Received: 3 October 2022 | Accepted: 1 November 2022

DOI: 10.1002/pamm.202200062

Development of a simple substitute model to describe the normal force of fluids in narrow gaps

Raphael Bilz[1,*] and **Kristin M. de Payrebrune**[1]

[1] Institute for Computational Physics in Engineering, Technische Universität Kaiserslautern, Gottlieb-Daimler-Straße 44, 67663 Kaiserslautern, Germany

Fluids in narrow gaps are employed frequently in many applications. The motivation for their use is diverse and ranges from hydrodynamic lubrication in plain bearings to the transport of hard particles into the working gap for the purpose of machining workpiece surfaces in lapping processes. Depending on the focus of the analysis, it may be useful to investigate the entire pressure field or to calculate only individual quantities. For example, in sophisticated simulations it may be of interest to know the resulting force of a fluid as a function of the external system state in order to describe its damping characteristics. Especially for the simulation of flows in narrow gaps, the Reynolds equation is a convenient choice, which, in contrast to the more general Navier-Stokes equations, can lead to considerable savings in computational time because no three-dimensional discretization is required, but only a two-dimensional discretization. However, if not a highly detailed pressure field is of interest, but only simple relations such as the resulting force as a function of distance and velocity, and if this relation to be evaluated many times for different parameter combinations over a wide range of values, the use of a robust substitute model is a good choice. This article deals with the creation of such a substitute model based on the Reynolds equation taking cavitation into account.

1 Introduction

Fluids in narrow gaps can have a significant influence on the surrounding system. The influence includes changed friction and wear properties as well as heat transfer behavior between the interfaces [1]. Thus, for the targeted use of fluid, it is necessary to know the effects of its properties.

Modeling of fluids is often based on the Navier-Stokes equations. For a fluid in a narrow gap between the interfaces of two adjacent bodies, the generalized Reynolds equation can be derived from the Navier-Stokes equations [2]. There are various representations of the Reynolds equation [2] as well as nonlinear material models to represent fluid behavior in narrow gaps as accurately as possible. Sophisticated models of elastohydrodynamic or even thermo-elastohydrodynamic effects may be suitable for the simulation of hydrodynamic journal bearings, but such models also require more parameters than simple models. In this publication, we use a representation of the Reynolds equation that relies only on a few parameters that are usually available from literature. Based on this, a substitute model for the calculation of the normal force is constructed, which increases the efficiency compared to the direct numerical calculation in case of frequent calculation.

2 Procedure

The representation of the Reynolds equation used here describes the laminar flow of an incompressible fluid with constant viscosity (i.e. Newtonian fluid) in a gap between two rigid bodies

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right)=6\eta\left(v_{1x}+v_{2x}\right)\frac{\partial h}{\partial x}+6\eta\left(v_{1y}+v_{2y}\right)\frac{\partial h}{\partial y}+12\eta\frac{\partial h}{\partial t}\quad \text{in } \Omega\subset\mathbb{R}^2 \quad [2]. \tag{1}$$

The quantity $p = p(x, y)$ describes the pressure distribution, $h = h(x, y)$ the (vertical) distance between the interfaces of two bodies, $v_{1x} = v_{1x}(x, y)$ and $v_{1y} = v_{1y}(x, y)$ resp. $v_{2x} = v_{2x}(x, y)$ and $v_{2y} = v_{2y}(x, y)$ the horizontal velocity components of body 1 resp. body 2, and η the dynamic viscosity of the fluid. In this paper, the time derivative $\frac{\partial h}{\partial t}$ is a (constant) input parameter $\frac{\partial h}{\partial t} = v$, which means the two bodies move uniformly toward or away from each other and the differential equation under consideration is independent of time t. On the boundary of the gap $\partial\Omega$ the pressure $p(x, y)$ takes the ambient pressure p_s. In addition, a non-mass-conserving cavitation model is used that constrains the occurring pressure downward to the vapor pressure p_v [2].

