Gross–Ooguri Phase Transition at Zero and Finite Temperature: Two Circular Wilson Loop Case

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Abstract

In the context of AdS/CFT correspondence the two Wilson loop correlator is examined at both zero and finite temperatures. On the basis of an entirely analytical approach we have found for Nambu–Goto strings the functional relation $dS_c^{(Reg)}/dL = 2\pi k$ between Euclidean action S_c and loop separation L with integration constant k, which corresponds to the analogous formula for point–particles. The physical implications of this relation are explored in particular for the Gross–Ooguri phase transition at finite temperature.

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Recently much attention has been paid to the Wilson loop correlator in large–N gauge theory. This interest results mainly from the fact that AdS/CFT duality [1] [2] makes it more tractable to understand this highly nontrivial quantum field theory effect through a classical description of the string configuration in the AdS background. Using this AdS/CFT duality Maldacena [3] was able to calculate for the first time the expectation value of the rectangular Wilson loop operator at zero temperature and found that the interquark potential exhibits the Coulomb type behavior expected from conformal invariance of the gauge theory, as well as indications of the screening of the charge. Furthermore Witten [4] has shown that the AdS/CFT duality can be used to explore finite temperature behavior of gauge theory by compactifying the AdS Euclidean time on a circle of radius $\propto 1/\text{temperature}$.

Maldacena's computational technique has already been extended to the finite temperature case by replacing the AdS metric by a Schwarzschild–AdS metric [5,6] which implies the same boundary conditions [7]. The main differences of the finite temperature case from the zero temperature one are (1) the presence of a maximal separation distance between quarks, and (2) the appearance of a cusp (or bifurcation point) in the plot of interquark potential–vs–interquark distance. These differences strongly suggest that there is a hidden functional relation between physical quantities, and indeed such a relation has been derived explicitly in Ref. [8].

In this letter we show that there is a similar relation in the two Wilson loop case. It is known [9] that the correlation function of two circular Wilson loops implies a phase transition analogous to that between the catenoid as minimal solution of the area connecting two concentric circles [10] and the associated discontinuous Goldschmit solution. In fact, this Gross–Ooguri(GO) phase transition takes place due to the instability of the classical string configuration. The GO phase transition at zero temperature has been examined in more detail in Ref. [11] by solving the equations of motion of the Nambu–Goto string action.

In this letter we will approach the GO phase transition at zero temperature entirely analytically, which enables us to derive a functional relation $dS_c^{Reg}/dL = 2\pi k$ between the regularized Euclidean classical action S_c^{Reg} and the separation L of the Wilson loops, k

being a constant of integration. We will also approach the GO phase transition at finite temperature and examine the physical implications of the functional relation at the finite temperature phase transition.

We use the classical Nambu–Goto action for a string world sheet

$$S_{NG} = \int d\tau d\sigma \sqrt{det g_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu}. \tag{1}$$

where $g_{\mu\nu}$ is the Euclidean $AdS_5 \times \mathbf{S}^5$ metric. We want to study the theory of a type IIB string with coordinates $X^\mu(\tau, \sigma)$ which ends on the near extremal D3–brane, as the dual of an $SU(N)$ gauge theory in the sense of refs. [1,3], the case of large N in the gauge theory corresponding to weak coupling perturbative string theory in the semiclassical approximation, and so to supergravity expanded about an $AdS_5 \times \mathbf{S}^5$ background. The quantity under the square root is the induced metric on the world sheet of the Nambu–Goto string.

The near extremal Euclidean Schwarzschild–AdS_5 metric of a $D3$–brane (with constant dilaton, and the S^5 coordinates eliminated) [12] is given in Poincaré coordinates by

$$ds_E^2 = \left[U^2 \left(f(U) dt^2 + \sum_{i=1}^{3} dx_i dx_i \right) + \frac{f(U)^{-1}}{U^2} dU^2 \right]. \tag{2}$$

Here t is Euclidean time of target space, and we choose $R_{ads} = \alpha' = 1$ for simplicity, where R_{ads} and α' are the radius of AdS_5 and Regge slope respectively [1]. U is the holographic coordinate [1]. The function $f(U)$ is given by $f(U) = 1 - U_T^4/U^4$ where U_T^2 is a parameter proportional to the energy density above extremality on the brane [1]; U_T is proportional to the external temperature T defined by $T = U_T/\pi R_{ads}$ [6] which enters through the periodic identification of $t \to t + 1/T$ to make the horizon at $U = U_T$ regular [9]. The world volume coordinates t, x_i of the brane can be regarded as coordinates of the dual $3+1$ dimensional gauge theory.

For simplicity again, we choose cylindrical coordinates $dx_i dx_i = dx^2 + dr^2 + r^2 d\phi^2$, which results in

$$ds_E^2 = \frac{1}{z^2} \left[(1 - U_T^4 z^4) dt^2 + \frac{dz^2}{1 - U_T^4 z^4} + dx^2 + dr^2 + r^2 d\phi^2 \right] \tag{3}$$

where $z \equiv 1/U$. Thus the AdS boundary is located at $z = 0$.

