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Vorwort

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Construction of discrete shell models by geometric finite differences

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ABSTRACT

In the presented work, we make use of the strong reciprocity between kinematics and geometry to build a geometrically nonlinear, shearable low order discrete shell model of *Cosserat type* defined on *triangular* meshes, from which we deduce a rotation–free *Kirchhoff type* model with the triangle vertex positions as degrees of freedom. Both models behave physically plausible already on very coarse meshes, and show good convergence properties on regular meshes. Moreover, from the theoretical side, this deduction provides a common geometric framework for several existing models.

1 INTRODUCTION

By adopting a differential geometric view, we can reformulate very concisely the elastic energy of an inextensible one-director shell model as proposed by Simo et al. [4] in terms of differential entities of the undeformed and deformed midsurfaces – the first fundamental forms, $\bar{I}$ and I, the generalized second fundamental forms, $\bar{II}$ and II (which involve the directors), and a differential one-form σ_n, that accounts for the shearing:

$$W_{\text{Cosserat}} = \frac{1}{2}\int_{\bar{S}} \Big(\underbrace{\frac{t}{4}\|I-\bar{I}\|_M^2}_{\text{stretching}} + \underbrace{\frac{t^3}{12}\|II_n-\bar{II}\|_M^2}_{\text{bending}} + \underbrace{t\|\sigma_n\|_{M_s}^2}_{\text{shearing}} \Big)\mathrm{d}\bar{A}\,. \tag{1}$$

Here and throughout, barred quantities refer to the undeformed state, t is the thickness and $\|\cdot\|_M$ and $\|\cdot\|_{M_s}$ are material-depending norms.

From this parametrization-free energy expression, we will first deduce a discrete shear-deformable shell model for triangle meshes with piecewise constant membrane and bending strains, and treating shearing as a discrete one-form in the sense of finite element exterior calculus [1]. This discrete model is then successively reduced to a Kirchhoff model with an additional rotational degree of freedom on edges, and subsequently to an even simpler model involving only the vertex positions of the mesh. Numerical tests for both the discrete Cosserat shell (DCS) and the rotation-free discrete Koiter shell (DKS) model are presented in the last section to illustrate the satisfactory behaviour of both models on coarse regular meshes.

2 SMOOTH COSSERAT ENERGY

Let $\bar{\boldsymbol{\mathcal{S}}} = (\bar{\mathcal{S}}, \bar{n}, t)$ be the undeformed configuration of a shell, where the undeformed director field, $\bar{n}$, equals the unit normal field of the undeformed midsurface, $\bar{\mathcal{S}}$. A deformation of $\bar{\boldsymbol{\mathcal{S}}}$ is a map

$$\begin{aligned} \mathbf{\Phi} = (\phi, \underline{n}) : \quad & \bar{\mathcal{S}} \times [-\frac{t}{2}, \frac{t}{2}] \to \mathbb{R}^3 \\ & (x, \xi)_{\bar{\boldsymbol{\mathcal{S}}}} \mapsto \phi(x) + \xi \underline{n}(x) \ , \end{aligned} \tag{2}$$

such that ϕ is a diffeomorphism onto its image and $\underline{n} : \bar{\mathcal{S}} \to \mathbb{S}^2$ is a differentiable unit vector field ($\mathbb{S}^2$ denoting the unit sphere). The resulting deformed configuration is then given by $\boldsymbol{\mathcal{S}} = (\mathcal{S} := \phi(\bar{\mathcal{S}}), n := \underline{n} \circ \phi^{-1}, t)$. Thus $\underline{n}$ is the *pullback* of the deformed director field n, to the undeformed midsurface.