2.1 Analytical solutions for selected scenarios

In the following, some analytical solutions are given for the reaction force as a function of the (minimum) gap height h_0 and the vertical velocity v for an incompressible Newtonian fluid without considering cavitation. Thereby the adjacent bodies do not move horizontally $v_{1x} = v_{2x} = v_{1y} = v_{2y} = 0$. A sketch of the process under consideration is shown in Fig. 1.

* Corresponding author: e-mail raphael.bilz@mv.uni-kl.de, phone +49 (0) 631 205 3205

The force between a plane and a sphere with radius $R \gg h_0$ equals

$$F_{sphere}\left(h_0, v\right) = -\frac{6\pi\eta R^2 v}{h_0} \quad [3]. \tag{2}$$

The force between two plane circular disks of radius $r \gg h_0$ is given by equation

$$F_{disc}\left(h_0, v\right) = -\frac{3\pi\eta r^4 v}{2h_0^3} \quad [3]. \tag{3}$$

The force between two plane, parallel, rectangular plates of dimensions $a \times b$ with $a, b \gg h_0$ can be expressed by

$$F_{rect}\left(h_0, v\right) = -\left(\sum_{m=1}^{\infty}\sum_{m=1}^{\infty}\frac{\left[(-1)^{m+n} - (-1)^m - (-1)^n + 1\right]^2}{m^2 n^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}\right)\cdot\frac{48ab\eta v}{\pi^6 h_0^3} \tag{4}$$

according to [4][1]. In all three analytical solutions it can be seen from the negative sign that the reaction force F acts in the opposite direction of the velocity v. This means that the fluid damps the system in the cases considered.

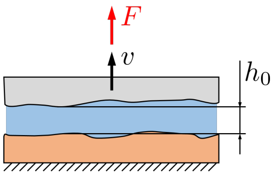

Fig. 1 Sketch of the analytically investigated relationship between the reaction force F, the vertical velocity v and the minimum gap height h_0 between the two bodies.

2.2 Numerical solution of the Reynolds equation

For its numerical solution, the Reynolds equation is discretized using finite differences and solved using Jacobi iterations [5]. The Reynolds equation Eq. (1) can be discretized after applying the chain rule to the terms

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) = h^3\frac{\partial^2 p}{\partial x^2} + 3h^2\frac{\partial h}{\partial x}\frac{\partial p}{\partial x} \quad \text{and} \quad \frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right) = h^3\frac{\partial^2 p}{\partial y^2} + 3h^2\frac{\partial h}{\partial y}\frac{\partial p}{\partial y} \tag{5}$$

with the finite differences

$$\begin{aligned} \frac{\partial \xi}{\partial x} &\approx \frac{\xi(x_{i+1}, y_j) - \xi(x_{i-1}, y_j)}{2\Delta x} & \qquad \frac{\partial^2 \xi}{\partial x^2} &\approx \frac{\xi(x_{i+1}, y_j) - 2\xi(x_i, y_j) + \xi(x_{i-1}, y_j)}{\Delta x^2} \\ \frac{\partial \xi}{\partial y} &\approx \frac{\xi(x_i, y_{j+1}) - \xi(x_i, y_{j-1})}{2\Delta y} & \qquad \frac{\partial^2 \xi}{\partial y^2} &\approx \frac{\xi(x_i, y_{j+1}) - 2\xi(x_i, y_j) + \xi(x_i, y_{j-1})}{\Delta y^2} \end{aligned}. \tag{6}$$

The derivative $\frac{\partial h}{\partial t} = v$ is constant in time and space. Finally, the discretized Reynolds equation can be expressed in the form

$$\begin{aligned} &K_c(x_i, y_j)\cdot p(x_i, y_j) \\ &+K_{x-}(x_i, y_j)\cdot p(x_{i-1}, y_j) + K_{x+}(x_i, y_j)\cdot p(x_{i+1}, y_j) \\ &+K_{y-}(x_i, y_j)\cdot p(x_i, y_{j-1}) + K_{y+}(x_i, y_j)\cdot p(x_i, y_{j+1}) = B(x_i, y_j) \end{aligned} \tag{7}$$

