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Holographic Trace Anomaly and Cocycle of Weyl Group

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Abstract

The behavior of the divergent part of the bulk AdS/CFT effective action is considered with respect to the special finite diffeomorphism transformations acting on the boundary as a Weyl transformation of the boundary metric. The resulting 1-cocycle of the Weyl group is in full agreement with the 1-cocycle of the Weyl group obtained from the cohomological consideration of the effective action of the corresponding CFT.

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1 Introduction

The conformal anomaly plays an important role in the investigation of non-renormalization properties of supersymmetric theories in different dimensions. Investigation of the structure of the conformal or trace anomaly has a 20 years history [1], but its general structure was first discovered in [2] using a scaling property of the effective action.The cohomological properties of trace anomalies were considered in [3, 4] and the connection with 1-cocycles of the Weyl group was established. This cohomological object is important because it can be considered as the full effective action in a conformal flat background. The structure of the conformal anomaly can be considered in a general way using the Wess-Zumino consistency condition [5] and cocyclic properties of the effective action, the nonlocal terms of which generate the anomaly after local Weyl variation. The exact values of coefficients of different terms in the anomaly could be fixed by calculation in perturbation theory using the free field representation.

The AdS/CFT correspondence [6] provides an important tool for the calculation of conformal anomaly coefficients and conformal correlation functions in the large N limit of $\mathcal{N} = 4$ Super-Yang-Mills in $d = 4$ and superconformal theories in other dimensions. In this approach the anomaly appears after classical calculation of the on-shell $d + 1$ dimensional effective action in the form of logarithmically divergent terms [7]. Thus the AdS/CFT approach offers the possibility to investigate the effective action in the strong coupling limit. The Weyl transformation properties and some cohomological considerations in this approach were presented in [8]. The authors of the latter discovered the relation between the infinitesimal Weyl transformation of the boundary metric and a certain one-parameter family of $d + 1$ dimensional diffeomorphisms. The finite form of these diffeomorphisms was obtained in [9]. In this note we reproduce the 1-cocycle of the Weyl group in $d = 2, 4$ from investigation of the behavior of the divergent part of the AdS/CFT effective action with respect to these finite diffeomorphism transformations. We show that a cut-off regularization of the pure bulk contribution in the effective action (without boundary term) is responsible for the anomaly and the cocycle generation after applying the diffemorphism transformation, and correctly reproduces the 1-cocycle of the Weyl group (analogue of the Liouville action) in $d = 2$ and 4 dimensions. In Section 2 we review the cohomological structure of the effective action and the derivation of 1-cocycles of the Weyl group. In Section 3 we derive this cocycle from the Heningson-Skenderis approach using finite diffeomorphism transformations of bulk divergent terms.

2 Cocycle of the Weyl Group

In this section we review the cohomological properties of the CFT effective action in the external gravitational field. This consideration is based on the results of [10] , [4].

We consider the effective action for d-dimensional conformal matter φ in an external gravitational field:

$$W(g) = \ln \int D_g\varphi \exp\{-S_{CFT}(\varphi; g)\} \tag{1}$$

where $S_{CFT}(\varphi; g)$ is the classical Weyl and diffeomorphism invariant action for matter fields and where the Weyl transformation is defined as:

$$g_{ij} \to e^{2\sigma(x)} g_{ij}, \qquad \varphi \to e^{\Delta\sigma(x)}\varphi. \tag{2}$$

Here Δ is the conformal weight of the matter field. Then, for an infinitesimal σ, we can write the equation for the anomaly

$$\delta_\sigma W(g) = \int T_i^i \delta\sigma(x) \sqrt{g} d^{2k}x. \tag{3}$$

The Wess-Zumino consistency condition in the case of Weyl transformations is simply a statement of the symmetry of the second conformal variation of the effective action:

$$\frac{\delta^2 W(g)}{\delta\sigma(x)\sqrt{g}\delta\sigma(y)} = \frac{\delta^2 W(g)}{\delta\sigma(y)\sqrt{g}\delta\sigma(x)} \tag{4}$$

or, in other words,

$$\frac{\delta\mathcal{A}(x)}{\delta\sigma(y)} = \frac{\delta\mathcal{A}(y)}{\delta\sigma(x)} \tag{5}$$

where we set $T_i^i = \mathcal{A}(x)$.

