A HIERARCHY OF MODELS FOR MULTILANE VEHICULAR TRAFFIC I: MODELING

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Abstract

In the present paper multilane models for vehicular traffic are considered. A microscopic multilane model based on reaction thresholds is developed. Based on this model an Enskog like kinetic model is developed. In particular, care is taken to incorporate the correlations between the vehicles. From the kinetic model a fluid dynamic model is derived. The macroscopic coefficients are deduced from the underlying kinetic model. Numerical simulations are presented for all three levels of description in [10]. Moreover, a comparison of the results is given there.

1 Introduction

There are essentially three types of approaches towards the modeling of traffic flow phenomena. The first and most basic one concerns microscopic or follow the leader models, modeling the actual response of single cars to their predecessor, see, e.g., [2, 23, 3]. Macroscopic models based on fluid dynamic equations have been proposed by a large number of authors, see, e.g., [22, 16, 11, 7, 5, 20]. However some of these models have been subject to a considerable controversy, concerning their validity and applicability to traffic flow. Kinetic or Boltzmann-like models [18], [12, 15, 17], [14, 13] may present an intermediate step between the above two types of models. On the one side, they can be derived from microscopic considerations. On the other side, fluid dynamic models can be derived from kinetic traffic models as has been shown in a heuristic way, e.g., in [19, 17, 15, 4]. A multilane model has been considered, e.g., in [6]. In [21, 9] a new kinetic model is described and numerically investigated. For a survey on the different types of models, see [8].

In the above mentioned papers multilane effects are usually included in a cumulative way neglecting the exact dynamics of the multilane model. In this paper we are especially concerned with derivation procedures and the links between the different levels of the hierarchy for a full multilane model. Each level is derived from the lower one. The derivation is supported by numerical results. Quantities like the vehicle distributions, the distribution of the leading vehicles and the equilibrium values for mean velocity (fundamental diagram), traffic pressure, etc. are determined numerically on different levels and compared to each other.

The paper is organized in the following way: In Section 2 we describe a microscopic multilane model using so called reaction thresholds. The values of this thresholds are given by investigations of the behaviour of individual drivers. The basic assumption in this model is that the time scales allow an instantaneous treatment of the interactions. Based on this model we derive in Section 3 a new kinetic multilane model. Correlations between the vehicles are taken into account by an ansatz for the leading vehicle distribution. This distribution is also used to determine the probability for lane changing. From the multilane model a cumulative model is developed. Section 4 contains the derivation of fluid dynamic equations. The stationary solution of the homogeneous cumulative kinetic equation is used to determine the coefficients in the macroscopic equations. This leads to a better foundation of the macroscopic model and an explanation of the coefficients in the model. The links between the different models are summarized in Figure 1.

Numerical investigations of the equations derived in this part are presented in [10]. This work is in the following refered to as II.

2 The Microscopic Model

In this section we present a microscopic model based on the description of individual cars. The model is based on reaction thresholds. The cars change velocity and lanes instantaneously, once certain reaction thresholds are crossed, i.e. once the distance between a car and its following or leading car is becoming larger or smaller than the threshold distance. As long as no threshold is crossed, the cars move with their respective velocities in free motion. The thresholds usually depend on the velocities of the cars.

We consider a highway with N lanes. For the notations see Figure 2. The car under consideration is denoted by c. Leading car and follower on the same line are c_+ and c_-. On the left and right lanes they are denoted by c_{l_+}, c_{l_-} and c_{r_+}, c_{r_-}, respectively. Velocities are denoted using the same subscripts. Velocities before an interaction

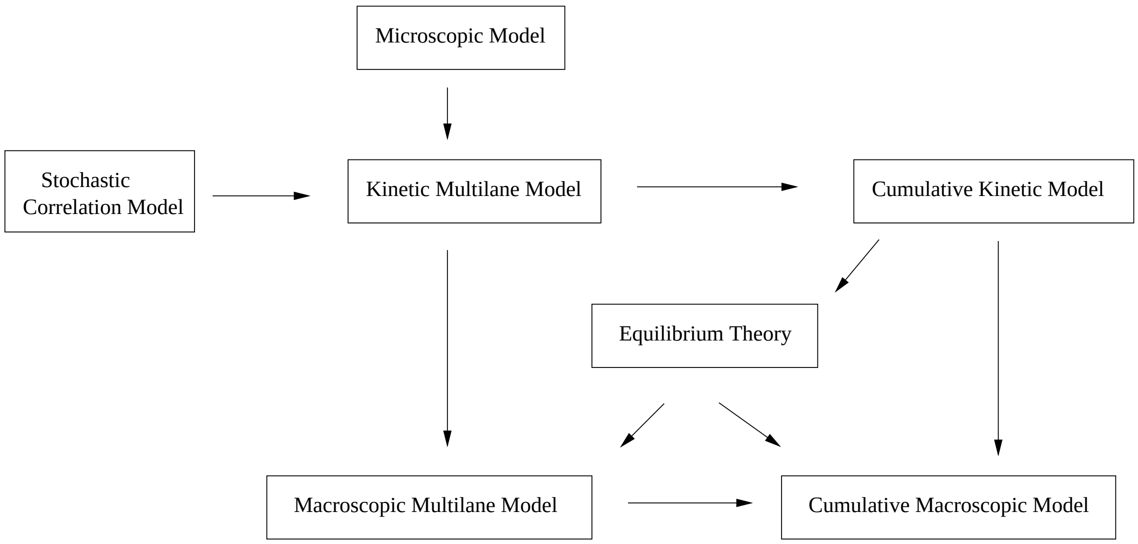

Figure 1: Links between models

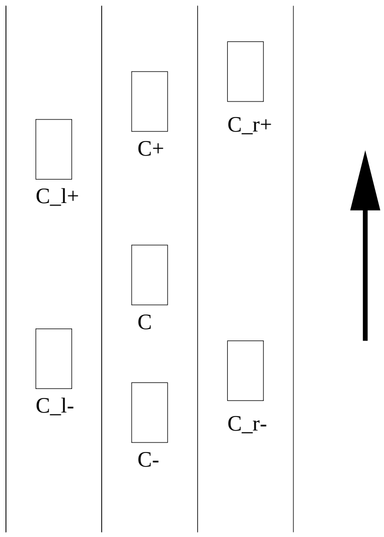

Figure 2: Notations

are denoted by v after the interaction by v'. The maximal velocity is denoted by w, i.e. the velocities range between 0 and w.

