Wave reflection and refraction in triclinic crystalline media

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Abstract:

In this paper, the reflection and refraction of a plane wave at an interface between .two half-spaces composed of triclinic crystalline material is considered. It is shown that due to incidence of a plane wave three types of waves namely quasi-P (qP), quasi-SV (qSV) and quasi-SH (qSH) will be generated governed by the propagation condition involving the acoustic tensor. A simple procedure has been presented for the calculation of all the three phase velocities of the quasi waves. It has been considered that the direction of particle motion is neither parallel nor perpendicular to the direction of propagation. Relations are established between directions of motion and propagation, respectively. The expressions for reflection and refraction coefficients of qP, qSV and qSH waves are obtained. Numerical results of reflection and refraction coefficients are presented for different types of anisotropic media and for different types of incident waves. Graphical representation have been made for incident qP waves and for incident qSV and qSH waves numerical data are presented in two tables.

Key words: Reflection, refraction, incident wave, triclinic medium, quasi-P,quasi-SV, quasi-SH

1. Introduction

The study of reflection and refraction phenomena of elastic waves is of considerable interest in the field of Seismology, in particular seismic prospecting as the Earth’s surface might be supposed of consist of different layers having different material properties. The elastic properties of a crystalline material depend on the internal structure of the material.

Effect of earthquake on artificial structures is of prime importance to engineers and architects. During an earthquake and similar disturbances a structure is excited into a more or less violent, with resulting oscillatory stresses, which depend both upon the ground vibration and physical properties of the structure. So, wave propagation in anisotropic medium plays a very important role in civil engineering and geophysics.

The propagation of body waves and surface waves in anisotropic media is fundamentally different from their propagation in isotropic media. In seismology anisotropy manifests itself most straightforwardly by a variation of the phase speed of seismic waves with their direction of propagation. A material displaying velocity anisotropy must have its effective elastic constants arranged in some form of crystalline symmetry. Cramplin [1977] has pointed out that the behaviour of both body and surface waves in anisotropic structures differs from that in isotropic structures, and variation of velocity with direction is only one of the anomalies which may occur. Within an anisotropic material three body waves propagate in any direction, having different and varying velocity and different and varying polarization. In highly anisotropic medium the P, SV and SH are coupled. This coupling introduces polarization anomalies which may be used to investigate anisotropy within the earth.

The problem of reflection and refraction of elastic waves have been discussed by several authors. Without going into the details of the research work in this field we mention the papers by Knott [1899], Gutenberg [1944], Achenbach [1976], Keith and Crampin [1977, 1977a, 1977b], Tolstoy [1982], Norris [1983], Pal and Chattopadhyay [1984], Auld [1990], Ogden and Sotirropoulos [1997,1998], Chattopadhyay and Rogerson [2001].

Crampin and Taylor [1971] studied surface wave propagation in examples of unlayered and multilayered anisotropic media, which is examined numerically with a program using as extension of the Thompson-Haskell matrix formulation. They studied some examples of surface wave propagation in anisotropic media to interpret a possible geophysical structure. Crampin [1975] showed that the surface waves have distinct particle motion when propagating in a structure having a layer of anisotropic material with certain symmetry relations.

In this paper we have studied the reflection and refraction of a plane wave at the interface of of two triclinic crystalline media. Relations have been established between directions of motion and propagation, respectively. Reflection and refraction coefficients due to incident qP,qSV and qSH waves have been computed for different types of anisotropic media. It has been observed that triclinic media plays a significant role in case of reflection and refraction.

2. Formulation of the problem

Consider a homogeneous triclinic medium having twenty one elastic constants. We assume $u_i = u_i(x_2, x_3, t)$, i=1,2,3. (1)

The stress-strain relations are

$$\begin{aligned}
\tau_{11} &= C_{11}e_{11} + C_{12}e_{22} + C_{13}e_{33} + C_{14}e_{23} + C_{15}e_{13} + C_{16}e_{12} \ , \\
\tau_{22} &= C_{12}e_{11} + C_{22}e_{22} + C_{23}e_{33} + C_{24}e_{23} + C_{25}e_{13} + C_{26}e_{12} \ , \\
\tau_{33} &= C_{13}e_{11} + C_{23}e_{22} + C_{33}e_{33} + C_{34}e_{23} + C_{35}e_{13} + C_{36}e_{12} \ , \\
\tau_{23} &= C_{14}e_{11} + C_{24}e_{22} + C_{34}e_{33} + C_{44}e_{23} + C_{45}e_{13} + C_{46}e_{12} \ , \\
\tau_{13} &= C_{15}e_{11} + C_{25}e_{22} + C_{35}e_{33} + C_{45}e_{23} + C_{55}e_{13} + C_{56}e_{12} \ , \\
\tau_{12} &= C_{16}e_{11} + C_{26}e_{22} + C_{36}e_{33} + C_{46}e_{23} + C_{56}e_{13} + C_{66}e_{12}
\end{aligned} \tag{2a}$$

where

$C_{ij} = C_{ji}$, $2e_{ij} = (u_{i,j} + u_{j,i})$ and u_i (i=1,2,3) are the displacement components.