The considerations of [2, 4] lead to the following general structure of the solution of the Wess-Zumino consistency condition in all (even) dimensions. For any local function of the metric $A(g)$ (i.e. the anomaly) the WZ consistency condition provides the following statement, concerning the structure of $A(g)$:

1) $\mathcal{A}(g)$ is a sum of the following terms with arbitrary coefficients:

a)Type **A**- Euler density,

b)Type **B**- Weyl-invariant polynomials over the Riemann tensor and its covariant derivatives,

c) Covariant total derivatives of polynomials over the Riemann tensor and its covariant derivatives.

We can add the following comment:

2) The latter type of the anomalies are the Weyl variations of local functionals of the metric. Taking into account the fact, that the definition of the measure in the functional

integral can always be changed by multiplying the measure by an exponential of the local functionals (counterterms) of the metric, one can deduce, that the third (i.e. c)) type of solutions of the WZ condition are in that sense inessential and might be called trivial below.

So we can classify the possible anomalies in d=2 and d=4 in the following way:

1) In $d = 2$ there is only a type **A** anomaly,

$$\mathcal{A}_2 = -\frac{c}{24\pi} R \tag{6}$$

where c is the central charge of the corresponding CFT_2;

2) In d=4 there are nontrivial type **A** and **B** anomalies,

$$\mathcal{A}_4 = \alpha E_4 + \beta I_4 \tag{7}$$

where E_4 is the Euler density and I_4 the square of the Weyl tensor,

$$E_4 = R^{ijkl} R_{ijkl} - 4R^{ij} R_{ij} + R^2, \tag{8}$$

$$I_4 = -C^{ijkl} C_{ijkl} = -R^{ijkl} R_{ijkl} + 2R^{ij} R_{ij} - \frac{1}{3} R^2. \tag{9}$$

The constants α and β depend upon the specific mode content of the theory and interaction.

We now consider the change of the measure in the functional integral for the conformal matter field φ in the external gravitational field under the finite Weyl transformation (2).

The measure in the functional integral changes in the following way:

$$D_{e^{2\sigma(x)} g} \varphi = D_g \varphi \exp S(\sigma; g) \tag{10}$$

This type of relation is very important since it is, for example, the starting point for the calculation of the critical exponent of $2d$ gravity [11].

The action $S(\sigma; g)$ in (10) has to satisfy some conditions. First, in the case of infinitesimal transformations $\delta\sigma(x)$ it has to reproduce the trace anomaly:

$$S(\delta\sigma(x); g_{ij}) = \int T_i^i \delta\sigma(x) \sqrt{g} d^{2k} x \tag{11}$$

Second, $S(\sigma; g)$ has to satisfy the following property, which follows from the application of (10) to the composition of two Weyl transformations σ_1 and σ_2:

$$S(\sigma_1 + \sigma_2; g) - S(\sigma_1; e^{2\sigma_2} g) - S(\sigma_2; g) = 0 \tag{12}$$

which means that $S(\sigma; g)$ is the 1-cocycle of the group of Weyl transformations [12].

On the other hand, the action $S(\sigma;g)$ coincides with the finite variation of the anomalous effective action, due to the properties (1) and (10). In other words

$$S(\sigma,g)=W(e^{2\sigma}g)-W(g), \tag{13}$$

and non-triviality of the cocycle $S(\sigma;g)$ follows from the fact that $W(g)$ is a non-local, $Diff(2k)$-invariant functional of $g_{\alpha\beta}$ in the case of type **A** and **B** anomalies. Thus, we can easily calculate the trivial cocycles (type c)) as a coboundary of local counterterms $W_0(g)$ which we shall call from now on 0-cochains:

$$S_0(\sigma,g)=\triangle W_0(g), \tag{14}$$

where we have defined the coboundary operator $\triangle$ on 0-cochains as the finite Weyl variation (13). Then we can define 1-cochains as local functions $W_1(\sigma,g)$ of group parameter and metric with coboundary operator:

$$\triangle W_1(\sigma_1,\sigma_2,g)=W_1(\sigma_1+\sigma_2;g)-W_1(\sigma_1;e^{2\sigma_2}g)-W_1(\sigma_2;g) \tag{15}$$

It is easy to see that $\triangle^2=0$ which is exactly the cocyclic property (12). One can generalize this construction on higher cohomologies of the Weyl group [12].