We introduce the following thresholds for lane changing to the left (H_L), lane changing to the right (H_R), braking (H_B), acceleration (H_A) and free driving (H_F) :

$$\begin{aligned} H_L(v) &= H_0 + vT_L \\ H_R(v) &= H_0 + vT_R \\ H_B(v) &= H_0 + vT_B \\ H_A(v) &= H_0 + \delta + vT_A \\ H_F &= H_0 + \delta + wT_F. \end{aligned}$$

T_L, T_R, T_B, T_A, T_F are the reaction times and H_0 denotes the minimal distance between the vehicles. These constants can be determined on an experimental basis, see, e.g., [8] for typical values. δ is a constant accounting for the fact that acceleration is done with a certain delay in comparison with braking. Moreover, we introduce the additional lines

$$\begin{aligned} H_L^S(v) &= H_0 + vT_L^S \\ H_R^S(v) &= H_0 + vT_R^S, \end{aligned}$$

denoting the space required on the left and right lane for a changing car. We assume the following ordering of the lines:

$$T_F \geq T_A > T_L > T_R > T_B$$

and

$$T_L^S, T_R^S \geq T_B.$$

In other words, braking takes place at a minimal safety distance, acceleration at a larger distance. For a changing car the available space must be at least such that braking is still possible.

In the following the possible interactions, taking place at the thresholds, are collected. We distinguish between two types of interactions. First interactions, where two lanes are involved, are considered.

Interaction 1 (Lane Changing to the left):

If the velocity of the car under consideration is larger than the velocity of the leading car $v > v_+$, i.e. the cars are approaching each other and the lane changing to the left line $H_L(v)$ is crossed, then the following takes place: The car will change to the left lane only if there is enough space on the left line, i.e. if

$$x_{l_+} - x > H_L^S(v), \qquad x - x_{l_-} > H_L^S(v_{l-}).$$

Moreover, c and c_- are accelerated after the lane change, if there is enough space:

$$v' = \begin{cases} \tilde{v} & \text{if } x_{l_+} - x > H_F \\ v & \text{else} \end{cases}, \qquad v'_- = \begin{cases} \tilde{v}_- & \text{if } x_+ - x_- > H_F \\ v_- & \text{else} \end{cases},$$

where $\tilde{v}, \tilde{v}_-$ are distributed due to a probability distribution of desired speeds of the drivers with density f_D, i.e., for example,

$$\tilde{v} = F_D^{-1}(\xi)$$

with $F_D(v) = \int_0^v f_D(\hat{v}) d\hat{v}$. Here ξ is a random variable uniformly distributed on $(0,1)$.

Interaction 2 (Lane Changing to the right):

If the velocity of the car under consideration is smaller than the velocity of the following car $v_- > v$ and the lane changing to the right line $H_R(v_-)$ is crossed, then the following takes place: The car will change to the right lane only if there is enough space on the right line, i.e. if

$$x_{r_+} - x > H_R^S(v), \qquad x - x_{r_-} > H_R^S(v_{r-}).$$

Moreover, c and c_- are accelerated after the lane change, if there is enough space:

$$v' = \begin{cases} \tilde{v} & \text{if } x_{r_+} - x > H_F \\ v & \text{else} \end{cases}, \qquad v'_- = \begin{cases} \tilde{v}_- & \text{if } x_+ - x_- > H_F \\ v_- & \text{else} \end{cases}$$

with $\tilde{v}, \tilde{v}_-$ as before.

The second type of interactions take place on one lane:

Interaction 3 (Braking):

If the velocity of the car under consideration is larger than the velocity of the leading car $v > v_+$ and the braking line $H_B(v)$ is crossed, then the following takes place: The car brakes in a range of velocities $[\beta v, v]$ below its actual velocity v. The new velocity is given by

$$v' = \beta v + \xi(v - \beta v), \ \beta < 1.$$

ξ is uniformly distributed in $[0,1]$. Braking is limited by the requirement that an acceleration is possible again, i.e. for all v, v' we require

$$H_A(v') > H_B(v) \quad \text{or} \quad \frac{T_B}{T_A} - \frac{\delta}{T_A w} < \beta < 1.$$

Interaction 4 (Acceleration I, Following):

If the velocity of the car under consideration is smaller than the velocity of the leading car $v < v_+$ and the acceleration line $H_A(v)$ is crossed, then the following takes place: The car accelerates in a range of velocities $[v, \alpha v]$ above its actual velocity v. The new velocity is given by

$$v' = v + \xi(min(w, v\alpha) - v), \ \alpha > 1.$$

Acceleration is limited by the requirement that braking is possible again, i.e. for all v, v' we require

$$H_B(v') < H_A(v) \quad \text{or} \quad 1 < \alpha < \frac{T_A}{T_B} + \frac{\delta}{T_B w}.$$

Interaction 5 (Acceleration II, Free):

If the velocity of the car under consideration is smaller than the velocity of the leading car $v < v_+$ and the acceleration line H_F is crossed, then the car accelerates. The new velocity v' (the desired velocity) is distributed due to the distribution function f_D, i.e.

$$v' = F_D^{-1}(\xi)$$

with F_D as before.

Remark 1:
The above microscopic interaction rules have to be changed in a suitable way for the first and last lane. In this case lane changing to the right or left, respectively, is not possible.

Remark 2:
The above model describes on the one hand cars in a following behaviour oscillating between braking and acceleration line. On the other hand cars driving freely accelerating to their desired velocities are described. Accidents are avoided by prohibiting a space of size $H_B(v)$ in front of each vehicle for the leading car. These facts will be used to set up an analytic model for the leading vehicle distribution, which will be used in the kinetic model in the next section.

Remark 3:
Obviously, a variety of other features could be included in the model. For example, lane changing to the right is not only caused by a following car but may be performed without any influence of another car. In general, lane changing can also be done in a spontaneous way without interactions with surrounding cars. In particular, this type of lane changing is important for inhomogeneous situations like the simulation of a reduction of lanes. We refer to II, Section 4 for an approach to include spontaneous lane changing into the macroscopic model.

Remark 4:
Finally we mention that to derive the kinetic equation in the next section we consider for simplicity a microscopic model without the additional acceleration terms in the lane changing interactions. Moreover, lane changing and braking lines are put together. This means we consider a model, where after reaching the braking line, the driver will - according to the above rules - first try to change the lane to the left, second - if this is not possible - the leading car will try to change to the right and third - if lane changing is not possible at all - the driver will brake.

3 The Kinetic Model

The second level of our hierarchy of models is given by a kinetic description, using the distribution function in space and velocity instead of a description of individual cars. We assume as in the last section a highway with N lanes. They are numbered by

$$\alpha = 1, \ldots, N.$$

The basic quantities in a kinetic approach are the single car distribution function and the leading vehicle distribution on each lane. The single car distribution function denoted by $f_\alpha(x,v)$ describes the number of cars at x with velocity v on lane α. The leading vehicle distribution denoted by $f_\alpha^{(2)}(x,v,h,v_+)$ describes the number of pairs of cars at x with velocity v and leading cars at $x+h$ with velocity v_+. Here and in the following we do not write explicitely the time dependence.