Let $\mathbf{v}, \mathbf{w} \in T\mathcal{S}$ be tangent vector fields of a shell's midsurface $\mathcal{S}$. We recall the usual first fundamental form I of $\mathcal{S}$, and introduce a *generalized* second fundamental form with respect to the director field n on $\mathcal{S}$:

$$I(\mathbf{v}, \mathbf{w}) \ = \ \langle \mathbf{v}, \mathbf{w} \rangle \ , \ \ II_n(\mathbf{v}, \mathbf{w}) \ = \ \frac{1}{2} \left(\langle \mathrm{d}n(\mathbf{v}), \mathbf{w} \rangle + \langle \mathbf{v}, \mathrm{d}n(\mathbf{w}) \rangle \right) \ , \tag{3}$$

where d denotes the (metric-free) Cartan outer derivative, and $\langle \ , \ \rangle$ stands for the usual inner product in $\mathbb{R}^3$. We use the subscript n in order to underline that the second fundamental form is a generalization of its classical counterparts of surface theory. Additionally, we define a one-form σ_n on the tangent space $T\mathcal{S}$ of the surface $\mathcal{S}$ by

$$(\sigma_n)_x : \ T_x\mathcal{S} \to \mathbb{R} \ , \ \ \mathbf{v} \mapsto \langle n(x), \mathbf{v} \rangle \tag{4}$$

for all $x \in \mathcal{S}$. Thus σ_n is the one-form associated with the projection of the director n to $T\mathcal{S}$.

In order to derive a Cosserat shell model using these differential entities, it is convenient to consider them on a fixed reference surface, which we chose to be the midsurface of the undeformed configuration. Specifically, for a deformed shell $\boldsymbol{\mathcal{S}}$ obtained by a deformation $(\phi, \underline{n})$, such that $\mathcal{S} = \phi(\bar{\mathcal{S}})$ and $n = \underline{n} \circ \phi^{-1}$, we can consider the pullback to the undeformed surface

$$\phi^* I = \mathrm{d}\phi^T \mathrm{d}\phi \ , \ \phi^* II_n = \frac{1}{2} \left(\mathrm{d}\underline{n}^T \mathrm{d}\phi + \mathrm{d}\phi^T \mathrm{d}\underline{n} \right) \ , \ \phi^* \sigma_n = \sigma_n \circ \mathrm{d}\phi \ ,$$

such that all the forms are defined on the reference configuration. In the following, as we will always consider these pullbacks, we will abuse notation and omit the pullback operator ϕ^*. Additionally, for simplicity, we will assume that the undeformed reference configuration is *unsheared*, *i.e.*, that directors and normals coincide.

Following Simo et al. [4], with respect to smallness assumptions and constitutive relations, we recover the energy for isotropic, homogenous, and (transversally) inextensible one-director shells, expressed in a compact, coordinate-free manner as

$$\mathcal{W}(\boldsymbol{\mathcal{S}}) = \frac{1}{2} \int_{\bar{\mathcal{S}}} \Big(\ \frac{t}{4} \| I - \bar{I} \|_M^2 + \frac{t^3}{12} \| II_n - \bar{II} \|_M^2 + t m_s \| \sigma_n \|^2 \ \Big) \mathrm{d}\bar{A} \ . \tag{5}$$

The norm $\| \cdot \|_M$ is a Frobenius norm weighted with a material tensor M, such that

$$\| \cdot \|_M^2 = \frac{E}{(1 - \nu^2)} (\ \nu \, \mathrm{tr}(\cdot)^2 + (1 - \nu) \, \mathrm{tr}(\cdot^2) \ )$$

and $m_s = \kappa \frac{E}{2(1+\nu)}$ is the shearing stiffness, both deduced by an asymptotic expansion of the three dimensional St. Venant-Kirchhoff model with respect to thickness. The material parameters E and ν denote Young's modulus and Poisson's ratio, while κ is the shear correction factor often set to $\frac{5}{6}$ in applications.

3 DISCRETE COSSERAT ENERGY

In the geometric formulation of the smooth energy (1), membrane and bending energies are described as first and generalized second fundamental forms, and shearing as differential one-form. Relying on methods from discrete differential geometry (DDG) [2, 10], we proceed by building the *discrete geometric* counterparts of these differential entities.

3.1 Discrete quadratic forms

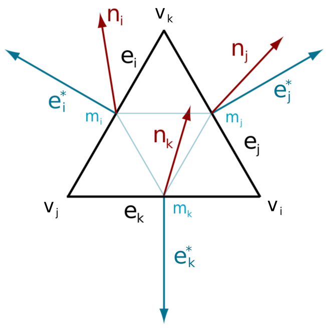


Figure 1.