using

$$\begin{aligned} K_c(x_i, y_j) &= -2h^3(x_i, y_j)\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right) \\ K_{x-}(x_i, y_j) &= \frac{h^2(x_i, y_j)}{4\Delta x^2}\left(4h(x_i, y_j) - 3h(x_{i+1}, y_j) + 3h(x_{i-1}, y_j)\right) \\ K_{x+}(x_i, y_j) &= \frac{h^2(x_i, y_j)}{4\Delta x^2}\left(4h(x_i, y_j) + 3h(x_{i+1}, y_j) - 3h(x_{i-1}, y_j)\right) \\ K_{y-}(x_i, y_j) &= \frac{h^2(x_i, y_j)}{4\Delta y^2}\left(4h(x_i, y_j) - 3h(x_i, y_{j+1}) + 3h(x_i, y_{j-1})\right) \\ K_{y+}(x_i, y_j) &= \frac{h^2(x_i, y_j)}{4\Delta y^2}\left(4h(x_i, y_j) + 3h(x_i, y_{j+1}) - 3h(x_i, y_{j-1})\right) \\ B(x_i, y_j) &= 6\eta\left[(v_{1x} + v_{2x})\frac{h(x_{i+1}, y_j) - h(x_{i-1}, y_j)}{2\Delta x} + (v_{1y} + v_{2y})\frac{h(x_i, y_{j+1}) - h(x_i, y_{j-1})}{2\Delta y} + 2v\right]. \end{aligned}$$

[1] D. Cheneler (personal communication, September 22, 2022) confirmed a typo in the original reference, here the correct forumla is displayed.

It is worth noting that the derivatives obtained using finite differences only exist in the region $\Omega \setminus \partial\Omega$, but not at $\partial\Omega$. Nevertheless, since at the boundary $\partial\Omega$ the pressure $p(x_i, y_j)$ is known, these values are not needed to solve the system of equations. Eq. (7) can be written as

$$p(x_i, y_j) = \frac{1}{K_c(x_i, y_j)} \big[B(x_i, y_j) - K_{x-}(x_i, y_j) \cdot p(x_{i-1}, y_j) - K_{x+}(x_i, y_j) \cdot p(x_{i+1}, y_j) - K_{y-}(x_i, y_j) \cdot p(x_i, y_{j-1}) - K_{y+}(x_i, y_j) \cdot p(x_i, y_{j+1})\big]. \quad (8)$$

To solve this system of equations, the ambient pressure $p^{(0)}(x_i, y_j) = p_s$ is given initially in the entire region Ω. For all interior nodes $\forall(x_i, y_j) \in \Omega \setminus \partial\Omega$, the pressure $p(x_i, y_j)$ is calculated by iteratively computing array $p^{(k+1)}(x_i, y_j)$ from step $k \to k+1$ using the iteration rule

$$p^{(k+1)}(x_i, y_j) = \frac{1}{K_c(x_i, y_j)} \big[B(x_i, y_j) - K_{x-}(x_i, y_j) \cdot p^{(k)}(x_{i-1}, y_j) - K_{x+}(x_i, y_j) \cdot p^{(k)}(x_{i+1}, y_j) - K_{y-}(x_i, y_j) \cdot p^{(k)}(x_i, y_{j-1}) - K_{y+}(x_i, y_j) \cdot p^{(k)}(x_i, y_{j+1})\big] \quad (9)$$

until convergence is achieved. After each iteration, the pressure is checked if it falls below the cavitation pressure $p^{(k+1)}(x_i, y_j) < p_v$. If this is the case, the pressure is set to $p^{(k+1)}(x_i, y_j) = p_v$ at the respective locations.