The nontrivial cocycles can be obtained from the solution of eq. (12) with condition (11). To obtain the solution we have to take $\sigma_2=\sigma$ and $\sigma_1=\delta\sigma$ and get the differential form of (12):

$$\delta S(\sigma;g)=S(\delta\sigma;e^{2\sigma}g)=\int A(R(e^{2\sigma}g))\delta\sigma\sqrt{g}d^{2k}x. \tag{16}$$

The explicit form of the solution for the two-dimensional case is the well-known Liouville action [13]

$$S_{d=2}(\sigma,g)=\frac{c}{24\pi}\int d^2x\sqrt{g}(g^{ij}\partial_i\sigma\partial_j\sigma-R\sigma). \tag{17}$$

We can restore this cocycle in the usual way using the variation of the non-local effective action:

$$S_{d=2}(\sigma,g)=\triangle\frac{-c}{96\pi}\int\sqrt{g}R\frac{1}{\sqrt{g}\Box}\sqrt{g}R. \tag{18}$$

In four dimensions the explicit form of the cocycle, corresponding to E_4, was first found in [10],

$$\begin{aligned}S_E(\sigma,g) \;&=\; \int d^4x\sqrt{g}\Big(2(\nabla_i\sigma\nabla^i\sigma)^2+4\nabla_i\sigma\nabla^i\sigma\nabla^2\sigma\\ &\quad -4(R^{ij}-\frac{1}{2}g^{ij}R)\nabla_i\sigma\nabla_j\sigma-\sigma E_4\Big).\end{aligned} \tag{19}$$

For type **B** anomaly in $d=4$ there is a linear nontrivial cocycle corresponding to the single invariant density $C_{ijkl}C^{ijkl}$:

$$S_C(\sigma,g)=\int C_{ijkl}C^{ijkl}\sigma(x)\sqrt{g}d^4x. \tag{20}$$

This expression satisfies the cocyclic property (12) and can appear in the Weyl transformation of the measure (10). All other freedom in the definition of the cocycle of the Weyl group is connected with the special choice of local counterterms in the regularized effective action. These counterterms will generate trivial cocycles corresponding to the type c) full derivative contribution in the anomaly. In d=4 we have only one local counterterm with independent Weyl variation and therefore here we can obtain only one trivial cocycle:

$$S_0(\sigma, g) = \triangle \int R^2 \sqrt{g} d^4x. \tag{21}$$

As was shown in [10] this trivial cocycle can be used to reduce the nontrivial one connected with the Euler density up to the second order in σ with the fourth-order conformal-invariant differential operator acting on a scalar field of zero conformal weight as a kinetic term and then can be expressed as a Weyl variation of some nonlocal effective action (analog of Liouville theory in d=4). The same but much more complicated structure exists also in d=6 [4] and d=8 [14]. Finally we want to describe the cocycle of the Weyl group for $\mathcal{N} = 4$ Super-Yang-Mills theory. It is well known that the anomaly of this theory vanishes for a Ricci-flat background [7] and has the following form:

$$\mathcal{A} = \alpha \left(E_{(4)} + I_{(4)}\right) = 2\alpha \left(\frac{1}{3}R^2 - R^{ij}R_{ij}\right), \quad \alpha = -\frac{N^2-1}{64\pi^2}. \tag{22}$$

We can easily solve our equation (16) and obtain the cocycle for the d=4, $\mathcal{N} = 4$ SYM anomaly or we can consider this cocycle as a sum of (19) and (20) corresponding to Euler density and Weyl invariant I_4,

$$\begin{aligned} S_{\alpha(E+I)}(\sigma, g) &= 2\alpha \int d^4x \sqrt{g} \left[(\nabla_i\sigma\nabla^i\sigma)^2 + 2\nabla_i\sigma\nabla^i\sigma\nabla^2\sigma \right. \\ &\left. -2(R^{ij} - \frac{1}{2}g^{ij}R)\nabla_i\sigma\nabla_j\sigma - \left(\frac{1}{3}R^2 - R^{ij}R_{ij}\right)\right]. \end{aligned} \tag{23}$$