A symmetric quadratic form Q in $\mathbb{R}^2$ is uniquely defined by its evaluation on a basis $\{\mathbf{v}, \mathbf{w}\}$ of the tangent plane, *i.e.*, by $Q(\mathbf{v}, \mathbf{v}), Q(\mathbf{w}, \mathbf{w}), Q(\mathbf{v}, \mathbf{w})$. By means of polarization, it can alternatively be determined by its evaluation in 3 non-parallel directions, for example on the edges of a triangle $\{\mathbf{v}, \mathbf{w}, (\mathbf{v} + \mathbf{w})\}$ in the tangent plane via: $Q(\mathbf{v}+\mathbf{w}, \mathbf{v}+\mathbf{w}) = Q(\mathbf{v}, \mathbf{v}) + Q(\mathbf{w}, \mathbf{w}) + 2\, Q(\mathbf{v}, \mathbf{w})$. This relation makes it easy to define a piecewise constant *discrete* quadratic form Q on a *triangular* mesh in the exact same way. If for each triangular face we consider the corresponding plane as its tangent plane, Q can be defined by prescribing its values along the triple of edge vectors $\mathbf{e}_i$, or similarly along the in-plane edge normals $\mathbf{e}_i^*$, which are edges rotated by 90 degrees in the triangle plane (see Figure 1). Some algebraic manipulations yield the closed form expression

$$\mathrm{Q} = -\frac{1}{8\mathrm{A}^2} \sum_{(ijk)} (\mathrm{Q}_i - \mathrm{Q}_j - \mathrm{Q}_k) \mathbf{e}_i^* \otimes \mathbf{e}_i^* = -\frac{1}{8\mathrm{A}^2} \sum_{(ijk)} (\mathrm{Q}_i^* - \mathrm{Q}_j^* - \mathrm{Q}_k^*) \mathbf{e}_i \otimes \mathbf{e}_i \ , \tag{6}$$

for the quadratic form Q, depending on its values $\mathrm{Q}_i := \mathrm{Q}(\mathbf{e}_i, \mathbf{e}_i)$ and $\mathrm{Q}_i^* := \mathrm{Q}(\mathbf{e}_i^*, \mathbf{e}_i^*)$, with A being the area of the triangle, (ijk) denoting a cyclic sum over edge indices and $\otimes$ being the dyadic product of vectors. This approach was already used to define discrete stretching and bending energies in [10] with a slightly different formula (i.e.: a missing numerical factor of $-\frac{1}{2}$).

3.2 Discrete first and second fundamental forms

This very simple algebraic expression allows to build constant strain and moment triangles out of one-dimensional directional strain quantities *consistently*. The smooth first fundamental form gives the squared distance of a tangent vector. Considering edges as finite difference approximations of tangent vectors, we define its discrete pendant in the analogous way, *i.e.*such that it assigns to each edge its squared length. This gives the expressions

$$\mathrm{I}_i \ := \ \mathrm{I}(\bar{e}_i) \ := \ \|e_i\|^2 \ , \ \ \bar{\mathrm{I}}_i \ := \ \bar{\mathrm{I}}(\bar{e}_i) \ := \ \|\bar{e}_i\|^2 \tag{7}$$

for the deformed and undeformed directional evaluations respectively. For the second fundamental form, we need to measure the change of directors along the edge. As directors are not attached to vertices, we make use of the particular geometry of triangles that the segment connecting edge midpoints of a triangle is parallel to the third edge and has half its length (fig. 1). This allows again to define the discrete generalized second fundamentel form along edges by a straight forward finite difference approach. We obtain:

$$\mathrm{II}_i \ := \ \mathrm{II}^e(\bar{e}_i) \ := \ 2\langle \underline{\mathrm{n}}_j - \underline{\mathrm{n}}_k, e_i \rangle \ , \ \ \bar{\mathrm{II}}_i \ := \ \bar{\mathrm{II}}(\bar{e}_i) \ := \ 2\langle \bar{\mathrm{n}}_j - \bar{\mathrm{n}}_k, \bar{e}_i \rangle \tag{8}$$