For reasons of symmetry we make the following *ansatz* for the minimal surface [11]

$$X^0 = t = 0,\ X^1/r = \phi = \sigma,\ X^2 = x(\tau),\ X^3 = r(\tau),\ X^4 = z(\tau). \tag{4}$$

Then (with elements $g_{\mu\nu}$ of eq.(3)) the Nambu–Goto action becomes

$$S_{NG} = 2\pi \int d\tau \frac{r}{z^2}\sqrt{\mathcal{L}_{\mathcal{T}}} \tag{5}$$

where

$$\mathcal{L}_{\mathcal{T}} = x'^2 + r'^2 + \frac{z'^2}{1 - U_T^4 z^4} \tag{6}$$

and the prime denotes differentiation with respect to τ. The Euler–Lagrange equations derived from action (5) are

$$\begin{aligned}
&\frac{r}{z^2}\frac{x'}{\sqrt{\mathcal{L}_{\mathcal{T}}}} = k, \\
&\frac{d}{d\tau}\left[\frac{rr'}{z^2\sqrt{\mathcal{L}_{\mathcal{T}}}}\right] - \frac{\sqrt{\mathcal{L}_{\mathcal{T}}}}{z^2} = 0, \\
&\frac{d}{d\tau}\left[\frac{rz'}{z^2\sqrt{\mathcal{L}_{\mathcal{T}}}(1 - U_T^4 z^4)}\right] + \frac{2r\sqrt{\mathcal{L}_{\mathcal{T}}}}{z^3} - \frac{2U_T^4 rzz'^2}{\sqrt{\mathcal{L}_{\mathcal{T}}}(1 - U_T^4 z^4)^2} = 0,
\end{aligned} \tag{7}$$

where k is the integration constant arising from the equation of motion for x.

With a gauge choice $\tau = x$ the equations of motion take the form

$$\begin{aligned}
r'^2 + \frac{z'^2}{1 - U_T^4 z^4} + 1 &= \frac{r^2}{k^2 z^4}, \\
r'' - \frac{r}{k^2 z^4} &= 0, \\
\frac{d}{d\tau}\left[\frac{z'}{1 - U_T^4 z^4}\right] + \frac{2r^2}{k^2 z^5} - \frac{2U_T^4 z^3 z'^2}{(1 - U_T^4 z^4)^2} &= 0.
\end{aligned} \tag{8}$$

We now assume that two circular Wilson loops are located at $x = \pm L/2$. Then we have to solve Eq.(8) with boundary conditions

$$\begin{aligned}
&r(-L/2) = r(L/2) = R, \\
&z(-L/2) = z(L/2) = \epsilon \approx 0.
\end{aligned} \tag{9}$$

Here R and L are respectively the radius of the circular Wilson loops and the distance between them. We also introduced the positive infinitesimal constant ϵ for the regularization of the minimal surface area later.

We first consider the zero temperature case ($U_T = 0$). This case has been analyzed in Ref. [11] partly analytically. Here we approach this case entirely analytically using various kinds of elliptic functions. This completely analytical approach enables one to derive a hidden functional relation explicitly.

For $U_T = 0$ Eqs.(8) become

$$\begin{aligned} r'^2 + z'^2 + 1 - \frac{r^2}{k^2 z^4} &= 0, \\ r'' - \frac{r}{k^2 z^4} &= 0, \\ z'' + \frac{2r^2}{k^2 z^5} &= 0, \end{aligned} \tag{10}$$

and after some manipulations can easily be shown to yield the equation $r^2 + z^2 + x^2 = a^2$, which is solved by

$$\begin{aligned} r &= \sqrt{a^2 - x^2}\cos\theta, \\ z &= \sqrt{a^2 - x^2}\sin\theta, \end{aligned} \tag{11}$$

where $a^2 \equiv R^2 + L^2/4$, and θ obeys

$$\theta' = \pm\frac{a}{a^2 - x^2}\sqrt{\frac{\cos^2\theta}{k^2 a^2 \sin^4\theta} - 1}. \tag{12}$$

Here we take the upper sign for $x \in [-L/2, 0]$ and the lower sign for $x \in [0, L/2]$.

Using the integral formula [13]

$$\int_t^b dt\sqrt{\frac{b-t}{(a-t)(t-c)(t-d)}} = (a-b)g\left[\Pi(\phi, \alpha^2, \kappa) - F(\phi, \kappa)\right] \tag{13}$$

where $a > b \geq t \geq c > d$, and

$$\begin{aligned} \kappa &= \sqrt{\frac{(b-c)(a-d)}{(a-c)(b-d)}}, \quad g = \frac{2}{\sqrt{(a-c)(b-d)}} \\ \alpha &= \sqrt{\frac{b-c}{a-c}}, \quad \phi = \sin^{-1}\sqrt{\frac{(a-c)(b-t)}{(b-c)(a-t)}} \end{aligned} \tag{14}$$

and Π and F are elliptic integrals of the third and first kinds respectively, one can show directly that

$$I(\theta) \equiv \int_0^\theta d\theta \frac{\sin^2\theta}{\sqrt{\cos^2\theta - k^2a^2\sin^4\theta}} \tag{15}$$
$$= \frac{1}{ka}\frac{\beta_+ - 1}{\sqrt{\beta_+ - \beta_-}}\left[\Pi\left(\sin^{-1}\sqrt{\frac{(\beta_+ - \beta_-)(1-\cos^2\theta)}{(1-\beta_-)(\beta_+ - \cos^2\theta)}}, \frac{1-\beta_-}{\beta_+ - \beta_-}, \kappa\right)\right.$$
$$\left. -F\left(\sin^{-1}\sqrt{\frac{(\beta_+ - \beta_-)(1-\cos^2\theta)}{(1-\beta_-)(\beta_+ - \cos^2\theta)}}, \kappa\right)\right]$$

where

$$\beta_\pm = \frac{(2k^2a^2+1) \pm \sqrt{1+4k^2a^2}}{2k^2a^2}, \tag{16}$$
$$\kappa = \sqrt{\frac{\beta_+(1-\beta_-)}{\beta_+ - \beta_-}}.$$