The iteration rule Eq. (9) corresponds to the Jacobi iteration [5]. Eq. (9) can be efficiently computed for a rectangular domain Ω without the need to explicitly express the system of equations in the form $\underline{\underline{A}} \cdot \underline{p} = \underline{b}$ or storing coefficient matrix $\underline{\underline{A}}$. The normal force F is obtained from the pressure distribution with

$$F = \Delta x \Delta y \cdot \sum_i \sum_j \left(p(x_i, y_j) - p_s\right). \quad (10)$$

The convergence criterion considers the relative change of the force F between two successive iteration steps. Convergence is achieved if

$$\frac{\left|F^{(k+1)} - F^{(k)}\right|}{\left|F^{(k)}\right| + 1 \cdot 10^{-10}} \leq 1 \cdot 10^{-10} \quad (11)$$

is satisfied in iteration $k+1$. The constant $1 \cdot 10^{-10}$ in the denominator is used for numerical stability in case of very small forces $F \to 0$.

2.3 Substitute model construction using two-step interpolation

For a pair of known arbitrary body interface geometries, the reaction force $\bar{\bar{F}}$ acting on these interfaces due to the fluid is calculated as a function of the minimum gap height h_0 and the vertical velocity v by interpolating the results of numerical simulations. For this purpose, the combinations $(h_0, v) \in H_0 \times V$ with

$$H_0 = \{h_0^{(1)}, h_0^{(2)}, \dots, h_0^{(k)}, \dots\} \quad (12)$$

$$V = \{v^{(1)}, v^{(2)}, \dots, v^{(l)}, \dots\} \quad (13)$$

are computed using the numerical solution described in subsection 2.2. Subsequently, an interpolation model can be created from the calculated tuples (h_0, v, F). To avoid oscillations in the interpolation function caused by higher order polynomials, linear interpolation is used. However, the analytic solutions have poles for $h_0 \to 0$, so direct linear interpolation between value tuples $(h_0^{(k)}, v^{(l)}, F(h_0^{(k)}, v^{(l)}))$ is inaccurate, especially for very small values of height h_0.

The analytical solutions $F_{sphere}(h_0, v)$, $F_{disc}(h_0, v)$ and $F_{rect}(h_0, v)$ motivate the relation

$$F(h_0, v) = a \cdot h_0^b \cdot v \quad (14)$$

with constants a and b and the (smallest) distance h_0 of the two bodies. However, this relationship does not account for cavitation or other effects, so it is only used for interpolation between the known supporting points.

If $v^* \in V$ is constant, the relation between $y_{v^*}(h_0) := \log(F(h_0, v = v^*))$ and $x(h_0) := \log(h_0)$ is linear according to the analytical Eq. (14). It follows

$$\underbrace{\log\left(F(h_0, v = v^*)\right)}_{=y_{v^*}(h_0)} = \log\left(a \cdot v^*\right) + b \cdot \underbrace{\log\left(h_0\right)}_{=x(h_0)}. \quad (15)$$

For each $v^* \in V$, a linear interpolation function $y_{v^*}^{\text{int}}(x)$ is created. Thus, the force can be determined as a function of the gap height h_0 with $v^* \in V$ in the form

$$\bar{F}(h_0, v = v^*) = \exp\left(y_{v^*}^{\text{int}}(\log(h_0))\right). \tag{16}$$

For a known combination (h_0, v), linear interpolation can then be performed between two successive values of $v^{(l)}, v^{(l+1)} \in V$ with $v^{(l)} \leq v \leq v^{(l+1)}$ and the two function values $\bar{F}(h_0, v^{(l)})$ and $\bar{F}(h_0, v^{(l+1)})$ to determine the force $\bar{\bar{F}}(h_0, v)$. In this way, an approximated function value $\bar{\bar{F}}(h_0, v)$ can be assigned for each $(h_0, v) \in [\min\{H_0\}, \max\{H_0\}] \times [\min\{V\}, \max\{V\}]$.

3 Results and Discussion

This section investigates the quality of the numerical and the substitute model. Subsection 3.1 verifies the numerical model using an analytical example. Subsection 3.2 compares the substitute model with the numerical solution concerning the accuracy between the interpolation support points.