3.3 Discrete Shearing

We treat shearing as a *discrete* differential one-form in the sense of Arnold et al. [1]. We will denote this discrete shear form by s_n. Such a one-form is completely defined over a triangle by its *edge values*, which in the Cosserat model represent the projection of the director to the *discrete tangent plane*. Instead of prescibing this tangent plane completely, for example by averaging the neighboring triangle planes, we will adopt an idea formulated in [2] and restrict it only partially by perceiving that on its *midpoint*, an edge approximates a tangent vector, and thereby *one direction* of the corresponding tangent plane, up to second order, independently of the shape of the triangles. This is a simple consequence of the approximation order of central finite differences. Hence, following [2] we will attach edge normals to edge *midpoints* and merely constrain them in edge direction, *i.e.*, require them to stay orthogonal to their associated edge. This provides a geometric motivation of the kinematic approach chosen for Morley's constant moment triangle (see [7]). Note that this construction of discrete normals can only be realized on edges. On faces as well as on vertices, there is no distinguished direction that approximates the tangent plane more accurately then an other.

In [2], this construction allows discrete normals to contribute to the minimization of the bending energy. We generalize this idea by introducing edge directors, which *(i)* contribute to reduce shearing energy and thereby approximate edge normals, and *(ii)* minimize bending energy governed by their differences. This brings us to define a *tangent plane on an edge* $\mathbf{e}_i$ as the plane that contains the edge $\mathbf{e}_i$, and minimizes the tangential projection $(\underline{\mathrm{n}}_i)^{\tan}$ of the edge director $\underline{\mathrm{n}}_i$. As a consequence, the position of the tangent plane depends on the rotational state of the director. The minimal projection of a director $\underline{\mathrm{n}}_i$ to all possible tangent planes is the projection

$$(\underline{\mathrm{n}}_i)^{\tan} \ := \ \langle \underline{\mathrm{n}}_i, \hat{\mathbf{e}}_i \rangle \hat{\mathbf{e}}_i \tag{9}$$

to the unit edge $\hat{\mathbf{e}}_i$ itself (Figure 2), which we use to define the edge values of the discrete shear form s_n.

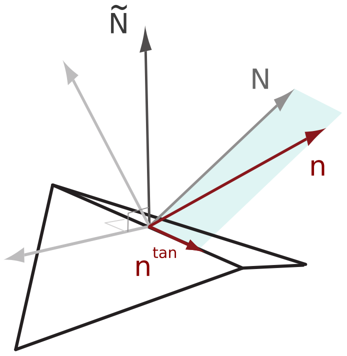


Figure 2. Rotating normals on edges

3.4 Discrete Cosserat energy

Altogether, we obtain the total elastic energy of a discrete Cosserat shell model (DCS) for isotropic homogenous materials:

$$\mathrm{W}(\mathbf{S}) = \frac{1}{2}\sum_{\mathrm{T}} \left(\mathrm{A}_\mathrm{T}(\underbrace{\frac{t}{4}\|\mathrm{I}_\mathrm{T} - \bar{\mathrm{I}}_\mathrm{T}\|^2_\mathrm{M}}_{\text{stretching}} + \underbrace{\frac{t^3}{12}\|\mathrm{II}_{n,\mathrm{T}} - \bar{\mathrm{II}}_\mathrm{T}\|^2_\mathrm{M}}_{\text{bending}}) + \underbrace{t m_s \int_\mathrm{T} \|\mathrm{s}_\mathrm{n}\|^2}_{\text{shearing}} \right) . \tag{10}$$

The sum is over triangular faces T of the discrete triangulated surface and A_T denotes the area of such a face. The material norm $\|\cdot\|^2_M$ and the coefficient m_s, as well as the extension to non-isotropic materials, is the same as in the remarkably similar smooth energy (1).

In terms of standard linear FE, the stretching energy can be interpreted as a conformal piecewise linear approximation of membrane strains and is equivalent to the popular constant strain triangle (CST). Our bending energy results from a piecewise linear non-conformal (Crouzeix-Raviart) interpolation of the directors based on midpoints, and the shearing field is a lowest order Raviart-Thomas interpolation of the directors' projection onto the edges. DCS may thus be seen as a geometric reformulation of the shell element TLLL considered by Flores et al. [3], which is the nonlienar version of the Reissner-Mindlin plate element presented in [8]. The latter element was analysed by Arnold and Falk [9] and proven to have a convergence order of $\mathcal{O}(\max h^2, t^2)$, where t is the thickness of the plate and h the average edge length.