The equations of motion without body forces are

$\tau_{ij,j} = \rho \ddot{u}_i$,i=1,2,3. (2b)

The following nonvanishing equations of motion are obtained after using equations (1) and (2)

$$(C_{55}\frac{\partial^2 u_1}{\partial x_3^2}+2C_{56}\frac{\partial^2 u_1}{\partial x_2\partial x_3}+C_{66}\frac{\partial^2 u_1}{\partial x_2^2})+\{C_{45}\frac{\partial^2 u_2}{\partial x_3^2}+(C_{46}+C_{25})\frac{\partial^2 u_2}{\partial x_2\partial x_3}+C_{26}\frac{\partial^2 u_2}{\partial x_2^2}\}$$

$$+\{C_{35}\frac{\partial^2 u_3}{\partial x_3^2}+(C_{36}+C_{45})\frac{\partial^2 u_3}{\partial x_2\partial x_3}+C_{46}\frac{\partial^2 u_3}{\partial x_2^2}\}=\rho\frac{\partial^2 u_1}{\partial t^2}\ , \qquad (3)$$

$$\{C_{45}\frac{\partial^2 u_1}{\partial x_3^2}+(C_{25}+C_{46})\frac{\partial^2 u_1}{\partial x_2\partial x_3}+C_{26}\frac{\partial^2 u_1}{\partial x_2^2}\}+\{C_{44}\frac{\partial^2 u_2}{\partial x_3^2}+2C_{24}\frac{\partial^2 u_2}{\partial x_2\partial x_3}+C_{22}\frac{\partial^2 u_2}{\partial x_2^2}\}$$

$$+\{C_{34}\frac{\partial^2 u_3}{\partial x_3^2}+(C_{23}+C_{44})\frac{\partial^2 u_3}{\partial x_2\partial x_3}+C_{24}\frac{\partial^2 u_3}{\partial x_2^2}\}=\rho\frac{\partial^2 u_2}{\partial t^2}, \qquad (4)$$

$$\{C_{35}\frac{\partial^2 u_1}{\partial x_3^2}+(C_{45}+C_{36})\frac{\partial^2 u_1}{\partial x_2\partial x_3}+C_{46}\frac{\partial^2 u_1}{\partial x_2^2}\}$$

$$+\{C_{34}\frac{\partial^2 u_2}{\partial x_3^2}+(C_{23}+C_{44})\frac{\partial^2 u_2}{\partial x_2\partial x_3}+C_{24}\frac{\partial^2 u_2}{\partial x_2^2}\}$$

$$+\{C_{33}\frac{\partial^2 u_3}{\partial x_3^2}+2C_{34}\frac{\partial^2 u_3}{\partial x_2\partial x_3}+C_{44}\frac{\partial^2 u_3}{\partial x_2^2}\}=\rho\frac{\partial^2 u_3}{\partial t^2}. \qquad (5)$$

Let $\vec{p}(0,p_2^{(n)},p_3^{(n)})$ denote the unit propagation vector, c_n is the phase velocity and k_n is the wavenumber of plane waves propagating in the x_2x_3-plane.

We consider plane wave solution of equations (3) to (5) as

$$\begin{pmatrix} u_1^{(n)} \\ u_2^{(n)} \\ u_3^{(n)} \end{pmatrix}=A_n\begin{pmatrix} d_1^{(n)} \\ d_2^{(n)} \\ d_3^{(n)} \end{pmatrix}\exp(i\eta_n) \qquad (6)$$