Using Eq.(15) one can integrate Eq.(12) completely. The final result for θ assumes the form

$$I(\theta) = \frac{1}{2ka}\ln\frac{(a+\frac{L}{2})(a \pm x)}{(a-\frac{L}{2})(a \mp x)}, \tag{17}$$

where the upper and lower signs correspond again to $x \in [-L/2, 0]$ and $x \in [0, L/2]$ respectively.

From Eqs.(12) and (17) one can show easily that $\theta(-L/2) = \theta(L/2) = 0$ and $\theta_0 \equiv \theta(0) = \cos^{-1}\sqrt{\beta_-}$, where the latter is the result of $\theta'(0) = 0$. Inserting $x = 0$ in Eq.(17) and realizing that $\sin^{-1}\sqrt{\frac{(\beta_+-\beta_-)(1-\cos^2\theta)}{(1-\beta_-)(\beta_+-\cos^2\theta)}} \to \frac{\pi}{2}$ at this point, one can derive

$$\mathcal{F} = \frac{1}{2}\ln\frac{a+\frac{L}{2}}{a-\frac{L}{2}} = \ln\frac{\sqrt{R^2+\frac{L^2}{4}}+\frac{L}{2}}{R} \tag{18}$$

where

$$\mathcal{F} = \frac{\beta_+ - 1}{\sqrt{\beta_+ - \beta_-}}\left[\Pi\left(\frac{1-\beta_-}{\beta_+ - \beta_-}, \kappa\right) - K(\kappa)\right] \tag{19}$$

and Π and K are complete elliptic integrals.

Hence the k–dependence of L is obtained by solving

$$L = (2\sinh \mathcal{F})R \tag{20}$$

numerically. Fig. 1 shows the k–dependence of L when $R = 1$. Fig. 1 indicates that there is a maximal distance L_* for the existence of the classical catenoid solution. If $L > L_*$, the classical catenoid solution becomes unstable and hence the physically relevant solution in this case becomes two discontinuous one Wilson loop solutions, which are the so–called Goldschmit discontinuous solutions.

The minimal surface is directly computed by calculating the classical action

$$S_c = 4\pi \int_{\frac{\epsilon}{R}}^{\theta_0} \frac{\cot^2\theta}{\sqrt{\cos^2\theta - k^2a^2\sin^4\theta}}. \tag{21}$$

Using the integral formulas [13]

$$\begin{aligned} &\int_t^b \frac{dt}{t-c}\sqrt{\frac{a-t}{(b-t)(t-c)(t-d)}} = \frac{a-b}{b-c} g \int_0^{u_1} nc^2 u du, \\ &\int nc^2 u du = \frac{1}{\kappa'^2}\left[\kappa'^2 u - E(u) + dnu \;\; tnu\right], \end{aligned} \tag{22}$$

where $a > b \geq t \geq c > d$,

$$\begin{aligned} &g = \frac{2}{\sqrt{(a-c)(b-d)}}, \;\; \kappa^2 \equiv 1 - \kappa'^2 = \frac{(b-c)(a-d)}{(a-c)(b-d)} \\ &\phi = \sin^{-1}\sqrt{\frac{(a-c)(b-t)}{(b-c)(a-t)}}, \;\; u_1 = sn^{-1}[\sin\phi] \end{aligned} \tag{23}$$

one can directly evaluate Eq.(21) which is

$$\begin{aligned} S_c &= \frac{4\pi R}{\epsilon} + S_c^{(Reg)}, \\ S_c^{(Reg)} &= 4\pi(1 + 4k^2a^2)^{\frac{1}{4}}\left[(1-\kappa^2)K(\kappa) - E(\kappa)\right], \end{aligned} \tag{24}$$

where κ is the same as that of Eq.(16).

The area of the discontinuous solution has been calculated in Ref. [14] using the special conformal transformation. Of course, this can also be evaluated directly by considering the case of one circular Wilson loop. Then the minimal surface of the discontinuous solution is

$$\begin{aligned} S_{dc} &= \frac{4\pi R}{\epsilon} + S_{dc}^{(Reg)}, \\ S_{dc}^{(Reg)} &= -4\pi. \end{aligned} \tag{25}$$

One should note that $S_c^{(Reg)}$ coincides with $S_{dc}^{(Reg)}$ at $k = 0$. This is expected from the first of Eqs.(7) which indicates that there is no propagation of the string along x when $k = 0$.