4 DERIVATION OF A ROTATION–FREE DISCRETE KIRCHOFF SHELL

4.1 Derivation of a discrete Kirchhoff shell model

When shearing is constrained to vanish, DCS provides a shell model satisfying a *discrete Kirchhoff hypothesis*. In this case, every director $\underline{\mathrm{n}}_i$ is forced to stay orthogonal to its corresponding edge $\mathbf{e}_i$ and is thus left with a single degree of freedom, which can be seen as a *turning angle* around the edge axis. This suggests the close relation of the discrete Kirchhoff limit of the DCS model to the combination of CST and Morley's triangular element (see [7]).

Let T be a triangle of the discrete surface, N its surface normal, and let denote α_i the angle between $\underline{\mathrm{n}}_i$ and N (see Fig. 3). Then we may write the director as $\underline{\mathrm{n}}_i = \cos(\alpha_i)\mathrm{N} + \sin(\alpha_i)\ \hat{\mathbf{e}}^*_i$ in terms of the normal N, the normalized in–plane edge normal $\hat{\mathbf{e}}^*_i := \mathbf{e}^*_i / l_i$ and the angle α_i, where $l_i = \|\mathbf{e}^*_i\| = \|\mathbf{e}^*_i\|$ denotes the edge length. Inserting this angle representation into the definition (8) of the values of the discrete 2^{nd} fundamental form on edges, we get the expression $\mathrm{II}_i - \mathrm{II}_j - \mathrm{II}_k = -(8\mathrm{A}_\mathrm{T}/l_i)\sin(\alpha_i)$, which we need to obtain II in closed form by the formula (6). Using the rescaled basis $\{l_i^2\ (\overline{\hat{\mathbf{e}}^*}_i \otimes \overline{\hat{\mathbf{e}}^*}_i)\}_{i=1,2,3}$, we obtain the

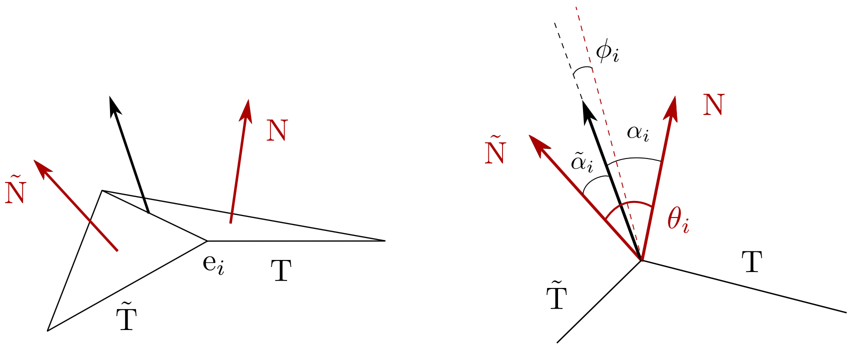


Figure 3. Angle notation in the derivation of the discrete Kirchhoff model

following discrete Kirchhoff limit of our second fundamental form:

$$\mathrm{II}_K \;:=\; \sum_{i=1}^{3} \frac{\sin(\alpha_i)}{\mathrm{h}_i/2}\, \overline{\hat{\mathbf{e}}^*}_i \otimes \overline{\hat{\mathbf{e}}^*}_i \,. \tag{11}$$

Here the height h_i of the triangle appears through the geometric relation $2\mathrm{A_T} = l_i \mathrm{h}_i$. This corresponds to a modified version of the discrete *midedge shape operator* (MSO) derived in [2] in the course of defining consistent discrete second fundamental forms on meshes. Indeed, if we denote by θ_i the *hinge angle* between the triangle normals adjacent to an edge $\mathbf{e}_i$, and require as compatibility condition over edges that $\theta_i = \alpha_i + \tilde{\alpha}_i$ holds, we can write α_i as $\alpha_i = \theta_i/2 + s_i\phi_i$, with the *half difference angle* $\phi_i = \frac{1}{2}(\alpha_i - \tilde{\alpha}_i)$ remaining as an independent variable per edge, and $s_i = \pm 1$. With this notation we may rewrite (11) equivalently as

$$\mathrm{II}_K \;=\; \sum_{i=1}^{3} \frac{1}{\mathrm{A_T} l_i} \sin(\theta_i/2 + s_i\phi_i)\, \overline{\hat{\mathbf{e}}^*}_i \otimes \overline{\hat{\mathbf{e}}^*}_i \,. \tag{12}$$

This recovers the MSO expression proposed in [2] for small absolute values $|\alpha_i|$ of the edge angles, as the angle α_i itself instead of $\sin(\alpha_i)$ was used there. As the hinge angle θ_i may take arbitrary values in the open interval $(-\pi, \pi)$, a close correspondence of the MSO expression and the Kirchhof limit II_K may be expected for small deformations of shallow shells only.