Integrating $f_\alpha^{(2)}$ over h and v_+we get

$$\int_0^w \int_0^\infty f_\alpha^{(2)}(x,v,h,v_+) dh dv_+ = f_\alpha(x,v).$$

Moreover,

$$\int_0^w f_\alpha(x,v) dv = \rho_\alpha(x)$$

where ρ_α denotes the density on lane α. Since the mean space available for each car is $\frac{1}{\rho_\alpha}$ we have

$$\frac{\int_0^w \int_0^w \int_0^\infty h f_\alpha^{(2)}(x,v,h,v_+) dh dv dv_+}{\int_0^w \int_0^w \int_0^\infty f_\alpha^{(2)}(x,v,h,v_+) dh dv dv_+} = \frac{1}{\rho_\alpha(x)}$$

or

$$\int_0^w \int_0^w \int_0^\infty h f_\alpha^{(2)}(x,v,h,v_+) dh dv dv_+ = 1. \tag{1}$$

A kinetic equation for the distribution function f_α uses the leading vehicle distribution $f_\alpha^{(2)}$ to describe the influence of the interactions. To obtain a closed equation for f_α we have to aproximate the leading vehicle distribution $f_\alpha^{(2)}$ in a suitable way using f_α and a correlation function. A connection between $f_\alpha^{(2)}$ and f_α is given by the following considerations:

Let $F_\alpha(x,v)$ denote the probability distribution in v of cars at x, i.e. $f_\alpha(x,v) = \rho_\alpha(x)F_\alpha(x,v)$. Moreover, we denote by $F_\alpha^+(v_+;h,v,x)$ the probability distribution in v_+ of the leading cars at distance h for cars at x with velocity v. $Q_\alpha(h;v,x)$ denotes the probability distribution of leading cars in h for a car at x with velocity v. Then

$$f_\alpha^{(2)}(x,v,h,v_+) = F_\alpha^+(v_+;h,v,x)Q_\alpha(h;v,x)f_\alpha(x,v). \tag{2}$$

We introduce now the following assumptions: The leading vehicles are distributed according to the probability distribution F_α at $x+h$:

$$F_\alpha^+(v_+;h,v,x) = F_\alpha(x+h,v_+).$$

Moreover, for Q_α we take the ansatz

$$Q_\alpha(h;v,x) = q(h;v,f_\alpha(x,\cdot)).$$

In the next subsection the space homogeneous case for a one lane situation is considered. An explicit expression for $q(h;v,f)$ is given there. This expression is then taken for the general case considered here.

As a consequence of (1) and (2) the above mentioned explicit expression for q has to fulfill

$$\int_0^\infty q(h;v,f_\alpha(x,\cdot))dh = 1$$

and

$$\int_0^w \int_0^\infty hq(h;v,f_\alpha(x,\cdot))F_\alpha(x,v)dhdv = \frac{1}{\rho_\alpha(x)}.$$

Finally this leads to the following approximation of $f_\alpha^{(2)}$:

$$f_\alpha^{(2)}(x,v,h,v_+) \sim q(h;v,f_\alpha(x,\cdot))F_\alpha(x+h,v_+)f_\alpha(x,v). \tag{3}$$

3.1 The Homogeneous Case

We consider first a space homogeneous one lane situation with vehicles having velocities that are distributed according to a given distribution function $f(v)$ with mass

$$\rho = \int_0^w f(v)dv.$$

Leading Vehicle Distribution

In this subsection we define $q(h; v, f)$, i.e. the probability density in h that for a vehicle with velocity v there is a leading vehicle with headway equal to h, if the velocities of the vehicles are distributed according to f.

We introduce the notation

$$< g >= \int_0^w g(v)F(v)dv.$$

for any function $g = g(v)$. F is defined as before by $f = \rho F$. Due to the last section q has to fulfill

$$\int_0^\infty q(h; v, f)dh = 1 \tag{4}$$

and

$$< \int_0^\infty hq(h; \cdot, f)dh >= \frac{1}{\rho}, \tag{5}$$

i.e. the average distance of the leading car is $\frac{1}{\rho}$.

We assume that one part $(0 \leq \lambda < 1)$ of the vehicles has a following behaviour, i.e. is moving between braking line H_B and acceleration lines H_A, and the other part $(1-\lambda)$ behaves independently from each other having a distance at least larger than the braking line H_B, compare Section 2, Remark 2. Then one obtains, see Part II Section 5.1 for a detailed discussion, the following expression for q:

$$\begin{aligned} q(h; v, f) &= (1-\lambda)\tilde{\rho}e^{-\tilde{\rho}(h-H_B(v))}\chi_{[H_B(v),\infty)}(h) \\ &\quad +\lambda\frac{1}{H_A(v)-H_B(v)}\chi_{[H_B(v),H_A(v)]}(h) \end{aligned} \tag{6}$$

where $\tilde{\rho}$ is determined using (5):

$$\tilde{\rho} = \frac{(1-\lambda)\rho}{1-\rho[(1-\lambda) < H_B > +\frac{\lambda}{2}(< H_B > + < H_A >)]}. \tag{7}$$

χ denotes the characteristic function. $\tilde{\rho}$ is the reduced density, since the available space for a free car is $[(1-\lambda) < H_B > +\frac{\lambda}{2}(< H_B > + < H_A >)]$.

The above expression takes into account a completely chaotic behaviour of the cars (λ near 0), as well a strongly correlated behaviour (λ near 1) which leads to a following behaviour. For (6), (7) we need

$$\rho < \frac{1}{(1-\lambda) < H_B > +\frac{\lambda}{2}(< H_B > + < H_A >)}.$$

This is a condition on the type of distribution functions f allowed in the model.

For a comparison of the averaged leading vehicle distribution $< q(h; \cdot, f) >$ and a leading vehicle distribution determined directly from the microscopic multilane model in Section 2 we refer to Part II, Section 2 and in particular to Figure II, 2.1.

Lane Changing Probabilities

In the following sections we need the probability that a lane change to the left and right is performed. This is again determined from a homogeneous situation. We assume that the velocities of the vehicles on the new lane are distributed according to the distribution function f. We consider a car with velocity v and determine the probability $P_Y(v,f), Y = L, R$ that lane changing is done, if the respective threshold is crossed, i.e. the probability that there is enough space on the other lane.