3.1 Comparison of the numerical with the analytical solution

This section compares the numerical solution with the analytical solution for a gap between two plane parallel interfaces of dimensions $a \times b = 2\,\text{mm} \times 2\,\text{mm}$. A fluid with dynamic viscosity $\eta = 0.001\,\text{Pa\,s}$ and vapor pressure $p_v = 0\,\text{Pa}$ is used. In addition, an ambient pressure $p_s = 101325\,\text{Pa}$, which is the standard pressure according to [6], is assumed. Both the numerical normal force F and the analytical normal force F_{rect} are calculated for the combinations $(h_0, v) \in H_0 \times V$ with

$$H_0 = \{0.01\,\text{mm}, 0.1\,\text{mm}, 1\,\text{mm}\} \tag{17}$$

$$V = \{-1\,\frac{\text{m}}{\text{s}}, -0.5\,\frac{\text{m}}{\text{s}}, 0\,\frac{\text{m}}{\text{s}}, 0.5\,\frac{\text{m}}{\text{s}}, 1\,\frac{\text{m}}{\text{s}}\}. \tag{18}$$

For the numerical solution, the computational domain is discretized by 257×257 grid points. The analytical solution is obtained by the sum of $(m, n) \in \{1, 2, \dots, 1000\} \times \{1, 2, \dots, 1000\}$ terms of the series in Eq. (4). The calculated normal forces and the relative difference between numerical and analytical solution are shown in Table 1. If cavitation occurred in the numerical simulation, the cells are colored orange. From the results presented, it can be seen that there is only an extremely small

Table 1: Numerically calculated normal force F, analytical normal force F_{rect} according to Eq. (4) and the relative difference $e := |1 - F/F_{rect}|$ as a function of the minimum gap height h_0 and vertical velocity v.

Num. Solution F	$v = -1\,\frac{\text{m}}{\text{s}}$	$v = -0.5\,\frac{\text{m}}{\text{s}}$	$v = 0\,\frac{\text{m}}{\text{s}}$	$v = 0.5\,\frac{\text{m}}{\text{s}}$	$v = 1\,\frac{\text{m}}{\text{s}}$
$h_0 = 1 \cdot 10^{-5}\,\text{m}$	$6.75 \cdot 10^{+4}\,\text{N}$	$3.37 \cdot 10^{+4}\,\text{N}$	$0\,\text{N}$	$-4.00 \cdot 10^{+1}\,\text{N}$	$-4.01 \cdot 10^{+1}\,\text{N}$
$h_0 = 1 \cdot 10^{-4}\,\text{m}$	$6.75 \cdot 10^{+1}\,\text{N}$	$3.37 \cdot 10^{+1}\,\text{N}$	$0\,\text{N}$	$-2.48 \cdot 10^{+1}\,\text{N}$	$-2.94 \cdot 10^{+1}\,\text{N}$
$h_0 = 1 \cdot 10^{-3}\,\text{m}$	$6.75 \cdot 10^{-2}\,\text{N}$	$3.37 \cdot 10^{-2}\,\text{N}$	$0\,\text{N}$	$-3.37 \cdot 10^{-2}\,\text{N}$	$-6.75 \cdot 10^{-2}\,\text{N}$

Analyt. Solution F_{rect}	$v = -1\,\frac{\text{m}}{\text{s}}$	$v = -0.5\,\frac{\text{m}}{\text{s}}$	$v = 0\,\frac{\text{m}}{\text{s}}$	$v = 0.5\,\frac{\text{m}}{\text{s}}$	$v = 1\,\frac{\text{m}}{\text{s}}$
$h_0 = 1 \cdot 10^{-5}\,\text{m}$	$6.75 \cdot 10^{+4}\,\text{N}$	$3.37 \cdot 10^{+4}\,\text{N}$	$0\,\text{N}$	$-3.37 \cdot 10^{+4}\,\text{N}$	$-6.75 \cdot 10^{+4}\,\text{N}$
$h_0 = 1 \cdot 10^{-4}\,\text{m}$	$6.75 \cdot 10^{+1}\,\text{N}$	$3.37 \cdot 10^{+1}\,\text{N}$	$0\,\text{N}$	$-3.37 \cdot 10^{+1}\,\text{N}$	$-6.75 \cdot 10^{+1}\,\text{N}$
$h_0 = 1 \cdot 10^{-3}\,\text{m}$	$6.75 \cdot 10^{-2}\,\text{N}$	$3.37 \cdot 10^{-2}\,\text{N}$	$0\,\text{N}$	$-3.37 \cdot 10^{-2}\,\text{N}$	$-6.75 \cdot 10^{-2}\,\text{N}$