where

$\bar{d}(d_1^{(n)},d_2^{(n)},d_3^{(n)})$ is the unit displacement vector and

$$\eta_n=k_n(x_2p_2^{(n)}+x_3p_3^{(n)}-c_nt). \qquad (7)$$

Inserting the expressions of (6) into the equations (3) to (5), we have

$$(S-\bar{c}^2)d_1^{(n)}+Td_2^{(n)}+Pd_3^{(n)}=0, \qquad (8)$$

$$Td_1^{(n)}+(Q-\bar{c}^2)d_2^{(n)}+Rd_3^{(n)}=0, \qquad (9)$$

$$Pd_1^{(n)}+Rd_2^{(n)}+(W-\bar{c}^2)d_3^{(n)}=0 \qquad (10)$$

where

$$\bar{c}^2=\frac{\rho c_n^2}{C_{44}},\bar{C}_{ij}=\frac{C_{ij}}{C_{44}}\ ,$$

$$S=\bar{C}_{55}p_3^2+2\bar{C}_{56}p_2p_3+\bar{C}_{66}p_2^2,$$

$$T=\bar{C}_{45}p_3^2+(\bar{C}_{46}+\bar{C}_{25})p_2p_3+\bar{C}_{26}p_2^2,$$

$$P=\bar{C}_{35}p_3^2+(\bar{C}_{36}+\bar{C}_{45})p_2p_3+\bar{C}_{46}p_2^2\ ,$$

$$Q=\bar{C}_{44}p_3^2+2\bar{C}_{24}p_2p_3+\bar{C}_{22}p_2^2\ ,$$

$$R=\bar{C}_{34}p_3^2+(\bar{C}_{23}+\bar{C}_{44})p_2p_3+\bar{C}_{24}p_2^2\ ,$$

$$W = \overline{C}_{33} p_3^2 + 2\overline{C}_{34} p_2 p_3 + \overline{C}_{44} p_2^2 . \tag{11}$$

From equations (8),(9) and (10), we obtain

$$\frac{d_3^{(n)}}{d_1^{(n)}} = \frac{T^2 - (Q - \bar{c}^2)(S - \bar{c}^2)}{P(Q - \bar{c}^2) - RT} \ , \tag{12}$$

$$\frac{d_2^{(n)}}{d_1^{(n)}} = \frac{R(S - \bar{c}^2) - PT}{P(Q - \bar{c}^2) - RT} \tag{13}$$

and

$$\frac{d_3^{(n)}}{d_2^{(n)}} = \frac{T^2 - (Q - \bar{c}^2)(S - \bar{c}^2)}{R(S - \bar{c}^2) - PT} . \tag{14}$$

The equations (12) to (14) may be used to calculate $\overline{d}$ in terms of $\overline{p}$.
Eliminating $d_1^{(n)}, d_2^{(n)}, d_3^{(n)}$ from (8),(9) and (10), we have

$$\bar{c}^6 + a_1 \bar{c}^4 + a_2 \bar{c}^2 + a_3 = 0 \tag{15}$$

where

$$a_1 = -(S + Q + W) \ ,$$

$$a_2 = QS + WS + QW - R^2 - T^2 - P^2 ,$$

$$a_3 = -(SQW - SR^2 - WT^2 + 2PTR - P^2 Q) . \tag{16}$$

Solving the equation (15), we will obtain the phase velocities of quasi-P(qP),quasi-SV(qSV) and quasi-SH(qSH) as

$$\rho c_L^2 = -2r \cos(\frac{\varphi}{3}) - \frac{a_1}{3} , \tag{17}$$

$$\rho c_{SV}^2 = 2r \cos(60^0 + \frac{\varphi}{3}) - \frac{a_1}{3} , \tag{18}$$

$$\rho c_{SH}^2 = 2r \cos(60^0 - \frac{\varphi}{3}) - \frac{a_1}{3} \tag{19}$$

where

$$2q = \frac{2a_1^3}{27} - \frac{a_1 a_2}{3} + a_3 , \ 3p = \frac{3a_2 - a_1^2}{3} \ ,$$

$$r = -\sqrt{|p|} \ , \ \varphi = \cos^{-1}(\frac{q}{r^3}) . \tag{20}$$

In isotropic case

$$C_{11} = C_{22} = C_{33} = \lambda + 2\mu \ ,$$

$$C_{12} = C_{13} = C_{23} = \lambda \ ,$$

$$C_{44} = C_{55} = C_{66} = \mu \tag{21}$$

and all other elastic constants are zero.
Substituting (21) in equations (17),(18) and (19) and after simplification, we obtain the following compressional velocity (c_L) and the repeated roots (c_{SV} and c_{SH}) for shear velocity as

$$c_L^2 = \frac{\lambda + 2\mu}{\rho} , \ c_{SV}^2 = c_{SH}^2 = \frac{\mu}{\rho} . \tag{22}$$

We solved the equation (15) and obtained three real roots of $\bar{c}^2$. The largest root is assigned to the phase velocity of qP waves, the second largest is the phase velocity of qSV waves and the lowest root for the phase velocity of qSH waves. The phase velocities of quasi-transverse waves (qSV and qSH) will not be identical in case of triclinic medium. The result was tested with different sets of data as mentioned in section 4. If any geophysical evidence exists that the qSH wave velocity is more than qSV wave velocity then the nature of the graphs of the reflected qSV and reflected qSH of this paper are to be interchanged. This method of solution for calculating the velocities of all the three quasi-waves is most general and will be helpful to identify the phase velocities for different types of anisotropy.