For lane changing, the distance after the lane change between the changing car with velocity v and its leading car on the new lane must be at least $H_Y^S(v), Y = L, R$. Moreover, the distance between the changing car and its follower on the new lane with velocity v' must be at least $H_Y^S(v'), Y = L, R$. The probability $p_Y(v, v', f)$ of a lane change of a car with velocity v having a follower on the new line with velocity v' is given by

$$p_Y(v, v', f) = 1 - \rho \int_0^{H_Y^S(v')+H_Y^S(v')} [1 - < Q(h; \cdot, f) >] dh \tag{8}$$

with the distribution function Q defined by

$$Q(h; v, f) = \int_0^h q(h'; v, f) dh'.$$

The derivation of this formula and the exact assumptions to obtain it are given in II, Section 5.2. Moreover, an explicit expression for P_Y using (6) is given there as well. The averaged version yields the desired probability for lane changing for a car with velocity v:

$$P_Y(v, f) = < p_Y(v, \cdot, f) > . \tag{9}$$

3.2 The Kinetic Multilane Model

The kinetic equation for the distibution functions $(f_1, \cdots, f_N)$ on the N lanes is obtained from similiar considerations as in the kinetic theory of gases. One determines the kinetic interaction operators, i.e. the gain (G) and loss (L) operators. This is done using the microscopic interactions as a basis combined with the standard procedure to derive kinetic equations. We get

$$\begin{aligned} \partial_t f_\alpha + v\partial_x f_\alpha &= \tilde{C}_\alpha^+(f_1^{(2)}, \cdots, f_N^{(2)}, f_1, \cdots, f_N) \\ &= (\tilde{G}_B^+ - \tilde{L}_B^+)(f_{\alpha-1}, f_\alpha^{(2)}, f_{\alpha+1}) \\ &\quad + (\tilde{G}_A^+ - \tilde{L}_A^+ + \tilde{G}_F^+ - \tilde{L}_F^+)(f_\alpha^{(2)}) \\ &\quad + [\tilde{G}_L^+(f_{\alpha-1}^{(2)}, f_\alpha) - \tilde{L}_R^+(f_{\alpha-1}, f_\alpha^{(2)}, f_{\alpha+1})](1 - \delta_{\alpha,1}) \\ &\quad + [\tilde{G}_R^+(f_\alpha, f_{\alpha+1}^{(2)}, f_{\alpha+2}) - \tilde{L}_L^+(f_\alpha^{(2)}, f_{\alpha+1})](1 - \delta_{\alpha,N}). \end{aligned} \tag{10}$$

$\delta_{i,j}$ denotes the Kronecker symbol. $f_\alpha^{(2)}(x,v,h,v_+)$ can be approximated, see (3), by

$$f_\alpha^{(2)}(x,v,h,v_+) \sim q(h;v,f_\alpha(x,\cdot))f_\alpha(x,v)F_\alpha(x+h,v_+) \tag{11}$$

with $\rho_\alpha = \int_0^w f_\alpha(x,v)dv$, $f_\alpha = \rho_\alpha F_\alpha$ and $q(h;v,f)$ the leading vehicle distribution defined in (6). This approach resembles Enskog's theory of a dense gas, see e.g., [1], rather than a Boltzmann type treatment. The necessity to do such an Enskog type approach is explained in detail in [9]. In particular, it is shown there that a Boltzmann type treatment leads to completely wrong results even for simple inhomogeneous situations.

In the following we use for $X = B, A, F$ the notation

$$q_X(v,f) = q(H_X(v),v,f)$$

to denote the correlation function. The probability $P_Y, Y = L, R$ for a lane change has been defined in (9). Additionally, we use the convention

$$P_L(v,f_{N+1}) = 0 = P_R(v,f_0).$$

The interaction terms appearing in (10) are stated and approximated using (11) in the following:

Interaction 1 (Lane changing to the left):

The car is changing to the left, if the braking line is reached and a lane change is possible (probability P_L).

Gainterm:

$$\tilde{G}_L^+(f_{\alpha-1}^{(2)},f_\alpha) = \int_{v>\hat{v}_+} P_L(v,f_\alpha(x))|v-\hat{v}_+|f_{\alpha-1}^{(2)}(x,v,H_B(v),\hat{v}_+)d\hat{v}_+$$

writing $f_\alpha(x)$ instead of $f_\alpha(x,\cdot)$. This is approximated by

$$\begin{aligned}&G_L^+(f_{\alpha-1},f_\alpha)\\ =& \int_{v>\hat{v}_+} P_L(v,f_\alpha(x))|v-\hat{v}_+|q_B(v,f_{\alpha-1}(x))f_{\alpha-1}(x,v)F_{\alpha-1}(x+H_B(v),\hat{v}_+)d\hat{v}_+.\end{aligned}$$

Lossterm:
With the same arguments one obtains an approximation of $\tilde{L}_L^+$:

$$\tilde{L}_L^+(f_\alpha^{(2)},f_{\alpha+1}) = \tilde{G}_L^+(f_\alpha^{(2)},f_{\alpha+1}) \sim G_L^+(f_\alpha,f_{\alpha+1}) = L_L^+(f_\alpha,f_{\alpha+1}).$$

Interaction 2 (Lane change to the right):

A car changes to the right if its follower reaches the braking line and is not able to overtake (change to the left). Moreover, a change to the right must be possible (probability P_R).

Gainterm:

$$
\begin{aligned}
& \tilde{G}_R^+(f_\alpha, f_{\alpha+1}^{(2)}, f_{\alpha+2}) \\
= & \int_{\hat{v}_- > v} P_R(v, f_\alpha(x))[1 - P_L(\hat{v}_-, f_{\alpha+2}(x - H_B(\hat{v}_-)))]|v - \hat{v}_-| \\
& f_{\alpha+1}^{(2)}(x - H_B(\hat{v}_-), \hat{v}_-, H_B(\hat{v}_-), v) d\hat{v}_-
\end{aligned}
$$

is approximated by

$$
\begin{aligned}
& G_R^+(f_\alpha, f_{\alpha+1}, f_{\alpha+2}) \\
= & \int_{\hat{v}_- > v} P_R(v, f_\alpha(x))[1 - P_L(\hat{v}_-, f_{\alpha+2}(x + H_B(\hat{v}_-)))]|v - \hat{v}_-| \\
& q_B(\hat{v}_-, f_{\alpha+1}(x - H_B(\hat{v}_-))) f_{\alpha+1}(x - H_B(\hat{v}_-), \hat{v}_-) F_{\alpha+1}(x, v) d\hat{v}_- .
\end{aligned}
$$

Lossterm:

$$
\begin{aligned}
& \tilde{L}_R^+(f_{\alpha-1}, f_\alpha^{(2)}, f_{\alpha+1}) = \tilde{G}_R^+(f_{\alpha-1}, f_\alpha^{(2)}, f_{\alpha+1}) \\
\sim \ & G_R^+(f_{\alpha-1}, f_\alpha, f_{\alpha+1}) = L_R^+(f_{\alpha-1}, f_\alpha, f_{\alpha+1}).
\end{aligned}
$$

Interaction 3 (Braking):

A car brakes, if it reaches the braking line and the driver is not able to change to the left and if the leading car cannot change to the right.