Rel. diff. e	$v = -1\,\frac{\text{m}}{\text{s}}$	$v = -0.5\,\frac{\text{m}}{\text{s}}$	$v = 0\,\frac{\text{m}}{\text{s}}$	$v = 0.5\,\frac{\text{m}}{\text{s}}$	$v = 1\,\frac{\text{m}}{\text{s}}$
$h_0 = 1 \cdot 10^{-5}\,\text{m}$	$0.005\,\%$	$0.005\,\%$	–	$99.88\,\%$	$99.94\,\%$
$h_0 = 1 \cdot 10^{-4}\,\text{m}$	$0.005\,\%$	$0.005\,\%$	–	$26.46\,\%$	$56.39\,\%$
$h_0 = 1 \cdot 10^{-3}\,\text{m}$	$0.005\,\%$	$0.005\,\%$	–	$0.005\,\%$	$0.005\,\%$

deviation of about $e = 0.005\,\%$ between the analytical solution and the numerical solution, provided that no cavitation occurs. Since cavitation is not considered in the analytical model, the deviations in these cases are correspondingly higher. Assuming that cavitation occurs in the entire domain, the (theoretically minimum possible) force $F_{cav} = -(p_s - p_v) \cdot ab = 40.53\,\text{N}$ must be reached. Force F approaches this theoretical minimum F_{cav} with decreasing gap height h_0 and increasing velocity v.

3.2 Comparison of the substitute model with numerical solutions

Building on subsection 3.1, a substitute model, as described in subsection 2.3, is created and compared to the direct numerical solution. It is obvious that the calculation speed of an interpolation model according to subsection 2.3 is several magnitudes faster compared to the numerical solution according to subsection 2.2. Therefore, the accuracy of the substitute model for values between the interpolation points is of particular interest. The numerical solution hardly differs from the analytical solution (in its range of validity), however, cavitation is included, which will also be investigated here. Therefore, in Table 2 the substitute model is evaluated in-between the grid points and compared with the numerical solution. If cavitation occurred in the numerical solution, Table 2 is colored orange like Table 1. If inter- or extrapolated values of $\bar{\bar{F}}$ and $\bar{e}$ solely base on simulation results with occuring cavitation, these cells are again colored orange in Table 2. In contrast, if thes values are based on a combination of results with and without cavitation, the corresponding cells are colored red. The linear interpolation models used in subsection 2.3 were thereby extended to include linear extrapolation. This allows the assessment of the substitute model's quality even outside the numerically investigated range. The deviation between the substitute model and

Table 2: Force $\bar{\bar{F}}$ calculated using the substitute model, numerically calculated normal force F and the relative difference $\bar{e} := |1 - \bar{\bar{F}}/F|$ as a function of the minimum gap height h_0 and vertical velocity v between the interpolation support points.