3. Solution of the problem

Consider a triclinic crystalline medium. The x_3-axis is taken along the free surface and x_2-axis is vertically downward. Plane wave is incident at the free boundary $x_2 = 0$. Incident qP or qSV or qSH waves will generate reflected qP, reflected qSV, reflected qSH waves and also refracted qP, refracted qSV, refracted qSH waves. It is also clear from the equations (3),(4) and (5) that all the displacement components are coupled. Let n=0,1,2,3,4,5,6 be assumed for incident wave, reflected qP, qSV, qSH and refracted qP, qSV, qSH waves respectively.

In the plane $x_2 = 0$, the displacements and stresses of incident and reflected waves are represented by

$u_j^{(n)} = A_n d_j^{(n)} \exp(i\overline{\eta}_n)$, j=1,2,3.

$\tau_{12}^{(n)} = P_{1n} i k_n A_n \exp(i\overline{\eta}_n)$,

$\tau_{22}^{(n)} = Q_n i k_n A_n \exp(i\overline{\eta}_n)$,

$$\tau_{23}^{(n)} = R_n i k_n A_n \exp(i\overline{\eta}_n) \tag{23}$$

where

$$P_{1n} = C_{26} p_2^{(n)} d_2^{(n)} + C_{36} p_3^{(n)} d_3^{(n)} + C_{46}\{d_2^{(n)} p_3^{(n)} + d_3^{(n)} p_2^{(n)}\} + C_{56} d_1^{(n)} p_3^{(n)} + C_{66} d_1^{(n)} p_2^{(n)}, \tag{24}$$

$$Q_n = C_{22} p_2^{(n)} d_2^{(n)} + C_{23} p_3^{(n)} d_3^{(n)} + C_{24}\{d_2^{(n)} p_3^{(n)} + d_3^{(n)} p_2^{(n)}\} + C_{25} d_1^{(n)} p_3^{(n)} + C_{26} d_1^{(n)} p_2^{(n)}, \tag{25}$$

$$R_n = C_{24} p_2^{(n)} d_2^{(n)} + C_{34} p_3^{(n)} d_3^{(n)} + C_{44}\{d_2^{(n)} p_3^{(n)} + d_3^{(n)} p_2^{(n)}\} + C_{45} d_1^{(n)} p_3^{(n)} + C_{46} d_1^{(n)} p_2^{(n)},$$

$\overline{\eta}_n = k_n (x_3 p_3^{(n)} - c_n t)$

and n=0,1,2,3,4,5,6. (26)

For n=4,5,6 the elastic constants C_{ij} to be replaced by $C_{ij}^{/}$ and accordingly equations (23) to (26) will be changed for the refracted waves in the upper half-space.

For incident plane waves

$p_2^{(0)} = -\cos\theta_0, \; p_3^{(0)} = \sin\theta_0, \; c_0 = c_I$.

For reflected qP waves

$p_2^{(1)} = \cos\theta_1,\, p_3^{(1)} = \sin\theta_1, c_1 = c_{L1}$.

For reflected qSV waves

$p_2^{(2)} = \cos\theta_2,\, p_3^{(2)} = \sin\theta_2, c_2 = c_T$.

For reflected qSH waves

$p_2^{(3)} = \cos\theta_3,\, p_3^{(3)} = \sin\theta_3, c_3 = c_{T1}$

For refracted qP waves

$p_2^{(4)} = -\cos\theta_4,\, p_3^{(4)} = \sin\theta_4, c_1 = c_L^{/}$.

For refracted qSV waves

$p_2^{(5)} = -\cos\theta_5,\, p_3^{(5)} = \sin\theta_5, c_2 = c_T^{/}$.

For refracted qSH waves

$$p_2^{(6)} = -\cos\theta_6,\, p_3^{(6)} = \sin\theta_6, c_3 = c_{T1}^{/} \tag{27}$$

where $c_I, c_{L1}, c_T, c_{T1},, c_{L1}^{/}, c_T^{/}$ and $c_{T1}^{/}$ are the phase velocities of incident plane wave, reflected qP, reflected qSV, reflected qSH waves, refracted qP, refracted qSV and refracted qSH waves respectively.