3 Holographic Effective Action and Cocycle

We now try to derive the latter cocycle from the Heningson-Skenderis AdS construction. It is well known that the effective action of all maximally supersymmetric theories in d=3,4, and 6 can be derived from the on-shell d+1 dimensional gravitational action with asymptotically AdS classical solution [7]:

$$W = \frac{1}{2k_{d+1}^2} \int_{M_{d+1}} d^dx d\rho \sqrt{G} \left(\mathbf{R} + 2\Lambda\right) - \frac{1}{2k_{d+1}^2} \int_{\partial M_d} d^d\sigma \sqrt{\gamma} 2K, \tag{24}$$

$$\Lambda = -\frac{d(d-1)}{2}, \qquad K = D_\mu n^\mu. \tag{25}$$

The boundary term (with γ the induced metric, K the trace of the extrinsic curvature and n^μ a unit vector normal to the boundary) is necessary in order to obtain an action which depends only on first derivatives of the metric and to obtain a well-defined variational problem with Dirichlet boundary conditions for the usual Einstein equations. The metric $G_{\mu\nu}$ is degenerate on the boundary and the boundary metric is defined up to conformal transformations. To obtain the anomalous effective action following standard procedures [7] we have to solve the equation of motion for this action using the Fefferman-Graham [15] coordinate system for $d+1$ dimensional metrics $G_{\mu\nu}$:

$$
\begin{aligned}
ds^2 &= G^{\mu\nu}dx^\mu dx^\nu = \frac{d\rho^2}{4\rho^2} + \frac{g_{ij}(\rho,x)}{\rho}dx^i dx^j, \qquad (26)\\
\mu,\nu &= \rho, 1, 2, ...d; \qquad i,j = 1,2,...d,\\
g(x,\rho) &= g_{(0)} + \cdots + \rho^{d/2}g_{(d)} + h_{(d)}\rho^{d/2}\ln\rho + \cdots. \qquad (27)
\end{aligned}
$$

The equations of motion determine $g_{(n)}(x)$ in terms of $g_{(0)}(x)$:

$$
\begin{aligned}
g_{(2)ij} &= \frac{1}{d-2}(R^{(0)}_{ij} - \frac{1}{2(d-1)}R^{(0)}g_{(0)ij}), \qquad (28)\\
g_{(4)ij} &= \cdots.
\end{aligned}
$$

We then have to insert this expansion of g into the on-shell gravitational action (24) and perform the integration over ρ. However, the on-shell action diverges because the boundary metric is degenerate. We therefore we have to regularize the action using a restriction on the ρ integral with some infrared cut-off $\rho \geq \varepsilon$, and evaluate the boundary term at $\rho = \varepsilon$. Then

$$
S = \frac{1}{2k^2_{d+1}}\int d^dx \int_{\rho\geq\varepsilon} d\rho \frac{d}{\rho^{\frac{d}{2}+1}}\sqrt{g(x,\rho)} - \frac{1}{k^2_{d+1}}\int_{\partial M^\varepsilon_d} d^d\sigma\sqrt{\gamma}K. \qquad (29)
$$

The resulting on-shell effective action contains divergences as poles $\frac{1}{\varepsilon^n}$, a logarithmic divergence and a finite part (which is the essential effective action if we apply the minimal renormalization scheme):

$$
W_{reg} = \frac{1}{2k^{d+1}}\int d^dx\sqrt{\det g_{(0)}}\left(\varepsilon^{-d/2}\mathbf{a}_{(0)} + \ldots - \ln\varepsilon\,\mathbf{a}_{(d)}\right) + W_{finite}. \qquad (30)
$$

From this expression we can easily recognize the anomalous behavior of the unknown W_{finite} by investigating the behavior of the divergent terms with coefficients $\mathbf{a}_{(n)}$, $n = 0, 1, ..., d$ with respect to scale transformation of the metric [7], $\delta g_{(0)} = 2\delta\sigma g_{(0)}$ and $\delta\varepsilon = 2\delta\sigma\varepsilon$, with *constant* $\delta\sigma$. Taking into account that the entire action (30) is invariant and all negative powers of ε terms are scale invariant, one can derive from variation of the logarithmically divergent term that the holographic anomaly is:

$$
\mathcal{A} = -\frac{1}{k^{d+1}}\mathbf{a}_{(d)}. \qquad (31)
$$

The explicit expressions of the logarithmically divergent terms [7, 17] in $d=2$ and $d=4$ are: $\mathbf{a}_{(2)} = \frac{1}{2}R$ and $\mathbf{a}_{(4)} = -R^{ij}R_{ij}/8 + R^2/24$. The value of the gravitational constant in $d=5$ can be obtained from [16],