Closely related to the Morley triangle — conceptually as well as analytically — are so-called *rotation–free shell elements*, which determine the *a priori* free angle on edge midpoints by the vertex positions of the neighboring triangles. Various types of models deduce a constant moment triangle by a *linear superposition* $\sum_{i=1}^{3} \kappa_i\, \overline{\hat{\mathbf{e}}^*}_i \otimes \overline{\hat{\mathbf{e}}^*}_i$ of one-dimensional normal curvatures κ_i across edges. In the view of Gaerdsback and Tibert [7], this superposition ansatz seems to be ad hoc, as they state that *"... no exact relation exists ... "* between scalar normal curvature quantities and 2D curvature tensors. However, the geometric perspective of our construction not only provides the derivation of such a relation through (11), it also reveils that the discrete Morley type curvature quantities $\kappa_i = 2\sin(\alpha_i)/h_i$ result from consistent finite difference expressions evaluated between edge midpoints, *i.e. along* edge directions, not perpendicular to them. Moreover, a discrete second fundamental form with normal curvatures $\{\tilde{\kappa}_i\}_{i=1,2,3}$ evaluated *accross* edges, *i.e.* along the $\mathbf{e}_i^*$ directions, would necessarily result in $\tilde{\mathrm{II}}_K = -1/(8\mathrm{A_T}) \sum_{(ijk)} (\kappa_i - \kappa_j - \kappa_k)\, \overline{\hat{\mathbf{e}}}_i \otimes \overline{\hat{\mathbf{e}}}_i$ according to (6), which is completely different from the previous expression.

4.2 Rotation–free approximation of the discrete Kirchhoff shell model

As the triangle normals N and $\tilde{\mathrm{N}}$ adjacent to an edge $\mathbf{e}_i$, which both depend on the four vertices of the pair of triangles sharing the edge only (see Fig. 3), implicitely determine the hinge angle θ_i via the relations $\cos(\theta_i) = \langle \mathrm{N}, \tilde{\mathrm{N}} \rangle$ and $\sin(|\theta_i|) = \|\mathrm{N} \times \tilde{\mathrm{N}}\|$, the hinge angles depend on triangle vertices only. Therefore we obtain a *rotation–free approximation* of the discrete Kirchhoff shell model derived in the previous subsection as the Kirchhoff limit of the DCS model by eliminating the angle variable ϕ_i from (12) by an approximating expression that likewise depends on adjecent vertices only.

One example of such a rotation–free approximation is the one proposed by Sabourin and Brunet [11] in the derivation for their triangular *S3* element, which turns out to be equivalent to postulate a difference angle $2\phi_i \sim \theta_i$ proportional to the hinge angle, with the relative difference $(\mathrm{A_T} - \tilde{\mathrm{A}}_\mathrm{T})/(\mathrm{A_T} + \tilde{\mathrm{A}}_\mathrm{T}) = 2\phi_i/\theta_i$ of adjacent triangle areas as proportionality factor. For well designed triangulations one may assume that all triangles have approximately the same size, such that the proportionality factor becomes very small, i.e.: $|\phi_i|/|\theta_i| \ll 1$. Based on this assumption, we further simplify the approximation of [11] by setting all difference angles ϕ_i exactly to zero, such that $\alpha_i = \tilde{\alpha}_i = \theta_i/2$ holds, and we may substitute $\sin(\alpha_i) = \sin(\theta_i/2)$ into (11). In the same way, the assumed approximate equality of triangle areas $\mathrm{A_T} \approx \tilde{\mathrm{A}}_\mathrm{T}$ implies the approximate equality of the corresponding heights $h_i \approx \tilde{h}_i$, which we approximate by their arithmetic mean $\bar{h}_i := \frac{1}{2}(h_i + \tilde{h}_i) = (\mathrm{A_T} + \tilde{\mathrm{A}}_\mathrm{T})/l_i$. Next we replace the hinge angle function $2\sin(|\theta_i|/2) = \|\mathrm{N} - \tilde{\mathrm{N}}\|$, which remains bounded by its maximum value 2 even in the case $|\theta_i| = \pi$ corresponding to a degenerate configuration, by the alternative expression $2\tan(|\theta_i|/2) = 2\|\mathrm{N} \times \tilde{\mathrm{N}}\|/(1 + \langle \mathrm{N}, \tilde{\mathrm{N}} \rangle)$.