The boundary conditions at $x_2 = 0$ are

$$u_1^{(0)} + u_1^{(1)} + u_1^{(2)} + u_1^{(3)} = u_1^{(4)} + u_1^{(5)} + u_1^{(6)}, \tag{28}$$

$$u_2^{(0)} + u_2^{(1)} + u_2^{(2)} + u_2^{(3)} = u_2^{(4)} + u_2^{(5)} + u_2^{(6)}, \tag{29}$$

$$u_3^{(0)} + u_3^{(1)} + u_3^{(2)} + u_3^{(3)} = u_3^{(4)} + u_3^{(5)} + u_3^{(6)}, \tag{30}$$

$$\tau_{12}^{(0)} + \tau_{12}^{(1)} + \tau_{12}^{(2)} + \tau_{12}^{(3)} = \tau_{12}^{(4)} + \tau_{12}^{(5)} + \tau_{12}^{(6)}, \tag{31}$$

$$\tau_{22}^{(0)} + \tau_{22}^{(1)} + \tau_{22}^{(2)} + \tau_{22}^{(3)} = \tau_{22}^{(4)} + \tau_{22}^{(5)} + \tau_{22}^{(6)}, \tag{32}$$

$$\tau_{23}^{(0)} + \tau_{23}^{(1)} + \tau_{23}^{(2)} + \tau_{23}^{(3)} = \tau_{23}^{(4)} + \tau_{23}^{(5)} + \tau_{23}^{(6)}. \tag{33}$$

Using the boundary conditions and the equations (23) to (26), we obtain,

$$\begin{aligned}
&A_0 d_1^{(0)} \exp\{ik_0(x_3 p_3^{(0)} - c_I t)\} + A_1 d_1^{(1)} \exp\{ik_1(x_3 p_3^{(1)} - c_{L1} t)\} \\
&+ A_2 d_1^{(2)} \exp\{ik_2(x_3 p_3^{(2)} - c_T t)\} + A_3 d_1^{(3)} \exp\{ik_3(x_3 p_3^{(3)} - c_{T1} t)\} \\
&= A_4 d_1^{(4)} \exp\{ik_4(x_3 p_3^{(4)} - c_{L1}^{/} t)\} + A_5 d_1^{(5)} \exp\{ik_5(x_3 p_3^{(5)} - c_T^{/} t)\} \\
&P_{10} A_0 k_0 \exp\{ik_0(x_3 p_3^{(0)} - c_I t)\} + P_{11} A_1 k_1 \exp\{ik_1(x_3 p_3^{(1)} - c_{L1} t)\} \\
&+ A_6 d_1^{(6)} \exp\{ik_6(x_3 p_3^{(0)} - c_{T1}^{/} t)\}
\end{aligned} \tag{34}$$

$$\begin{aligned}
&A_0 d_2^{(0)} \exp\{ik_0(x_3 p_3^{(0)} - c_I t)\} + A_1 d_2^{(1)} \exp\{ik_1(x_3 p_3^{(1)} - c_{L1} t)\} \\
&+ A_2 d_2^{(2)} \exp\{ik_2(x_3 p_3^{(2)} - c_T t)\} + A_3 d_2^{(3)} \exp\{ik_3(x_3 p_3^{(3)} - c_{T1} t)\} \\
&= A_4 d_2^{(4)} \exp\{ik_4(x_3 p_3^{(4)} - c_{L1}^{/} t)\} + A_5 d_2^{(5)} \exp\{ik_5(x_3 p_3^{(5)} - c_T^{/} t)\} \\
&+ A_6 d_2^{(6)} \exp\{ik_6(x_3 p_3^{(0)} - c_{T1}^{/} t)\}
\end{aligned} \tag{35}$$

$$\begin{aligned}
&A_0 d_3^{(0)} \exp\{ik_0(x_3 p_3^{(0)} - c_I t)\} + A_1 d_3^{(1)} \exp\{ik_1(x_3 p_3^{(1)} - c_{L1} t)\} \\
&+ A_2 d_3^{(2)} \exp\{ik_2(x_3 p_3^{(2)} - c_T t)\} + A_3 d_3^{(3)} \exp\{ik_3(x_3 p_3^{(3)} - c_{T1} t)\} \\
&= A_4 d_3^{(4)} \exp\{ik_4(x_3 p_3^{(4)} - c_{L1}^{/} t)\} + A_5 d_3^{(5)} \exp\{ik_5(x_3 p_3^{(5)} - c_T^{/} t)\} \\
&+ A_6 d_3^{(6)} \exp\{ik_6(x_3 p_3^{(0)} - c_{T1}^{/} t)\}
\end{aligned} \tag{36}$$

$$P_{10} A_0 k_0 \exp\{ik_0(x_3 p_3^{(0)} - c_I t)\} + P_{11} A_1 k_1 \exp\{ik_1(x_3 p_3^{(1)} - c_{L1} t)\}$$