Fig. 2 shows the L–dependence of $S_c^{(Reg)}$ and $S_{dc}^{(Reg)}$. One should note that $S_c^{(Reg)}$ merges smoothly with $S_{dc}^{(Reg)}$ at $L = 0$, which indicates again that $S_c^{(Reg)} = S_{dc}^{(Reg)}$ at $k = 0$. The appearance of the cusp in $S_c^{(Reg)}$ at $L = L_*$ strongly suggests that there is a hidden functional relation [8] in the case of the two Wilson loop correlator.

In order to derive this relation explicitly we differentiate $S_c^{(Reg)}$ and L with respect to the elliptic modulus κ. This is straightforwardly achieved using the various derivative formulas of elliptic functions [13]. The final relations are simply

$$\begin{aligned} \frac{dS_c^{(Reg)}}{d\kappa} &= -\frac{4\pi\kappa}{(2\kappa^2-1)^{\frac{3}{2}}}\left[K(\kappa) - 2E(\kappa)\right], \\ \frac{dL}{d\kappa} &= -\frac{2a}{\kappa'\sqrt{2\kappa^2-1}}\left[K(\kappa) - 2E(\kappa)\right]. \end{aligned} \tag{26}$$

One should note that the coefficients of the complete elliptic integrals in the brackets coincide with each other, which, thus yields

$$\frac{dS_c^{(Reg)}}{dL} = 2\pi k. \tag{27}$$

As observed in Ref. [8], this relation has a close analogy with the point–particle formula $dS_E/dP = \mathcal{E}$, where S_E, P and $\mathcal{E}$ are Euclidean action, period and energy of the classical point–particle. It is worthwhile noting that Eq.(27) and a condition $S_c^{(Reg)} = S_{dc}^{(Reg)}$ at $L = 0$ determine completely the L–dependence of $S_c^{(Reg)}$ from the k–dependence of L since Eq.(27) says that $S_c^{(Reg)} = 2\pi \int k dL$ up to the constant.

We now turn to the GO phase transition at finite temperature. In this case it seems impossible to attack Eqs.(8) analytically. However, the analysis of the zero temperature case shows how one can solve Eqs.(8) numerically. In fact, one can solve the second and third of Eqs.(8) simultaneously using the first equation as a relation of boundary conditions at $x = 0$. Solving these coupled differential equations completely determines L and R. If R is fixed by R_0, the only step that remains is to select the solutions which yield $R = R_0$.

Fig. 3 shows the k-dependence of L at finite temperature when $R = 1$. Fig. 3 indicates that the peak point moves to the right, and the maximum distance of the Wilson loop L_* becomes larger when the temperature increases.

Next we consider the minimal surface area in the finite temperature case. For this quantity the numerical approach is not a useful tool in view of the divergent term which arises in the course of the calculation. It is, in fact, a formidable task to regularize the minimal surface in the numerical technique. However, assuming that Eq.(27) also holds in the finite temperature case, one can conjecture the L–dependence of $S_c^{(Reg)}$ from Fig. 3 since Eq.(27) tells us that $S_c^{(Reg)} = 2\pi \int kdL$ up to a constant. To fix this constant we need $S_{dc}^{(Reg)}$ of the finite temperature case.

However, even with the numerical technique the computation of $S_{dc}^{(Reg)}$ is not an easy problem – again in view of the divergent term. In order to evaluate $S_{dc}^{(Reg)}$ at finite temperatures we observe that the minimal surface becomes

$$S_c = 4\pi k \left[rr' \mid_{x=\frac{L}{2}} - \int_0^{\frac{L}{2}} dx r'^2 \right]. \tag{28}$$

This is obtained from Eq.(5) and Eq.(8) and by performing an appropriate partial integration. If one examines the behavior of $r(x)$ and $z(x)$ as $x \to L/2$, it is easy to show that the second term in Eq.(28) is finite. Hence the divergence of S_c is contained in the first term of Eq.(28). In the zero temperature case, for example, one can derive the asymptotic behavior of $r(x)$ and $z(x)$ for $x \approx L/2$:

$$\begin{aligned} r &\approx R - \frac{1}{2R}\left(\frac{3R}{kL}\right)^{\frac{2}{3}} y^{\frac{4}{3}} + \frac{y^2}{2R} + \cdots, \\ z &\approx \left(\frac{3R}{kL}\right)^{\frac{1}{3}} y^{\frac{2}{3}} - \frac{3}{5RkL} y^2 + \frac{1}{2R^2}\left(\frac{3R}{kL}\right)^{\frac{1}{3}} y^{\frac{8}{3}} + \cdots, \end{aligned} \tag{29}$$

where $y^2 \equiv L^2/4 - x^2$. It is important to note that the coefficient of y^2 in r is not determined by direct expansion but from the relation $r^2 + z^2 + x^2 = a^2$, which does not have a counterpart in the finite temperature case. Hence in the zero temperature case $rr' \mid_{x=\frac{L}{2}}$ becomes

$$rr' \mid_{x=\frac{L}{2}} = \frac{R}{k\epsilon} - \frac{L}{2} \tag{30}$$

which yields same divergence term $4\pi R/\epsilon$ and $S_c^{(Reg)} = -2\pi k L - 4\pi k \int_0^{\frac{L}{2}} dx r'^2$. If one plots this numerically, one can reproduce $S_c^{(Reg)}$ in Fig. 2. The important point one should note is that the finite term in $4\pi k r r' \mid_{x=\frac{L}{2}}$, $i.e. -2\pi k L$, becomes zero in the limit $k \to 0$. Thus the limit $k \to 0$ of $S_c^{(Reg)}$, which is nothing but $S_{dc}^{(Reg)}$, originates from the second term of Eq.(28). We believe this property is maintained in the finite temperature case. Then one can plot the temperature–dependence of $S_{dc}^{(Reg)}$ which is shown in Fig. 4. Fig. 4 completely determines the shift constant in $S_c^{(Reg)} = 2\pi \int k dL$. Fig. 5 shows $S_c^{(Reg)}$ at various temperatures. As explained the appearance of cusp in $S_c^{(Reg)}$ indicates the non-monotonic behavior of k-dependence of L.