Gainterm:
We define P_B, the probability for braking, as

$$
\begin{aligned}
& P_B(v, v_+, f_{\alpha-1}(x + H_B(v)), f_{\alpha+1}(x)) \\
= & [1 - P_L(v, f_{\alpha+1}(x))][1 - P_R(v_+, f_{\alpha-1}(x + H_B(v)))].
\end{aligned}
$$

One obtains an approximation of

$$
\begin{aligned}
& \tilde{G}_B^+(f_{\alpha-1}, f_\alpha^{(2)}, f_{\alpha+1}) \\
= & \int\int_{\hat{v} > \hat{v}_+} P_B(\hat{v}, \hat{v}_+, f_{\alpha-1}(x + H_B(v)), f_{\alpha+1}(x))|\hat{v} - \hat{v}_+| \\
& \sigma_B(v, \hat{v}) f_\alpha^{(2)}(x, \hat{v}, H_B(\hat{v}), \hat{v}_+) d\hat{v} d\hat{v}_+
\end{aligned}
$$

by

$$
\begin{aligned}
& G_B^+(f_{\alpha-1}, f_\alpha, f_{\alpha+1}) \\
= & \int\int_{\hat{v} > \hat{v}_+} P_B(\hat{v}, \hat{v}_+, f_{\alpha-1}(x + H_B(v)), f_{\alpha+1}(x))|\hat{v} - \hat{v}_+| \\
& \sigma_B(v, \hat{v}) q_B(\hat{v}, f_\alpha(x)) f_\alpha(x, \hat{v}) F_\alpha(x + H_B(\hat{v}), \hat{v}_+) d\hat{v} d\hat{v}_+
\end{aligned}
$$

with

$$\sigma_B(v,\hat{v}) = \frac{1}{\hat{v}(1-\beta)}\chi_{[\beta\hat{v},\hat{v}]}(v).$$

Lossterm:

$$\begin{aligned} & \tilde{L}_B^+(f_{\alpha-1}, f_\alpha^{(2)}, f_{\alpha+1}) \\ = & \int_{v>\hat{v}_+} P_B(v,\hat{v}_+, f_{\alpha-1}(x+H_B(v)), f_{\alpha+1}(x))|v-\hat{v}_+|f_\alpha^{(2)}(x,v,H_B(v),\hat{v}_+)d\hat{v}_+ \end{aligned}$$

is approximated by

$$\begin{aligned} & L_B^+(f_{\alpha-1}, f_\alpha, f_{\alpha+1}) \\ = & \int_{v>\hat{v}_+} P_B(v,\hat{v}_+, f_{\alpha-1}(x+H_B(v)), f_{\alpha+1}(x))|v-\hat{v}_+| \\ & q_B(v, f_\alpha(x))f_\alpha(x,v)F_\alpha(x+H_B(v),\hat{v}_+)d\hat{v}_+. \end{aligned}$$

Interaction 4 (Acceleration):

The car accelerates, if the acceleration line is reached.

Gainterm:

$$\tilde{G}_A^+(f_\alpha^{(2)}) = \int\int_{\hat{v}<\hat{v}_+} |\hat{v}-\hat{v}_+|\sigma_A(v,\hat{v})f_\alpha^{(2)}(x,\hat{v},H_A(\hat{v}),\hat{v}_+)d\hat{v}d\hat{v}_+$$

is approximated by

$$G_A^+(f_\alpha) = \int\int_{\hat{v}<\hat{v}_+} |\hat{v}-\hat{v}_+|\sigma_A(v,\hat{v})q_A(\hat{v}, f_\alpha(x))f_\alpha(x,\hat{v})F_\alpha(x+H_A(\hat{v}),\hat{v}_+)d\hat{v}d\hat{v}_+$$

with

$$\sigma_A(v,\hat{v}) = \frac{1}{\min(w,\alpha\hat{v})-\hat{v}}\chi_{[\hat{v},\min(w,\alpha\hat{v})]}(v).$$

Lossterm:

$$\tilde{L}_A^+(f_\alpha^{(2)}) = \int_{v<\hat{v}_+} |v-\hat{v}_+|f_\alpha^{(2)}(x,v,H_A(v),\hat{v}_+)d\hat{v}_+$$

is approximated by

$$L_A^+(f_\alpha) = \int_{v<\hat{v}_+} |v-\hat{v}_+|q_A(v, f_\alpha(x))f_\alpha(x,v)F_\alpha(x+H_A(v),\hat{v}_+)d\hat{v}_+.$$

Interaction 5 (Free acceleration):

Using q_F, H_F and

$$\sigma_F(v,\hat{v}) = f_D(v)$$

instead of q_A, H_A and σ_A one defines G_F^+ and L_F^+ in the same way as G_A^+ and L_A^+, respectively.

Using the above approximations the kinetic equation reads for $\alpha = 1, \cdots, N$:

$$\begin{aligned}
\partial_t f_\alpha + v\partial_x f_\alpha &= C_\alpha^+(f_1, \cdots, f_N) \\
&= (G_B^+ - L_B^+)(f_{\alpha-1}, f_\alpha, f_{\alpha+1}) \\
&\quad +(G_A^+ - L_A^+ + G_F^+ - L_F^+)(f_\alpha) \\
&\quad +[G_L^+(f_{\alpha-1}, f_\alpha) - L_R^+(f_{\alpha-1}, f_\alpha, f_{\alpha+1})](1-\delta_{\alpha,1}) \\
&\quad +[G_R^+(f_\alpha, f_{\alpha+1}, f_{\alpha+2}) - L_L^+(f_\alpha, f_{\alpha+1})](1-\delta_{\alpha,N}).
\end{aligned} \tag{12}$$

3.3 A Cumulative Kinetic Model

We derive in this subsection a cumulative model from the multilane model above. The homogeneous version of this model will in the following section be used to derive macroscopic coefficients. The basic assumption underlying the following derivation is that traffic is homogenized over all lanes.

In the model derived in this section the lane changing interactions influence the dynamics only by reducing the number of braking interactions in the cumulative model. This is similar to the standard kinetic models used in traffic flow, see, e.g., [14, 19, 15, 21]. However here the dynamics in the cumulative model is derived from the multilane model. This is done by introducing a probability for braking in the equations according to the lane changing rules derived above.