$$\frac{1}{2\kappa_5^2}=\frac{N^2}{8\pi^2}, \tag{32}$$

where N is the number of coincident $D3$ branes. It is easy to see that inserting this expression into (32), we can recognize the anomalous coefficient of $\mathcal{N}=4$ SYM in the large N limit. The origin of this anomaly generation is also well-known [8]. There are special $d+1$ dimensional diffeomorphisms of asymptotic AdS space-time parameterized with one scalar function $\sigma(x)$ acting on the boundary as a Weyl transformation of the metric $g_{(0)}$.These diffeomorphisms leave the form of the Fefferman-Graham metric invariant. Solving this condition we can write down the following important formulas for such diffeomorphisms [9] in the form of power series in ρ':

$$\rho = \rho' e^{-2\sigma(x')}+\sum_{k=2} a_{(k)}(x')\rho'^k, \qquad x^i = x'^i+\sum_{k=1} a^i_{(k)}(x')\rho'^k, \tag{33}$$

$$a_{(2)} = -\frac{1}{2}(\partial\sigma)^2 e^{-4\sigma}, \qquad a_{(3)}=\frac{1}{4}e^{-6\sigma}\left(\frac{3}{4}(\partial\sigma)^2+\partial^i\sigma\partial^j\sigma g_{(2)ij}\right), \tag{34}$$

$$a^i_{(1)} = \frac{1}{2}\partial^i\sigma e^{-2\sigma},\ a^i_{(2)}=-\frac{1}{4}e^{-4\sigma}\left(\partial_k\sigma g^{ik}_{(2)}+\frac{1}{2}\partial^i\sigma(\partial\sigma)^2+\frac{1}{2}\Gamma^i_{kl}\partial^k\sigma\partial^l\sigma\right). \tag{35}$$

The nice property of these diffeomorphisms is that the transformation rules for $g_{(n)}$ are the usual Weyl transformations of metric $g_{(0)}$:

$$g'_{(0)ij} = e^{2\sigma}g_{(0)ij}, \tag{36}$$

$$g'_{(2)ij} = g_{(2)ij}+\nabla_i\nabla_j\sigma-\nabla_i\sigma\nabla_j\sigma+\frac{1}{2}(\nabla\sigma)^2 g_{(0)ij}, \tag{37}$$

$$g'_{(2)ij} = \cdots.$$

The important point here is that the anomalies are described only by the logarithmic divergence originating only from the bulk integral. The interesting point about the boundary extrinsic curvature term is the following: *there is no contribution from this term in the anomaly, because there is no ρ integral there, and there is no ε independent R^2 order contribution from this term* . We can easily check this from the definition of the boundary term at the point $\rho=\varepsilon$ [7]:

$$\frac{1}{2k^{d+1}}\int_{\partial M_d} d^d\sigma\sqrt{\gamma}2D_\mu n^\mu=\frac{1}{2k^{d+1}}\int d^dx\frac{1}{\rho^{d/2}}\left(-2d\sqrt{\det g(x,\rho)}+4\rho\partial_\rho\sqrt{\det g(x,\rho)}\right)|_{\rho=\epsilon}, \tag{38}$$

using the trivial identity

$$(-2d+4\rho\partial_\rho)\rho^{d/2}=0. \tag{39}$$

In addition we can note that the coefficient $h_{(d)}$ in the expansion of the metric is traceless and there is no room for a contribution in the anomaly from this term also. The absence of an ε independent term is important because this R^2 type term could be considered like a local counterpart to the effective action and produce after Weyl variation a trivial ($\Box R$) contribution in the anomaly. We can now try to apply the special diffeomorphisms (33) to the divergence part of the effective action (30), and we can expect to obtain our cocycle (23) constructed earlier from the solution of the cohomological equation (12). Before doing that we note that the definitions of boundary and boundary term are invariant with respect to $d+1$ dimensional diffeomorphisms. Indeed, the definition of the regularized boundary $X^{\mu}(\sigma) = (x^i = \sigma^i, \ \rho = \varepsilon)$ will change after diffeomorphism to $X'^{\mu}(\sigma) = (x'^i = x'^i(\sigma, \varepsilon), \rho' = \rho'(\sigma, \varepsilon))$ where $x'^i(x, \rho), \rho'(x, \rho)$ are diffeomorphism functions inverse to those defined in (33). But the boundary term depends on the $d+1$ dimensional covariant divergence of the normal vector $D'_{\mu} n'^{\mu} = D_{\mu} n^{\mu}$ and on the induced metric $\gamma_{ij}(\sigma)$ which is also invariant:

$$\gamma_{ij}(\sigma) = G_{\mu\nu} \frac{\partial X^{\mu}}{\partial \sigma^i} \frac{\partial X^{\nu}}{\partial \sigma^i} = G'_{\mu\nu} \frac{\partial X'^{\mu}}{\partial \sigma^i} \frac{\partial X'^{\nu}}{\partial \sigma^i}. \tag{40}$$