Subs. model $\bar{\bar{F}}$	$v = -1.25\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = -0.75\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = -0.25\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = 0.25\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = 0.75\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = 1.25\,\frac{\mathrm{m}}{\mathrm{s}}$
$h_0 = 1 \cdot 10^{-5.5}\,\mathrm{m}$	$2.67 \cdot 10^{+6}\,\mathrm{N}$	$1.60 \cdot 10^{+6}\,\mathrm{N}$	$5.33 \cdot 10^{+5}\,\mathrm{N}$	$-2.54 \cdot 10^{+1}\,\mathrm{N}$	$-4.88 \cdot 10^{+1}\,\mathrm{N}$	$-4.48 \cdot 10^{+1}\,\mathrm{N}$
$h_0 = 1 \cdot 10^{-4.5}\,\mathrm{m}$	$2.67 \cdot 10^{+3}\,\mathrm{N}$	$1.60 \cdot 10^{+3}\,\mathrm{N}$	$5.33 \cdot 10^{+2}\,\mathrm{N}$	$-1.58 \cdot 10^{+1}\,\mathrm{N}$	$-3.29 \cdot 10^{+1}\,\mathrm{N}$	$-3.58 \cdot 10^{+1}\,\mathrm{N}$
$h_0 = 1 \cdot 10^{-3.5}\,\mathrm{m}$	$2.67 \cdot 10^{+0}\,\mathrm{N}$	$1.60 \cdot 10^{+0}\,\mathrm{N}$	$5.33 \cdot 10^{-1}\,\mathrm{N}$	$-4.57 \cdot 10^{-1}\,\mathrm{N}$	$-1.16 \cdot 10^{+0}\,\mathrm{N}$	$-1.66 \cdot 10^{+0}\,\mathrm{N}$
$h_0 = 1 \cdot 10^{-2.5}\,\mathrm{m}$	$2.67 \cdot 10^{-3}\,\mathrm{N}$	$1.60 \cdot 10^{-3}\,\mathrm{N}$	$5.33 \cdot 10^{-4}\,\mathrm{N}$	$6.22 \cdot 10^{-4}\,\mathrm{N}$	$-2.24 \cdot 10^{-3}\,\mathrm{N}$	$-4.22 \cdot 10^{-3}\,\mathrm{N}$

Num. Solution F	$v = -1.25\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = -0.75\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = -0.25\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = 0.25\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = 0.75\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = 1.25\,\frac{\mathrm{m}}{\mathrm{s}}$
$h_0 = 1 \cdot 10^{-5.5}\,\mathrm{m}$	$2.67 \cdot 10^{+6}\,\mathrm{N}$	$1.60 \cdot 10^{+6}\,\mathrm{N}$	$5.33 \cdot 10^{+5}\,\mathrm{N}$	$-4.02 \cdot 10^{+1}\,\mathrm{N}$	$-4.02 \cdot 10^{+1}\,\mathrm{N}$	$-4.02 \cdot 10^{+1}\,\mathrm{N}$
$h_0 = 1 \cdot 10^{-4.5}\,\mathrm{m}$	$2.67 \cdot 10^{+3}\,\mathrm{N}$	$1.60 \cdot 10^{+3}\,\mathrm{N}$	$5.33 \cdot 10^{+2}\,\mathrm{N}$	$-3.66 \cdot 10^{+1}\,\mathrm{N}$	$-3.82 \cdot 10^{+1}\,\mathrm{N}$	$-3.88 \cdot 10^{+1}\,\mathrm{N}$
$h_0 = 1 \cdot 10^{-3.5}\,\mathrm{m}$	$2.67 \cdot 10^{+0}\,\mathrm{N}$	$1.60 \cdot 10^{+0}\,\mathrm{N}$	$5.33 \cdot 10^{-1}\,\mathrm{N}$	$-5.33 \cdot 10^{-1}\,\mathrm{N}$	$-1.60 \cdot 10^{+0}\,\mathrm{N}$	$-2.67 \cdot 10^{+0}\,\mathrm{N}$
$h_0 = 1 \cdot 10^{-2.5}\,\mathrm{m}$	$2.67 \cdot 10^{-3}\,\mathrm{N}$	$1.60 \cdot 10^{-3}\,\mathrm{N}$	$5.33 \cdot 10^{-4}\,\mathrm{N}$	$-5.33 \cdot 10^{-4}\,\mathrm{N}$	$-1.60 \cdot 10^{-3}\,\mathrm{N}$	$-2.67 \cdot 10^{-3}\,\mathrm{N}$