$$+P_{12}A_2k_2\exp\{ik_2(x_3p_3^{(2)}-c_Tt)\}+P_{13}A_3k_3\exp\{ik_3(x_3p_3^{(3)}-c_{T1}t)\}$$
$$=P_{14}A_4k_4\exp\{ik_4(x_3p_3^{(4)}-c_{L1}^{/}t)\}+P_{15}A_5k_5\exp\{ik_5(x_3p_3^{(5)}-c_T^{/}t)\}$$
$$+\ P_{16}A_6k_6\exp\{ik_6(x_3p_3^{(6)}-c_{T1}^{/}t)\}=0, \tag{37}$$
$$Q_0A_0k_0\exp\{ik_0(x_3p_3^{(0)}-c_It)\}+Q_1A_1k_1\exp\{ik_1(x_3p_3^{(1)}-c_{L1}t)\}$$
$$+Q_2A_2k_2\exp\{ik_2(x_3p_3^{(2)}-c_Tt)\}+Q_3A_3k_3\exp\{ik_3(x_3p_3^{(3)}-c_{T1}t)\}$$
$$=Q_4A_4k_4\exp\{ik_4(x_3p_3^{(4)}-c_{L1}^{/}t)\}+Q_5A_5k_5\exp\{ik_5(x_3p_3^{(5)}-c_T^{/}t)\}$$
$$+\ Q_6A_6k_6\exp\{ik_6(x_3p_3^{(6)}-c_{T1}^{/}t)\}=0, \tag{38}$$
$$R_0A_0k_0\exp\{ik_0(x_3p_3^{(0)}-c_It)\}+R_1A_1k_1\exp\{ik_1(x_3p_3^{(1)}-c_{L1}t)\}$$
$$+R_2A_2k_2\exp\{ik_2(x_3p_3^{(2)}-c_Tt)\}+R_3A_3k_3\exp\{ik_3(x_3p_3^{(3)}-c_{T1}t)\}$$
$$=R_4A_4k_4\exp\{ik_4(x_3p_3^{(4)}-c_{L1}^{/}t)\}+R_5A_5k_5\exp\{ik_5(x_3p_3^{(5)}-c_T^{/}t)\}$$
$$+\ R_6A_6k_6\exp\{ik_6(x_3p_3^{(6)}-c_{T1}^{/}t)\}=0, \tag{39}$$

The above equations are valid for all values of x_3 and t. Therefore, we have

$$k_0(x_3\sin\theta_0-c_It)=k_1(x_3\sin\theta_1-c_{L1}t)=k_2(x_3\sin\theta_2-c_Tt)=k_3(x_3\sin\theta_3-c_{T1}t)$$
$$=k_4(x_3\sin\theta_4-c_{L1}^{/}t)=k_5(x_3\sin\theta_5-c_T^{/}t)=k_6(x_3\sin\theta_6-c_{T1}^{/}t) \tag{40}$$

which gives

$$k_0c_I=k_1c_{L1}=k_2c_T=k_3c_{T1}=k_4c_{L1}^{/}=k_5c_T^{/}=k_6c_{T1}^{/}=k, \tag{41}$$

and

$$k_0\sin\theta_0=k_1\sin\theta_1=k_2\sin\theta_2=k_3\sin\theta_3=k_4\sin\theta_4=k_5\sin\theta_5=k_6\sin\theta_6=\omega \tag{42}$$

where k and ω are apparent wave number and circular frequency respectively. The amplitude ratios of qP,qSV and qSH are denoted by $\frac{A_1}{A_0}$, $\frac{A_2}{A_0}$, $\frac{A_3}{A_0}$, $\frac{A_4}{A_0}$, $\frac{A_5}{A_0}$,and $\frac{A_6}{A_0}$.

Solving the equations (34)-(39), the reflection and refraction coefficients of qP, qSV and qSH may be obtained as

$$\frac{A_1}{A_0}=\frac{D_1}{D_0},\frac{A_2}{A_0}=\frac{D_2}{D_0},\frac{A_3}{A_0}=\frac{D_3}{D_0},\frac{A_4}{A_0}=\frac{D_4}{D_0},\frac{A_5}{A_0}=\frac{D_5}{D_0},\frac{A_6}{A_0}=\frac{D_6}{D_0} \tag{43}$$

where

$$D_0=\begin{vmatrix} a_1 & a_2 & a_3 & -a_4 & -a_5 & -a_6 \\ b_1 & b_2 & b_3 & -b_4 & -b_5 & -b_6 \\ c_1 & c_2 & c_3 & -c_4 & -c_5 & -c_6 \\ e_1 & e_2 & e_3 & -e_4 & -e_5 & -e_6 \\ f_1 & f_2 & f_3 & -f_4 & -f_5 & -f_6 \\ g_1 & g_2 & g_3 & -g_4 & -g_5 & -g_6 \end{vmatrix},$$