Thus we can deduce that there is no contribution to the cocycle from the boundary term, just as there is no contribution from this term in the anomaly. Then we can concentrate on the pure bulk contribution in the effective action:

$$W^{bulk}_{reg} = \frac{1}{2k^2_{d+1}} \int d^d x \int_{\rho \geq \varepsilon} d\rho \frac{d}{\rho^{\frac{d}{2}+1}} \sqrt{g(x, \rho)}. \tag{41}$$

After integration in $d = 2, 4$ we obtain the divergent part of the action:

$$W^{bulk}_{d=2} = \frac{1}{2k^2_{d+1}} \int d^d x \sqrt{g_{(0)}(x)} \left(\frac{2}{\varepsilon} - \ln \varepsilon \frac{1}{2} R \right), \tag{42}$$

$$W^{bulk}_{d=4} = \frac{1}{2k^2_{d+1}} \int d^d x \sqrt{g_{(0)}(x)} \left(\frac{2}{\varepsilon^2} + \frac{1}{3\varepsilon} R - \ln \varepsilon \left(\frac{1}{24} R^2 - \frac{1}{8} R^{ij} R_{ij} \right) \right). \tag{43}$$

We can now apply diffeomorphisms (33) to the bulk effective action (41) and calculate the cocycle of the Weyl group because this transformation reproduces on the boundary the usual Weyl transformation of the boundary metric. For this one should note that the integrand of (41) is the covariant volume $\sqrt{G}$, so that we get the following transformation of our parameters.Instead of initial coordinates x, ρ , metric $g(x, \rho) = g[g_0(x, \rho)]$ and integration limit $\rho = \varepsilon$, we have now $x', \rho', g'(x', \rho') = g'[g'_0(x', \rho') = e^{2\sigma(x')} g_0(x')]$ and an x'-dependent integration limit $\rho' = f(x', \varepsilon)$, where $f(x', \varepsilon)$ is the solution with respect to ρ' of the equation

$$\varepsilon = \rho(x', \rho') = \rho' e^{-2\sigma(x')} + \sum_{k=2} a_{(k)}(x') \rho'^k \tag{44}$$

which follows from (33) to (37). The solution of (44) is

$$\rho' = f(x', \varepsilon) = \varepsilon e^{2\sigma(x')} + \sum_{k=2} b_{(k)}(x') \varepsilon^k, \tag{45}$$

$$
\begin{aligned}
b_{(2)}(x') &= -a_{(2)}(x')e^{6\sigma(x')} = \frac{e^{2\sigma(x')}}{2}\partial_i\sigma(x')\partial_j\sigma(x')g^{ij}_{(0)}, & (46)\\
b_{(3)}(x') &= 2a^2_{(2)}(x')e^{10\sigma(x')} - a_{(2)}(x')e^{8\sigma(x')}\\
&= \frac{e^{2\sigma(x')}}{4}\left(\frac{5}{4}(\partial\sigma)^4 - \partial_i\sigma(x')\partial_j\sigma(x')g^{ij}_{(2)}\right), & (47)\\
b_{(4)}(x') &= \cdots.
\end{aligned}
$$

Finally we have to calculate the cocycle by performing the following transformation of the divergent part of the bulk effective action

$$
\begin{aligned}
S(\sigma, g_{(0)}) &= \lim_{\varepsilon\to 0}\left[\frac{1}{2k^2_{d+1}}\int d^dx' \int_{\rho'\geq f(x',\varepsilon)} d\rho' \frac{d}{\rho'^{\frac{d}{2}+1}}\sqrt{g'(x',\rho')}\right.\\
&- \left.\frac{1}{2k^2_{d+1}}\int d^dx \int_{\rho\geq\varepsilon} d\rho \frac{d}{\rho^{\frac{d}{2}+1}}\sqrt{g(x,\rho)}\right]. & (48)
\end{aligned}
$$