In summary, we analyzed above the Gross–Ooguri phase transition at zero and finite temperatures. We obtained the functional relation $dS_c^{(Reg)}/dL = 2\pi k$ which is the Nambu–Goto string analogue of the formula relating Euclidean action to period and energy of a classical point–particle [15]. It is interesting to investigate this point–particle analogy in more detail by calculating the spectrum of the fluctuation operator. This work is in progress.

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FIGURES

FIG. 1. k-dependence of L at zero temperature

FIG. 2. L-dependence of $S_c^{(reg)}$ and $S_{dc}^{(reg)}$ at zero temperature

FIG. 3. k-dependence of L at finite temperature

FIG. 4. temperature-dependence of $S_{dc}^{(reg)}$

FIG. 5. L-dependence of $S_c^{(reg)}$ at finite temperature

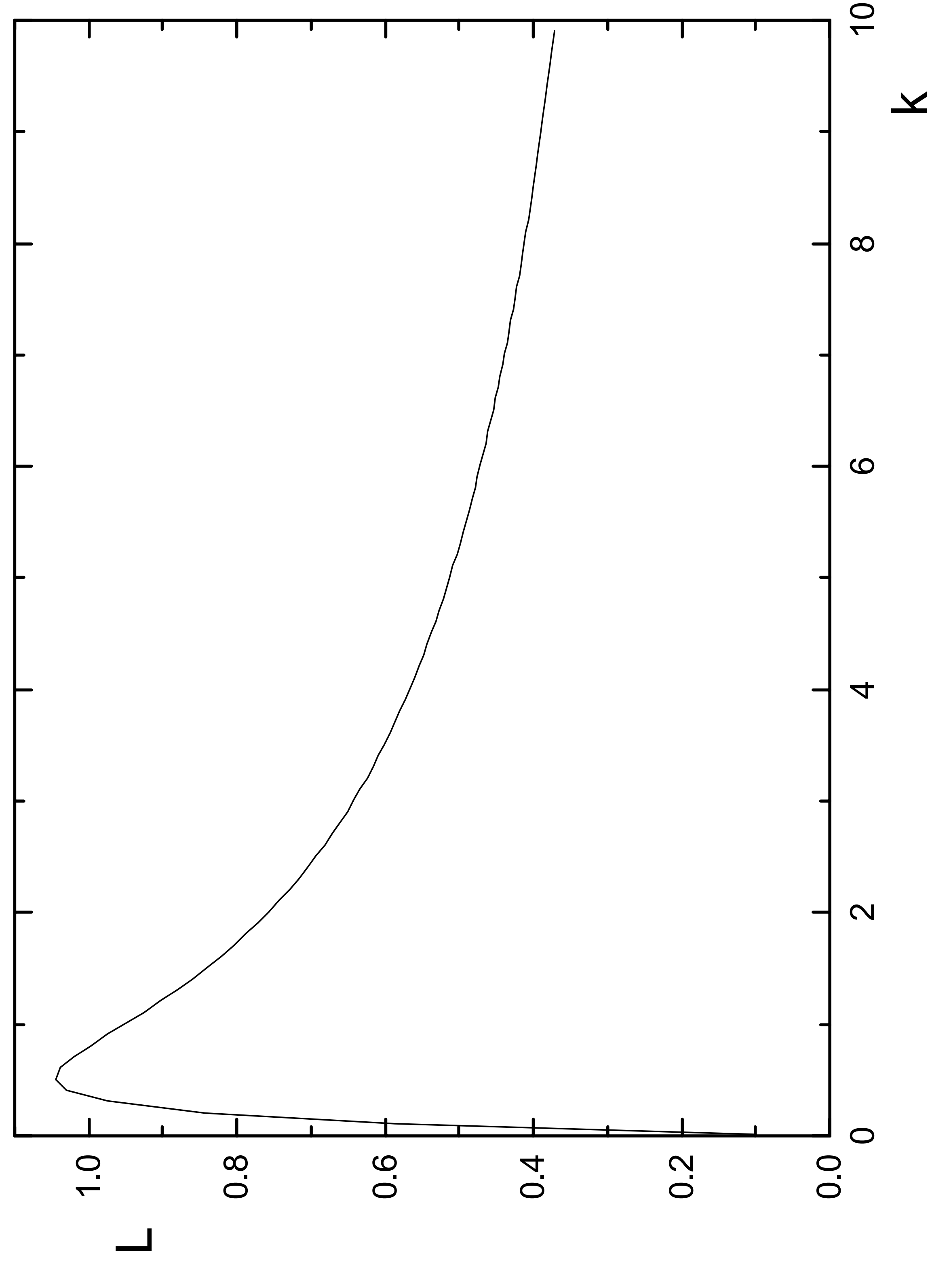

Fig. 1

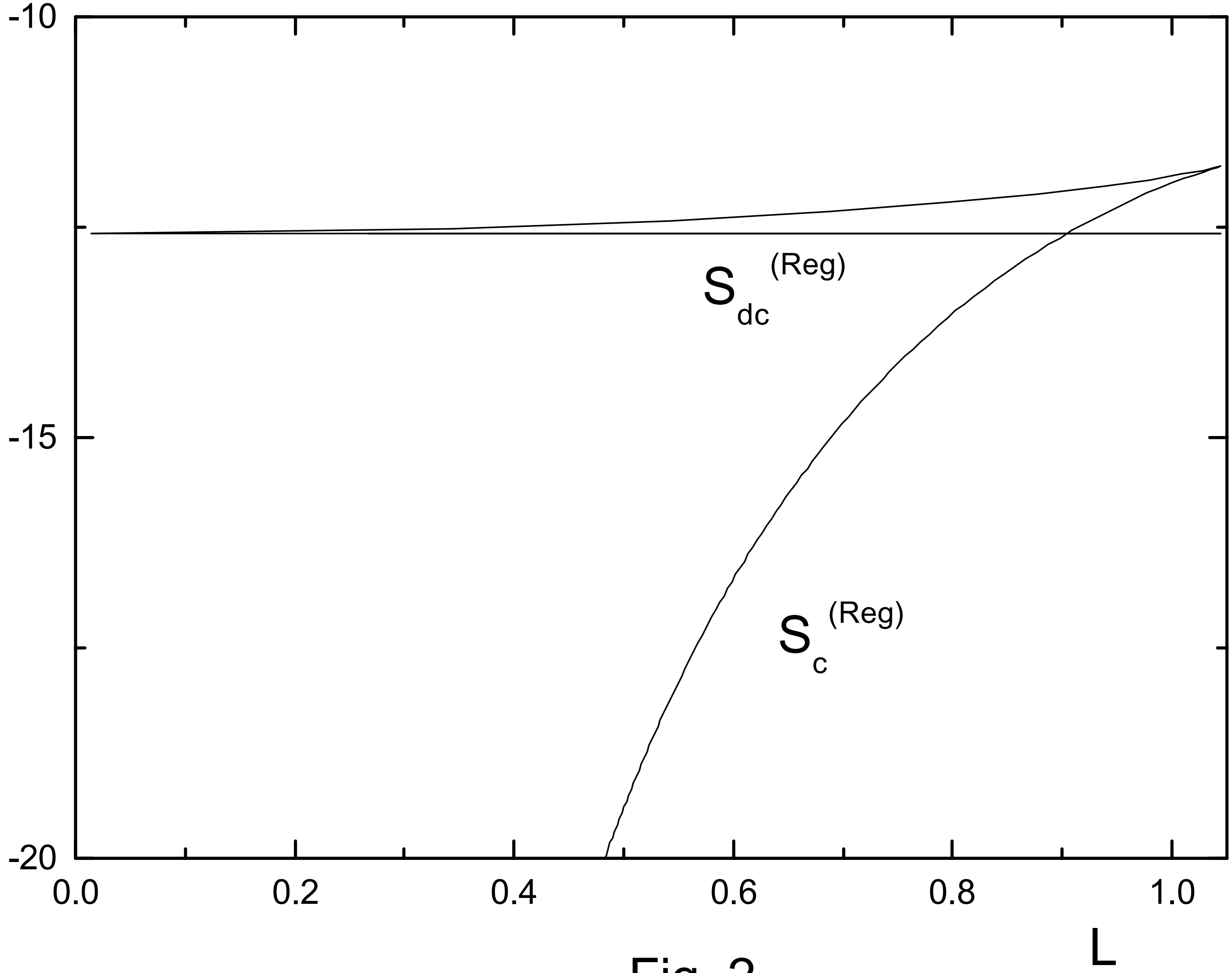

Fig. 2

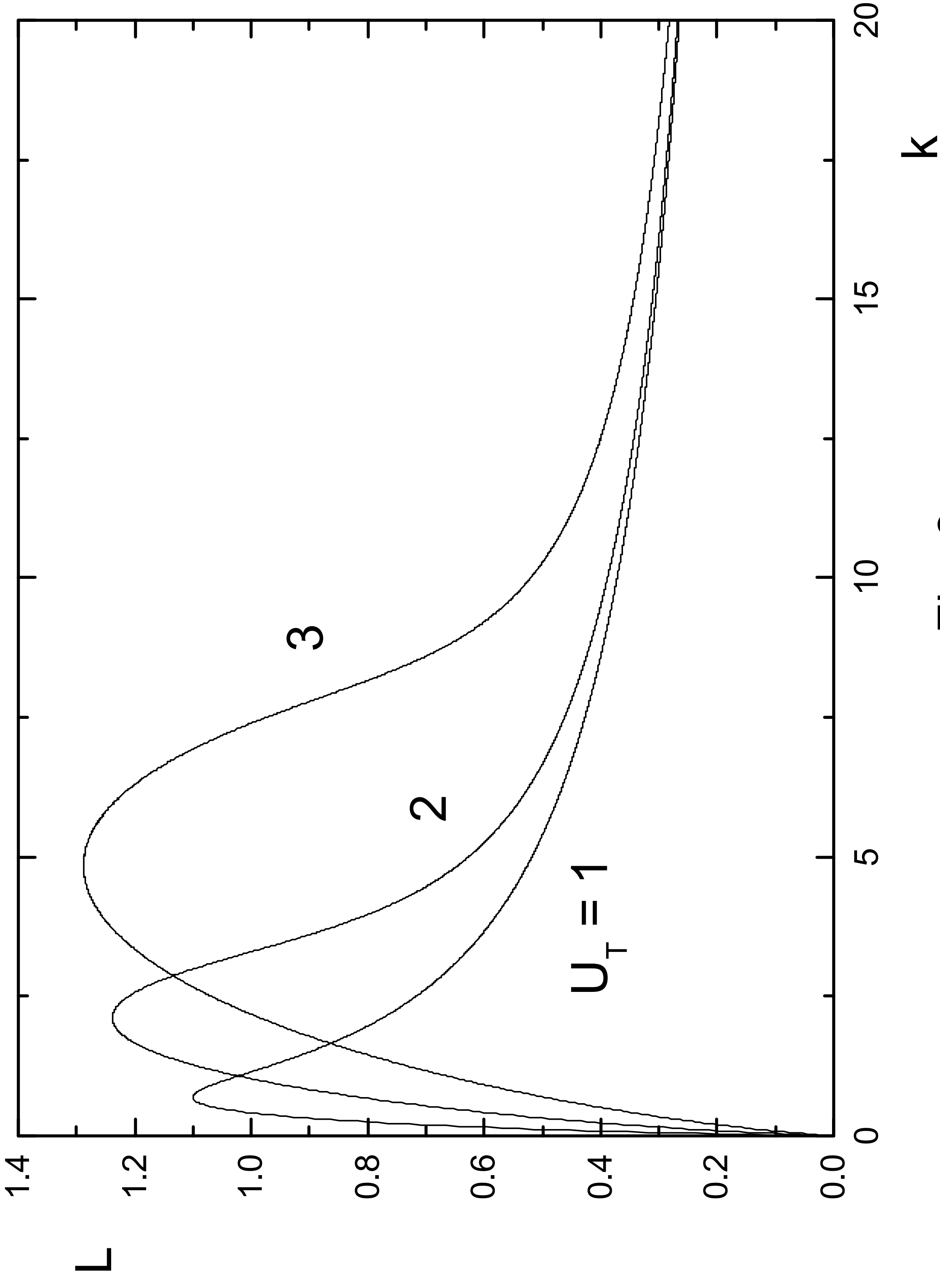

Fig. 3

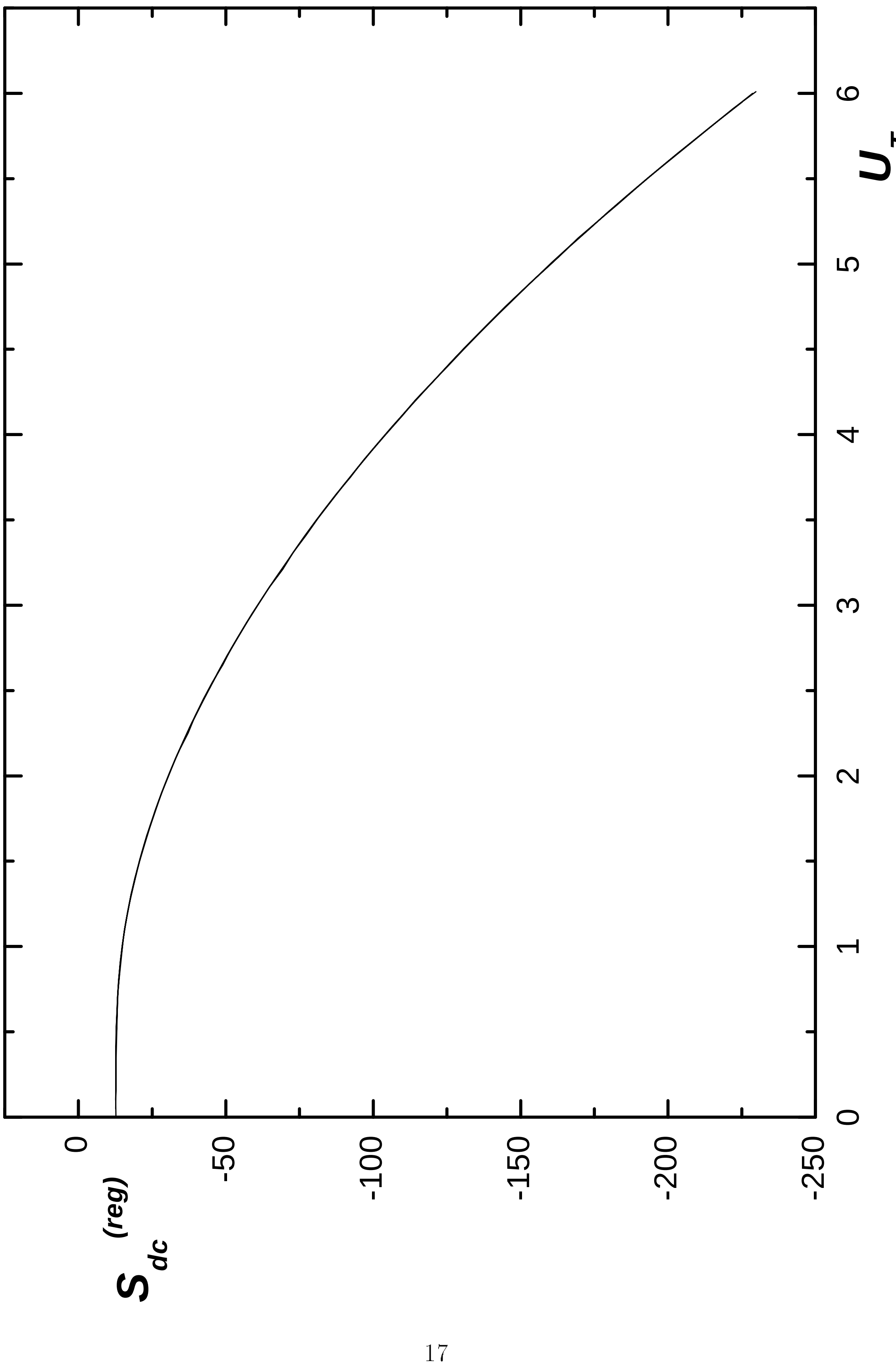

Fig. 4

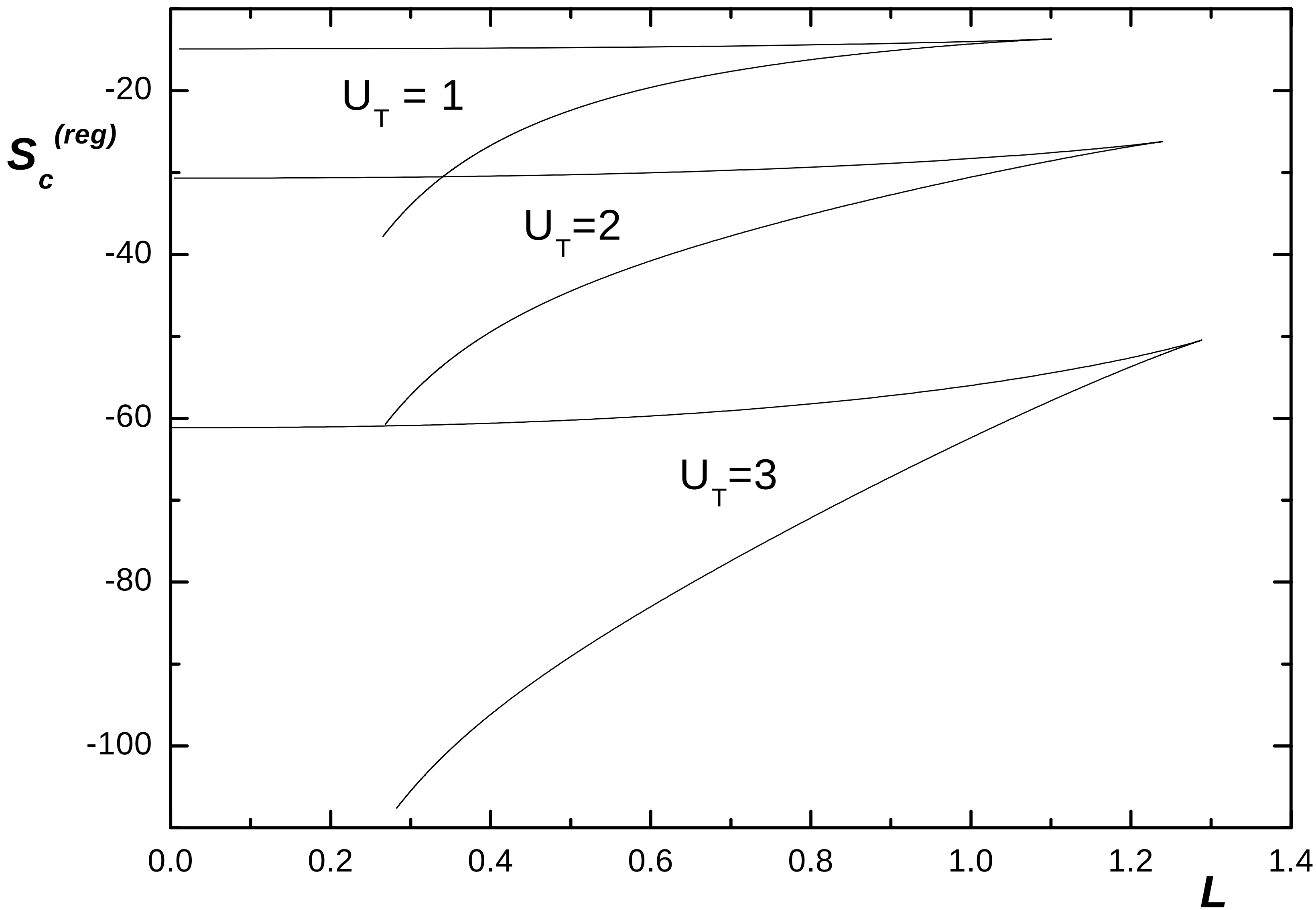

Fig. 5