$$D_1 = \begin{vmatrix} -1 & a_2 & a_3 & -a_4 & -a_5 & -a_6 \\ -1 & b_2 & b_3 & -b_4 & -b_5 & -b_6 \\ -1 & c_2 & c_3 & -c_4 & -c_5 & -c_6 \\ -1 & e_2 & e_3 & -e_4 & -e_5 & -e_6 \\ -1 & f_2 & f_3 & -f_4 & -f_5 & -f_6 \\ -1 & g_2 & g_3 & -g_4 & -g_5 & -g_6 \end{vmatrix},$$

$$D_2 = \begin{vmatrix} a_1 & -1 & a_3 & -a_4 & -a_5 & -a_6 \\ b_1 & -1 & b_3 & -b_4 & -b_5 & -b_6 \\ c_1 & -1 & c_3 & -c_4 & -c_5 & -c_6 \\ e_1 & -1 & e_3 & -e_4 & -e_5 & -e_6 \\ f_1 & -1 & f_3 & -f_4 & -f_5 & -f_6 \\ g_1 & -1 & g_3 & -g_4 & -g_5 & -g_6 \end{vmatrix},$$

$$D_3 = \begin{vmatrix} a_1 & a_2 & -1 & -a_4 & -a_5 & -a_6 \\ b_1 & b_2 & -1 & -b_4 & -b_5 & -b_6 \\ c_1 & c_2 & -1 & -c_4 & -c_5 & -c_6 \\ e_1 & e_2 & -1 & -e_4 & -e_5 & -e_6 \\ f_1 & f_2 & -1 & -f_4 & -f_5 & -f_6 \\ g_1 & g_2 & -1 & -g_4 & -g_5 & -g_6 \end{vmatrix},$$

$$D_4 = \begin{vmatrix} a_1 & a_2 & a_3 & -1 & -a_5 & -a_6 \\ b_1 & b_2 & b_3 & -1 & -b_5 & -b_6 \\ c_1 & c_2 & c_3 & -1 & -c_5 & -c_6 \\ e_1 & e_2 & e_3 & -1 & -e_5 & -e_6 \\ f_1 & f_2 & f_3 & -1 & -f_5 & -f_6 \\ g_1 & g_2 & g_3 & -1 & -g_5 & -g_6 \end{vmatrix},$$

$$D_5 = \begin{vmatrix} a_1 & a_2 & a_3 & -a_4 & -1 & -a_6 \\ b_1 & b_2 & b_3 & -b_4 & -1 & -b_6 \\ c_1 & c_2 & c_3 & -c_4 & -1 & -c_6 \\ e_1 & e_2 & e_3 & -e_4 & -1 & -e_6 \\ f_1 & f_2 & f_3 & -f_4 & -1 & -f_6 \\ g_1 & g_2 & g_3 & -g_4 & -1 & -g_6 \end{vmatrix},$$

$$D_6 = \begin{vmatrix} a_1 & a_2 & a_3 & -a_4 & -a_5 & -1 \\ b_1 & b_2 & b_3 & -b_4 & -b_5 & -1 \\ c_1 & c_2 & c_3 & -c_4 & -c_5 & -1 \\ e_1 & e_2 & e_3 & -e_4 & -e_5 & -1 \\ f_1 & f_2 & f_3 & -f_4 & -f_5 & -1 \\ g_1 & g_2 & g_3 & -g_4 & -g_5 & -1 \end{vmatrix}$$

and

$$a_i = \frac{d_1^{(i)}}{d_1^{(0)}}\,, \quad b_i = \frac{d_2^{(i)}}{d_2^{(0)}}\,, \quad c_i = \frac{d_3^{(i)}}{d_3^{(0)}}\,, \quad ,\text{i=1,2…,6}$$

$$e_i = \frac{\overline{P}_{1i}}{\overline{P}_{10}}\frac{k_i}{k_0}\,,\ f_i = \frac{\overline{Q}_i}{\overline{Q}_0}\frac{k_i}{k_0}\,,\ g_i = \frac{\overline{R}_i}{\overline{R}_0}\frac{k_i}{k_0}\,,\ \text{i=1,2,3,…6}$$

$$\overline{P}_{1i} = \frac{P_{1i}}{C_{44}}, \overline{Q}_i = \frac{Q_i}{C_{44}}, \overline{R}_i = \frac{R_i}{C_{44}},\ \text{i=0,1,2,3,…6.} \tag{39}$$

4. Numerical Calculations and Discussions

Numerical calculations were performed for incident qP, qSV and qSH waves with different types of anisotropic data. We have considered eight hypothetical data in case of Data-1 and twelve hypothetical data in case of Data-3 to get the effect of 21 elastic constants.

