general model can be analyzed with regard to the subsequent error in the optimal objective value. For the approximation problem two different approaches are described, the sandwich procedure and the greedy algorithm. Both of these approaches lead - for fixed epsilon - to polynomial approximation algorithms with accuracy epsilon for solving the general model considered in this paper.
Keywords:
Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 S., 2000)

19. A. Becker

A Review on Image Distortion Measures

Within this paper we review image distortion measures. A distortion measure is a criterion that assigns a "quality number" to an image. We distinguish between mathematical distortion measures and those distortion measures in-cooperating a priori knowledge about the imaging devices (e. g. satellite images), image processing algorithms or the human physiology. We will consider representative examples of different kinds of distortion measures and are going to discuss them.
Keywords:
Distortion measure, human visual system
(26 S., 2000)

20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn

Polyhedral Properties of the Uncapacita*ted Multiple Allocation Hub Location Problem*

We examine the feasibility polyhedron of the uncapacitated hub location problem (UHL) with multiple allocation, which has applications in the fields of air passenger and cargo transportation, telecommunication and postal delivery services. In particular we determine the dimension and derive some classes of facets of this polyhedron. We develop some general rules about lifting facets from the uncapacitated facility location (UFL) for UHL and projecting facets from UHL to UFL. By applying these rules we get a new class of facets for UHL which dominates the inequalities in the original formulation. Thus we get a new formulation of UHL whose constraints are all facet–defining. We show its superior computational performance by benchmarking it on a well known data set.
Keywords:
integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 S., 2000)

21. H. W. Hamacher, A. Schöbel

Design of Zone Tariff Systems in Public Transportation

Given a public transportation system represented by its stops and direct connections between stops, we consider two problems dealing with the prices for the customers: The fare problem in which subsets of stops are already aggregated to zones and "good" tariffs have to be found in the existing zone system. Closed form solutions for the fare problem are presented for three objective functions. In the zone problem the design of the zones is part of the problem. This problem is NP hard and we therefore propose three heuristics which prove to be very successful in the redesign of one of Germany's transportation systems.
(30 S., 2001)

22. D. Hietel, M. Junk, R. Keck, D. Teleaga:

The Finite-Volume-Particle Method for Conservation Laws

In the Finite-Volume-Particle Method (FVPM), the weak formulation of a hyperbolic conservation law is discretized by restricting it to a discrete set of test functions. In contrast to the usual Finite-Volume approach, the test functions are not taken as characteristic functions of the control volumes in a spatial grid, but are chosen from a partition of unity with smooth and overlapping partition functions (the particles), which can even move along prescribed velocity fields. The information exchange between particles is based on standard numerical flux functions. Geometrical information, similar to the surface area of the cell faces in the Finite-Volume Method and the corresponding normal directions are given as integral quantities of the partition functions.
After a brief derivation of the Finite-Volume-Particle Method, this work focuses on the role of the geometric coefficients in the scheme.
(16 S., 2001)

23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel

Location Software and Interface with GIS and Supply Chain Management

The objective of this paper is to bridge the gap between location theory and practice. To meet this objective focus is given to the development of software capable of addressing the different needs of a wide group of users. There is a very active community on location theory encompassing many research fields such as operations research, computer science, mathematics, engineering, geography, economics and marketing. As a result, people working on facility location problems have a very diverse background and also different needs regarding the software to solve these problems. For those interested in non-commercial applications (e. g. students and researchers), the library of location algorithms (LoLA can be of considerable assistance. LoLA contains a collection of efficient algorithms for solving planar, network and discrete facility location problems. In this paper, a detailed description of the functionality of LoLA is presented. In the fields of geography and marketing, for instance, solving facility location problems requires using large amounts of demographic data. Hence, members of these groups (e. g. urban planners and sales managers) often work with geographical information too s. To address the specific needs of these users, LoLA was inked to a geographical information system (GIS) and the details of the combined functionality are described in the paper. Finally, there is a wide group of practitioners who need to solve large problems and require special purpose software with a good data interface. Many of such users can be found, for example, in the area of supply chain management (SCM). Logistics activities involved in strategic SCM include, among others, facility location planning. In this paper, the development of a commercial location software tool is also described. The too is embedded in the Advanced Planner and Optimizer SCM software developed by SAP AG, Walldorf, Germany. The paper ends with some conclusions and an outlook to future activities.
Keywords:
facility location, software development, geographical information systems, supply chain management.
(48 S., 2001)

24. H. W. Hamacher, S. A. Tjandra

Mathematical Modelling of Evacuation Problems: A State of Art

This paper details models and algorithms which can be applied to evacuation problems. While it concentrates on building evacuation many of the results are applicable also to regional evacuation. All models consider the time as main parameter, where the travel time between components of the building is part of the input and the overall evacuation time is the output. The paper distinguishes between macroscopic and microscopic evacuation models both of which are able to capture the evacuees' movement over time.
Macroscopic models are mainly used to produce good lower bounds for the evacuation time and do not consider any individual behavior during the emergency situation. These bounds can be used to analyze existing buildings or help in the design phase of planning a building. Macroscopic approaches which are based on dynamic network flow models (minimum cost dynamic flow, maximum dynamic flow, universal maximum flow, quickest path and quickest flow) are described. A special feature of the presented approach is the fact, that travel times of evacuees are not restricted to be constant, but may be density dependent. Using multicriteria optimization priority regions and blockage due to fire or smoke may be considered. It is shown how the modelling can be done using time parameter either as discrete or continuous parameter.
Microscopic models are able to model the individual evacuee's characteristics and the interaction among evacuees which influence their movement. Due to the corresponding huge amount of data one uses simulation approaches. Some probabilistic laws for individual evacuee's movement are presented. Moreover ideas to model the evacuee's movement using cellular automata (CA) and resulting software are presented.
In this paper we will focus on macroscopic models and only summarize some of the results of the microscopic approach. While most of the results are applicable to general evacuation situations, we concentrate on building evacuation.
(44 S., 2001)

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