The cumulative model is obtained from the multilane one by assuming that the distribution function f_α is the same on all lanes and by summing the equations over all lanes $1, \cdots, N$. We consider the cumulative distribution functions

$$f = f_1 = \cdots = f_N = \frac{1}{N}\sum_{\alpha=1}^{N} f_\alpha, \qquad F = F_1 = \cdots = F_N = \frac{1}{N}\sum_{\alpha=1}^{N} F_\alpha.$$

This means that $Nf(x,v)$ is the total distribution function on the highway and $f = \rho F$, where ρ is the average density per lane.

The probability P_B for braking on lane α is due to the last section

$$\begin{aligned}
&P_B(v, v_+, f_{\alpha-1}(x + H_B(v)), f_{\alpha+1}(x)) \\
= \;&[1 - P_L(v, f_{\alpha+1}(x))][1 - P_R(v_+, f_{\alpha-1}(x + H_B(v)))]
\end{aligned}$$

with the conventions

$$P_L(v, f_{N+1}) = 0 = P_R(v, f_0).$$

Averaging over all lanes gives the cumulative braking probability which we denote by P_B^C as

$$\begin{aligned} & P_B^C(v, v_+, f(x + H_B(v)), f(x)) \\ = \; & \frac{1}{N}(1 - P_L(v, f(x))) + \frac{1}{N}[1 - P_R(v_+, f(x + H_B(v)))] \\ + \; & \frac{N-2}{N}[1 - P_L(v, f(x))][1 - P_R(v_+, f(x + H_B(v)))]. \end{aligned}$$

These considerations yield the following equation for the cumulative model:

$$\partial_t f + v \partial_x f = C_C^+(f)$$

with

$$C_C^+(f) = G_B^+(f) - L_B^+(f) + G_A^+(f) - L_A^+(f) + G_F^+(f) - L_F^+(f), \tag{13}$$

where with a slight abuse of notation we have defined

$$\begin{aligned} G_B^+(f) \; = \; & \int \int_{\hat{v} > \hat{v}_+} P_B^C(\hat{v}, \hat{v}_+, f(x + H_B(\hat{v})), f(x)) |\hat{v} - \hat{v}_+| \sigma_B(v, \hat{v}) \\ & q_B(\hat{v}, f(x)) f(x, \hat{v}) F(x + H_B(\hat{v}), \hat{v}_+) d\hat{v} d\hat{v}_+ \end{aligned}$$

and

$$\begin{aligned} L_B^+(f) \; = \; & \int_{v > \hat{v}_+} P_B^C(v, \hat{v}_+, f(x + H_B(v)), f(x)) |v - \hat{v}_+| \\ & q_B(v, f(x)) f(x, v) F(x + H_B(v), \hat{v}_+) d\hat{v}_+. \end{aligned}$$

$G_A^+, L_A^+, G_F^+, L_F^+$ are defined as before.

Remark:
This model should be compared with the one developed in [9] or with the standard model in [19]. In the present case the terms σ_A, σ_B defined by the microscopic interactions rules are not depending on the local density ρ but only on the behaviour of a single driver, i.e. no macroscopic effects are included in these terms.

4 The Fluid Dynamic Model

The third and last level of the hierarchy is given by a fluid dynamic description on the basis of the moments density and velocity. To derive a macroscopic equation we proceed similarly to the gas kinetic case:

4.1 Balance Equations

We start the derivation of fluid dynamic equations by multiplying the inhomogeneous kinetic equation (12) with the property $\phi(v)$ and integrating it with respect to v. One obtains the following set of balance equations:

$$\partial_t \int_0^w \phi f_\alpha dv + \partial_x \int_0^w v\phi f_\alpha dv = \int_0^w \phi(v) C_\alpha^+(f)(x,v,t)dv.$$

We define the density n_α^ϕ of the property ϕ as

$$n_\alpha^\phi = \int_0^w \phi f_\alpha dv.$$

The important point in deriving fluid dynamic equations from kinetic Enskog equations is to identify clearly the flux and the source terms in the equation. The flux of ϕ due to the kinetic advection part is as usual

$$q_\alpha^\phi = \int_0^w v\phi f_\alpha dv.$$

However, there is a second contribution to the flux coming from the Enskog collison term due to the finite size of the interaction thresholds. To obtain this flux we separate the Enskog interaction term into a local interaction term and a deviation from the local term:

$$C_\alpha^+ = C_\alpha - (C_\alpha - C_\alpha^+),$$

where the local term C_α is defined by

$$\begin{aligned} C_\alpha(f_1,\cdots,f_N) &= (G_B - L_B)(f_{\alpha-1}, f_\alpha, f_{\alpha+1}) \\ &+ (G_A - L_A + G_F - L_F)(f_\alpha) \\ &+ [G_L(f_{\alpha-1}, f_\alpha) - L_R(f_{\alpha-1}, f_\alpha, f_{\alpha+1})](1-\delta_{\alpha,1}) \\ &+ [G_R(f_\alpha, f_{\alpha+1}, (f_{\alpha+2}) - L_L(f_\alpha, f_{\alpha+1})](1-\delta_{\alpha,N}). \end{aligned}$$

The gain and loss terms G_B, L_B, etc. are defined as G_B^+, L_B^+, etc. without a shift in the x-dependence, i.e., all functions appearing depend only on x. We mention that we do not proceed here exactly as in Enskogs theory of a dense gas. The fact that the velocities are only positive requires a slightly different treatment.

Rewriting the balance equations we get

$$\partial_t n_\alpha^\phi + \partial_x q_\alpha^\phi + E_\alpha^\phi = S_\alpha^\phi$$

with Enskog flux term

$$E_\alpha^\phi = \int_0^w \phi(v)[C_\alpha(f)(x,v,t) - C_\alpha^+(f)(x,v,t)]dv$$

and source term

$$S_\alpha^\phi = \int_0^w \phi(v) C_\alpha(f)(x,v,t) dv. \tag{14}$$

Using $\phi(v) = 1$ and $\phi(v) = v$ one obtains equations for the traffic flow density

$$n_\alpha^1 = \rho_\alpha = \int_0^w f_\alpha dv$$

and the traffic flux

$$n_\alpha^v = \rho_\alpha u_\alpha = \int_0^w v f_\alpha dv.$$

$u_\alpha = n_\alpha^v / n_\alpha^1$ denotes the mean velocity. For $\phi(v) = 1$ we get the continuity equations

$$\partial_t \rho_\alpha + \partial_x(\rho_\alpha u_\alpha) + E_\alpha^1 = S_\alpha^1.$$

Moreover, for $\phi(v) = v$ the acceleration equations

$$\partial_t(\rho_\alpha u_\alpha) + \partial_x(p_\alpha + \rho_\alpha u_\alpha^2) + E_\alpha^v = S_\alpha^v \tag{15}$$

are obtained with the 'traffic pressure'

$$p_\alpha = \int_0^w (v - u_\alpha)^2 f_\alpha dv.$$

To obtain closed equations for ρ_α and u_α one has to specify the dependence of $p_\alpha, S_\alpha^1, S_\alpha^v$ and E_α^1, E_α^v on ρ_α and u_α.