The following cases have been considered:

Data-1: The 13 elastic constants for the case of AT-cut quartz are (Tiersten[1969])

$C_{11} = 86.74GPa, C_{22} = 129.77GPa, C_{33} = 102.83GPa$,

$C_{12} = -8.25GPa, C_{13} = 27.15GPa, C_{14} = -3.66GPa,$

$C_{23} = -7.42GPa, C_{24} = 5.7GPa, C_{34} = 9.92GPa,$

$C_{44} = 38.61GPa, C_{55} = 68.81GPa, C_{66} = 29.01GPa, C_{56} = 2.53GPa$,

$\rho = 2.649 gm/cm^3$.

To test the effect of triclinic structures, we have considered the following hypothetical values of the constants:

$C_{15} = C_{16} = C_{25} = C_{26} = C_{35} = C_{36} = C_{45} = C_{46} = 7.5$GPa.

Data-2: The 13 elastic constants of Data-1 case and

$C_{15} = C_{16} = C_{25} = C_{26} = C_{35} = C_{36} = C_{45} = C_{46} = 0.5$ GPa

Data-3: The 9 elastic constants for Rochelle salt (Auld [1990]) are

$C_{11} = 28.0GPa, C_{22} = 41.4GPa, C_{33} = 39.4GPa,$

$C_{44} = 6.66GPa, C_{55} = 2.85GPa, C_{66} = 9.6GPa,$

$C_{12} = 17.4GPa, C_{13} = 15.0GPa, C_{23} = 19.7GPa.$

$\rho' = 2.7 gm/cm^3$.

We have considered alongwith the set of Data-3, the following hypothetical data

$C_{14} = C_{34} = C_{24} = C_{56} = 0.5$ GPa

and
$C_{15} = C_{16} = C_{25} = C_{26} = C_{35} = C_{36} = C_{45} = C_{46} = 0.5$ Gpa.
Curve-1 has been drawn considering the values of the upper layer as Data-1 and for the lower layer as Data 3. Curve-2 has been drawn with the set of vaules for the upper layer as Data-3 and the lower layer as Data-2.
Figures 1 to 6 have been drawn for incident qP waves. It has been observed that the existence of the angles for amplitude ratios of reflected and refracted waves are upto 68^0. Curves 1 and 2 have some jumps at certain angles for each diagrams from 1 to 6. Due to want of practical data the actual behaviour cannot be presented but idea can be made by considering these hypothetical data about the behaviour of reflection and refraction of waves in a triclinic media which is highly anisotropic in nature.

Table-1

The reflection coefficients for incident qSV waves for the set of data of curve-1 as mentioned above:

θ in degrees	A_1/A_0	A_2/A_0	A_3/A_0	A_4/A_0	A_5/A_0	A_6/A_0
0	0.5011	-0.7285	-0.7182	0.1916	0.0303	-0.1682
10	0.8549	-.7938	-0.9305	0.0236	-0.0082	0.0416
20	-0.13409	-2.4629	1.7378	-0.5539	-0.0201	0.6569
30	0.3268	-3.0822	2.4645	-0.4448	0.0258	0.4792
40	0.5807	-3.3594	2.4821	2.4825	0.1714	-2.6587
50	-0.9412	3.1518	-2.2028	-2.6552	-0.2639	2.7961
60	1.1948	-2.7522	-1.9326	1.5351	0.2678	-1.47136
70	1.5806	-2.5766	-1.8422	1.3135	0.2564	-.9914

Table-2

The reflection coefficients for incident qSH waves for the set of data of curve-1 as mentioned above:

θ in degrees	A_1/A_0	A_2/A_0	A_3/A_0	A_4/A_0	A_5/A_0	A_6/A_0
0	-0.5187	-1.2574	0.9662	0.6328	0.0136	-0.4530
10	1.9077	0.5953	-3.2210	1.3696	0.0425	-1.2106
20	0.4372	-1.7627	0.7549	1.0272	0.0758	-0.9497
30	0.1879	-2.8943	2.3863	0.5420	0.0573	-0.4573
40	0.5246	-3.2924	2.7141	3.5593	0.1596	-3.6311
50	0.7984	-2.7937	2.2873	2.6008	0.2034	-2.5651
60	0.6717	-1.8051	-1.7599	1.2689	0.1439	-0.9705
70	-2.3998	2.6543	-0.0552	-0.6534	-0.3901	1.3588

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