Actually for calculation of the transformed action in $d = 2, 4$ we only have to replace in (42) to (43) ε by $f(x', \varepsilon)$ and $g_{(0)}$ by $g'_{(0)}$. We use the following formulas

$$
\begin{aligned}
\frac{1}{f(x',\varepsilon)} &= e^{-2\sigma(x')}\left[\frac{1}{\varepsilon} - \frac{1}{2}\partial_i\sigma(x')\partial_j\sigma(x')g^{ij}_{(0)} + \mathcal{O}(\varepsilon)\right], & (49)\\
\frac{1}{f(x',\varepsilon)^2} &= e^{-4\sigma(x')}\left[\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\partial_i\sigma(x')\partial_j\sigma(x')g^{ij}_{(0)}\right.\\
&+ \left.\frac{1}{2}\left(\frac{1}{4}(\partial\sigma)^4 + \partial_i\sigma(x')\partial_j\sigma(x')g^{ij}_{(2)}\right) + \mathcal{O}(\varepsilon)\right], & (50)\\
\mathbf{a}'_{(2)}(g'_{(0)}) &= e^{-2\sigma}\left(\mathbf{a}_{(2)}(g_{(0)}) + \Box\sigma\right), \quad d = 2, & (51)\\
\mathbf{a}'_{(4)}(g'_{(0)}) &= e^{-4\sigma}\left[\mathbf{a}_{(4)}(g_{(0)}) - \frac{1}{2}\nabla_i\left(\left(R^{ij}_{(0)} - \frac{1}{2}R_{(0)}g^{ij}_{(0)}\right)\partial_j\sigma\right.\right.\\
&+ \left.\left.\frac{1}{2}\nabla^j(\partial\sigma)^2 - \nabla^i\sigma\Box\sigma - \nabla^i\sigma(\partial\sigma)^2\right)\right] \quad d = 4. & (52)
\end{aligned}
$$

We find that in (48) all divergences will cancel for both $d = 2, 4$ cases (which is a nice indication of the correctness of our idea) and we obtain the following finite expressions for (48) in $d = 2, 4$

$$
\begin{aligned}
S^{d=2}(\sigma, g_{(0)}) &= \frac{1}{2k_3^2}\int d^2x\sqrt{g}(g^{ij}\partial_i\sigma\partial_j\sigma - R_{(0)}\sigma), & (53)\\
S^{d=4}(\sigma, g_{(0)}) &= \frac{1}{2k_5^2}\int d^4x\sqrt{g}\left[\frac{1}{4}(\partial\sigma)^4 + \frac{1}{2}\Box\sigma(\partial\sigma)^2 - \frac{1}{2}\left(R^{ij}_{(0)} - \frac{1}{2}R_{(0)}g^{ij}_{(0)}\right)\partial_i\sigma\partial_j\sigma\right.\\
&- \left.\frac{1}{4}\left(\frac{1}{3}R^2_{(0)} - R^{ij}_{(0)}R^{(0)}_{ij}\right)\right]. & (54)
\end{aligned}
$$

These expressions are in full agreement with (17) and (23) after using the standard relation $\frac{1}{2k_3^2} = \frac{c}{24\pi}$ and the $5d$ gravitational constant (32). Thus we have shown that finite diffeomorphisms (33) applied to bulk contributions of the AdS effective action generate cocycles of the Weyl group in dimensions 2 and 4 with the right form.

4 Conclusion

Above we considered the 1-cocycle of the Weyl group (i.e. integral trace anomaly) in the AdS/CFT approach. We have shown that special $d+1$ dimensional diffeomorphisms [8] of the finite form [9] applied to the divergent part of the AdS action originating from the bulk integral reproduce the 1-cocycle of the Weyl group corresponding to the correct anomaly in $d = 2, 4$. It will be interesting to extend this consideration to $d = 6$. This will be interesting because the renormalization properties of anomaly coefficients in AdS_7/CFT_6 still lack explanation because we still do not have a good explanation of the renormalization of the Euler density coefficient from the weak to the strong-coupling regimes [18] for the (2,0) tensor multiplet, like the difference in the R-symmetry anomaly structure for this multiplet in weak and strong-coupling regimes [19]. Thus any investigation in the field of anomalous behavior in the AdS/CFT picture will be interesting and important.

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