4.2 Closure Relations

There are a variety of possible closure relations, which could be borrowed from gas dynamics. We restrict here to the derivation of nonviscous fluid dynamic equations. As usual, to find closure relations for the balance equations one has to use the stationary solutions of the cumulative kinetic equation (16). All parameters of the fluid dynamic equation can be identified from these solutions. For the following compare the derivation of macroscopic equations in the case of a multicomponent gas with chemical reactions.

On each lane we consider the homogeneous cumulative equation for $f(v)$ derived from equation (13) given in Section 3.3:

$$\partial_t f = C_C(f) \tag{16}$$

with $C_C(f)$defined by

$$C_C(f) = G_B(f) - L_B(f) + G_A(f) - L_A(f) + G_F(f) - L_F(f).$$

$G_B, L_B, G_A, L_A, G_F, L_F$ have been defined above. For this kinetic equation $\rho = \int_0^w f(v)dv$ is constant in time. As before the definition $f(v) = \rho F(v)$ is used.

For the following arguments the homogeneous equation (16) must have a one parameter family of stationary equilibrium distributions $f^e(\rho, v)$ depending only on the density, i.e. for ρ fixed we have

$$f(v) \to f^e(\rho, v) \quad \text{for} \quad t \to \infty$$

no matter what the inital distribution of the homogenous equation is. See Part II for numerical experiments on this issue.

The fundamental diagram, i.e. the equilibrium mean velocity, is given by

$$u^e(\rho) = \frac{1}{\rho} \int_0^w v f^e(\rho, v) dv. \tag{17}$$

Equation (15) is now closed by the following procedure:

We approximate the traffic pressure p_α in (15) by its equilibrium value:

$$p_\alpha = \int_0^w (v - u_\alpha)^2 f_\alpha dv \sim \int_0^w (v - u^e(\rho))^2 f^e(\rho, v) dv = p^e(\rho_\alpha).$$

The Enskog terms E_α^ϕ are approximated in the following way: We linearize the expressions for E_α^ϕ in H and substitute the stationary distributions $f^e(\rho_\alpha, v)$ for f_α. This yields a contribution from each of the terms appearing in the definition of E_α^ϕ. E_α^ϕ is given by

$$E_\alpha^\phi = \sum_{X=L,R,B,A,F} E_X^\phi$$

with

$$E_X^\phi = \int_0^w \phi(v)[G_X - G_X^+]dv - \int_0^w \phi(v)[L_X - L_X^+]dv$$

for $X = L, R, B, A, F$. In the following the terms E_L^ϕ, E_R^ϕ due to lane changing are neglected. This means we neglect the nonlocal Enskog corrections due to the lane changing terms and keep the corrections due to acceleration and braking. This is justified by the numerical analysis of the corresponding interaction frequencies, see Part II. A further simplification arises, since the remaining terms with $\phi = 1$ are zero. We are left with E_α^v given by

$$E_\alpha^v = E_B^v + E_A^v + E_F^v.$$

The procedure is shown in detail for the term

$$\int_0^w v[G_B - G_B^+]dv.$$

The results for the other terms are stated without derivation .

$$
\begin{aligned}
& \int_0^w v[G_B(f_{\alpha-1}, f_\alpha, f_{\alpha+1}) - G_B^+(f_{\alpha-1}, f_\alpha, f_{\alpha+1})]dv \\
= & \int_0^w v \int\int_{\hat{v}>\hat{v}_+} |\hat{v} - \hat{v}_+| \sigma_B(v, \hat{v}) q_B(\hat{v}, f_\alpha(x)) \\
& [P_B(\hat{v}, \hat{v}_+ f_{\alpha-1}(x), f_{\alpha+1}(x)) f_\alpha(x, \hat{v}) F_\alpha(x, \hat{v}_+) \\
& - P_B(\hat{v}, \hat{v}_+ f_{\alpha-1}(x + H_B(\hat{v})), f_{\alpha+1}(x)) f_\alpha(x, \hat{v}) F_\alpha(x + H_B(\hat{v}), \hat{v}_+)] d\hat{v} d\hat{v}_+ dv
\end{aligned}
$$

The probability for braking is treated by using $P_B^C(\hat{v}, \hat{v}_+, f_\alpha(x), f_\alpha(x))$ instead of $P_B(\hat{v}, \hat{v}_+, f_{\alpha-1}(x+H_B(\hat{v})), f_{\alpha+1}(x))$. Moreover, we write $P_B^C(\hat{v}, \hat{v}_+, f)$ for $P_B^C(\hat{v}, \hat{v}_+, f, f)$. Using

$$
F_\alpha(x + H_B(v), \hat{v}_+) \sim F_\alpha(x, \hat{v}_+) + H_B(v) \partial_x F_\alpha(x, \hat{v}_+),
$$

we get for $\int_0^w v[G_B - G_B^+]dv$:

$$
\begin{aligned}
- & \int_0^w v \int\int_{\hat{v}>\hat{v}_+} P_B^C(\hat{v}, \hat{v}_+ f_\alpha(x)) |\hat{v} - \hat{v}_+| \sigma_B(v, \hat{v}) H_B(\hat{v}) q_B(\hat{v}, f_\alpha(x)) \\
& f_\alpha(x, \hat{v}) \partial_x F_\alpha(x, \hat{v}_+) d\hat{v} d\hat{v}_+ dv.
\end{aligned}
$$

Moreover, we introduce $f^e(\rho_\alpha(x), v)$ instead of $f_\alpha(x, v)$ and use

$$
f^e(\rho_\alpha, \hat{v}_+) = \rho_\alpha F^e(\rho_\alpha, \hat{v}_+).
$$

This yields the following final approximation for $\int_0^w v[G_B - G_B^+]dv$:

$$
\begin{aligned}
- \int_0^w v \int\int_{\hat{v}>\hat{v}_+} & P_B^C(\hat{v}, \hat{v}_+, f^e(\rho_\alpha(x))) |\hat{v} - \hat{v}_+| \sigma_B(v, \hat{v}) H_B(\hat{v}) q_B(\hat{v}, f^e(\rho_\alpha(x))) \\
& f^e(\rho_\alpha(x), \hat{v}) \partial_\rho F^e(\rho_\alpha(x), \hat{v}_+) d\hat{v} d\hat{v}_+ dv \ \partial_x \rho_\alpha.
\end{aligned}
$$