Rel. diff. $\bar{e}$	$v = -1.25\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = -0.75\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = -0.25\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = 0.25\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = 0.75\,\frac{\mathrm{m}}{\mathrm{s}}$	$v = 1.25\,\frac{\mathrm{m}}{\mathrm{s}}$
$h_0 = 1 \cdot 10^{-5.5}\,\mathrm{m}$	$< 0.0001\,\%$	$< 0.0001\,\%$	$< 0.0001\,\%$	$36.82\,\%$	$21.44\,\%$	$11.61\,\%$
$h_0 = 1 \cdot 10^{-4.5}\,\mathrm{m}$	$< 0.0001\,\%$	$< 0.0001\,\%$	$< 0.0001\,\%$	$56.93\,\%$	$13.88\,\%$	$7.66\,\%$
$h_0 = 1 \cdot 10^{-3.5}\,\mathrm{m}$	$< 0.0001\,\%$	$< 0.0001\,\%$	$< 0.0001\,\%$	$14.24\,\%$	$27.39\,\%$	$37.91\,\%$
$h_0 = 1 \cdot 10^{-2.5}\,\mathrm{m}$	$< 0.0001\,\%$	$< 0.0001\,\%$	$< 0.0001\,\%$	$16.61\,\%$	$39.82\,\%$	$58.39\,\%$

the numerical solution is negligible, as long as in the substitute model only supporting points are used where no cavitation occured. This applies to both the interpolated and extrapolated values. If, on the other hand, the interpolation or extrapolation is partly based on numerical simulations in which cavitation occurs, the deviations $\bar{e} > 50\,\%$ can be observed. In the range of pure cavitation, however, the error of interpolation and extrapolation decreases again. In order to ensure that the model is sufficiently accurate in areas where cavitation occurs, in particular including the transition between supporting points with cavitation and without cavitation, these must be resolved by using a sufficient number of supporting points.

This publication focuses on the principle procedure in establishing a substitute model, not in creating a substitute model that is as accurate as possible. For this reason, only a few grid points are used here, so that the numerically calculated forces between two grid points can differ by several powers of ten, cf. Table 1. Both the range covered and the number of supporting points should be selected reasonably, depending on the intended use.

4 Conclusion

In this paper, a substitute model is proposed that can describe the normal force of a fluid between two bodies. For this purpose, the Reynolds equation concerning an incompressible Newtonian fluid is used considering a simple cavitation model and solved numerically for different gap heights and vertical velocities. From this, a two-step interpolation model, the substitution model, is constructed.

Except for the regions where cavitation occurs, the numerical solution shows excellent agreement with the analytical solution for a squeezing flow between two plane square plates. The substitute model also agrees very well with the numerical solution, provided that all the grid points used are from a region without cavitation, both in the interpolation and extrapolation region. In contrast, in the transition and cavitation regions, an increased resolution is required by means of enough numerically calculated supporting points in order to achieve sufficient accuracy of the substitute model. The resolution can be increased successively in several steps by comparing specific values in the transition area with the numerical solution. If the deviation is unacceptably high, additional rows or columns of supporting points can be calculated numerically at the critical locations and subsequently a refined substitute model can be constructed.

In some applications, a load-bearing effect can already occur at a vertical velocity of $v = 0$ if the gap height is a function of the location and the two adjacent bodies perform tangential relative motions. In these cases, cavitation may still be expected even for negative vertical velocities.

The presented substitute model is suitable for estimation purposes and requires a negligible amount of resources for storage and evaluation once it has been created. Due to the high computational speed of the substitute model compared to the numerical model, the substitute model qualifies excellently as a submodel that has to be evaluated frequently and quickly.

Acknowledgements The present publication was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) — Project-ID 172116086 — SFB 926. Open access funding enabled and organized by Projekt DEAL.

References

[1] G. W. Stachowiak, Engineering Tribology (Elsevier, Oxford, 2014).
[2] D. Bartel, Simulation von Tribosystemen (Vieweg+Teubner Verlag, 2010).
[3] V. L. Popov, Kontaktmechanik und Reibung (Springer Berlin Heidelberg, 2015).
[4] D. Cheneler, Analysis of a coupled-mass microrheometer, in: Advances in Microfluidics, (InTech, 2012).
[5] Y. Saad, Iterative Methods for Sparse Linear Systems (Society for Industrial and Applied Mathematics, 2003).
[6] DIN 1343:1990-01, Referenzzustand, Normzustand, Normvolumen; Begriffe und Werte (Beuth Verlag GmbH, 1990).