The other terms are treated similarly. With a_B^e, a_A^e and a_F^e defined by

$$
\begin{aligned}
a_B^e(\rho) &= -\int\int_{\hat{v}>\hat{v}_+} P_B^C(\hat{v}, \hat{v}_+, f^e(\rho) |\hat{v} - \hat{v}_+| H_B(\hat{v}) q_B(\hat{v}, f^e(\rho)) f^e(\rho, \hat{v}) \partial_\rho F^e(\rho, \hat{v}_+) \\
& \quad [\int_0^w v \sigma_B(v, \hat{v}) dv - \hat{v}] d\hat{v}_+ d\hat{v} \\
a_A^e(\rho) &= -\int\int_{\hat{v}<\hat{v}_+} |\hat{v} - \hat{v}_+| H_A(\hat{v}) q_A(\hat{v}, f^e(\rho)) f^e(\rho, \hat{v}) \partial_\rho F^e(\rho, \hat{v}_+) \\
& \quad [\int_0^w v \sigma_A(v, \hat{v}) dv - \hat{v}] d\hat{v}_+ d\hat{v} \\
a_F^e(\rho) &= -\int\int_{\hat{v}<\hat{v}_+} |\hat{v} - \hat{v}_+| H_F q_F(\hat{v}, f^e(\rho)) f^e(\rho, \hat{v}) \partial_\rho F^e(\rho, \hat{v}_+) \\
& \quad [\int_0^w v \sigma_F(v, \hat{v}) dv - \hat{v}] d\hat{v}_+ d\hat{v}
\end{aligned}
$$

we obtain alltogether

$$E_\alpha^v \sim a^e(\rho_\alpha)\partial_x \rho_\alpha$$

with the definition

$$a^e(\rho) = a_B^e(\rho) + a_A^e(\rho) + a_F^e(\rho).$$

We introduce the integrated Enskog coefficent

$$A^e(\rho) = \int_0^\rho a^e(\rho) d\rho.$$

This gives finally

$$E_\alpha^v \sim \partial_x A^e(\rho_\alpha).$$

The source term S_α^ϕ defined in (14) is treated as for example in kinetic semiconductor theory, by using a relaxation time approximation for C_α:

$$\begin{aligned} C_\alpha \quad \sim \quad & \Big(\frac{1}{T_{\alpha-1}^L} f_{\alpha-1} - \frac{1}{T_\alpha^R} f_\alpha\Big)(1-\delta_{\alpha,1}) \\ & + \Big(\frac{1}{T_{\alpha+1}^R} f_{\alpha+1} - \frac{1}{T_\alpha^L} f_\alpha\Big)(1-\delta_{\alpha,N}) \\ & + \frac{1}{T^e(\rho_\alpha)} \left[f^e(\rho_\alpha) - f_\alpha\right], \end{aligned} \tag{18}$$

where

$$\begin{aligned} \frac{1}{T_\alpha^L} &= P_L^e(\rho_{\alpha+1})\nu_B^e(\rho_\alpha) \\ \frac{1}{T_\alpha^R} &= P_R^e(\rho_{\alpha-1})(1 - P_L^e(\rho_{\alpha+1}))\nu_B^e(\rho_\alpha) \\ \frac{1}{T^e(\rho_\alpha)} &= (\nu_B^e + \nu_A^e + \nu_F^e)(\rho_\alpha). \end{aligned}$$

ν_B^e is defined by

$$\nu_B^e(\rho) = \frac{1}{\rho} \int \int_{v > \hat{v}_+} P_B^C(v, \hat{v}_+, f^e(\rho)) |v - \hat{v}_+| q_B(v, f^e(\rho)) f^e(\rho, v) F^e(\rho, \hat{v}_+) d\hat{v}_+ dv$$

Moreover,

$$\nu_X^e(\rho) = \frac{1}{\rho} \int \int_{v < \hat{v}_+} |v - \hat{v}_+| q_X(v, f^e(\rho)) f^e(\rho, v) F^e(\rho, \hat{v}_+) d\hat{v}_+ dv$$

for $X = A, F$. The lane changing probabilities $P_Y^e, Y = L, R$ are given by averaging $P_Y(v, f), Y = L, R$ with the equilibrium distribution f^e, i.e. $P_Y^e(\rho)$ is defined by

$$P_Y^e(\rho) = \int_0^w P_Y(v, f^e(\rho)) F^e(\rho, v) dv.$$

Multiplication of (18) with $\phi(v)$ and integration over v then gives an approximation for the source terms S_α^ϕ.

Using the above approximations, one obtains fluid dynamic equations in the form

$$\begin{aligned} \partial_t \rho_\alpha + \partial_x(\rho_\alpha u_\alpha) &= (\frac{1}{T^L_{\alpha-1}}\rho_{\alpha-1} - \frac{1}{T^R_\alpha}\rho_\alpha)(1-\delta_{\alpha,1}) \\ &\quad +(\frac{1}{T^R_{\alpha+1}}\rho_{\alpha+1} - \frac{1}{T^L_\alpha}\rho_\alpha)(1-\delta_{\alpha,N}) \end{aligned} \tag{19}$$

and

$$\begin{aligned} &\partial_t(\rho_\alpha u_\alpha) + \partial_x(p^e(\rho_\alpha) + \rho_\alpha u_\alpha^2) + \partial_x(A^e(\rho_\alpha)) \\ = \; &(\frac{1}{T^L_{\alpha-1}}\rho_{\alpha-1}u_{\alpha-1} - \frac{1}{T^R_\alpha}\rho_\alpha u_\alpha)(1-\delta_{\alpha,1}) \\ &+(\frac{1}{T^R_{\alpha+1}}\rho_{\alpha+1}u_{\alpha+1} - \frac{1}{T^L_\alpha}\rho_\alpha u_\alpha)(1-\delta_{\alpha,N}) \\ &+\frac{1}{T^e(\rho_\alpha)}\rho_\alpha\left[u^e(\rho_\alpha) - u_\alpha\right] \end{aligned} \tag{20}$$

for $\alpha = 1, \cdots, N$. Terms on the right hand side describe sources and sinks due to lane changing. All equilibrium quantities are determined from the stationary solution of the homogeneous kinetic equations (16).

Remark:
The cumulative fluid dynamic equation derived from the cumulative kinetic model in Section 3.3 is

$$\begin{aligned} \partial_t \rho + \partial_x(\rho u) &= 0 \\ \partial_t(\rho u) + \partial_x(p^e(\rho) + \rho u^2) + \partial_x(A^e(\rho)) &= \frac{1}{T^e(\rho)}\rho\left[u^e(\rho) - u\right] \end{aligned}$$

ρ denotes the density per lane, i.e. $N\rho$ is the total density on the highway. This is obtained from the model above by adding all lanes and assuming that density und velocity are equal on all lanes.

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