Altogether, this yields the following rotation–free approximation of the discrete 2^{nd} fundamental form (11), which corresponds to a modified version of the *triangle average shape operator* introduced in [2]:

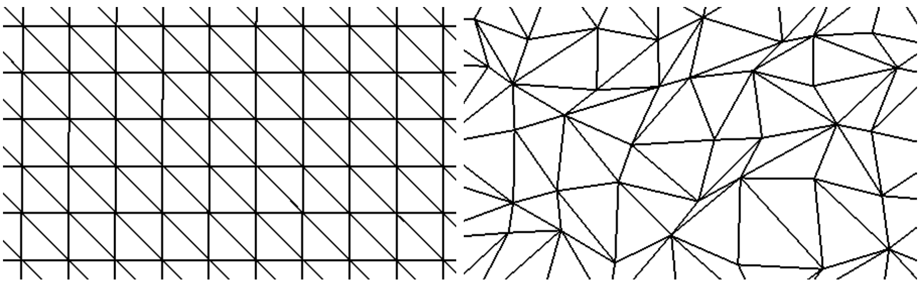
$$\tilde{\kappa}_i \;:=\; \sigma_i \frac{2}{\bar{h}_i} \tan\left(\frac{|\theta|_i}{2}\right) \;\longrightarrow\; \mathrm{II}_{DKS} \;:=\; \sum_{i=1}^{3} \tilde{\kappa}_i \, \overline{\hat{\mathbf{e}}^*}_i \otimes \overline{\hat{\mathbf{e}}^*}_i \,. \tag{13}$$

Here $\sigma_i = \mathrm{sgn}(\tilde{\kappa}_i)$ has to be defined consistently with the global choice of the orientation of all triangles in the mesh. We denote the *discrete Kirchhoff shell* model obtained by using CST in combination with the rotation–free approximation of the bending energy obtained from (13) as DKS model. Next we investigate both the DCS and DKS model using classical benchmarks from Belytschko's obstacle course.

5 BENCHMARKS

The models we used for static benchmarks are identical to those studied by Sze et al.[5] with ABAQUS's *S4R* elements, and we use their results as a reference solution for comparison. The static problems were solved by minimising the elastic energy with the conjugate gradient method. The magnitude of the load was gradually increased from zero to maximum usually with 5% steps providing deflection curves. Model and load symmetry (where applicable) were not exploited, the whole model was simulated.

Two triangulation methods were used for the tests: regular and strongly irregular. A regular mesh with triangles of similar size and vertices of equal valence, for example a quad mesh with diagonals in the same direction Fig. 5(a), produces a mesh which is very favourable for the DKS model. If the diagonals are not pointing in the same direction (randomised or alternated) or if there are other irregularities, the mesh will become less favourable. A strongly irregular mesh with close to degenerate triangles Fig. 5(b) was generated to test the DKS model under the most unfavourable conditions. The reason for that is to discover the limits of the model and to see how well it behaves under the "worst case scenario" circumstances.



(a) Regular (favourable) (b) Strongly irregular (unfavourable)

Figure 4. Triangulation methods used for benchmark models.

We refer to the test results produced with a regular triangulation as favourable and present them with green lines in the plots. Favourable meshes we used were of similar design but not identical to those in [5]. The test results produced with an irregular triangulation are refered to as unfavourable and presented with red lines. The numbers shown in the legends represent the numbers of DOF of the simulated model. However, we note that most of the DCS d.o.f. are associated to the directors and not to the vertices, making the DCS meshes appear coarser than the DKS meshes with a similar number of DOF.

5.1 Slit annular plate

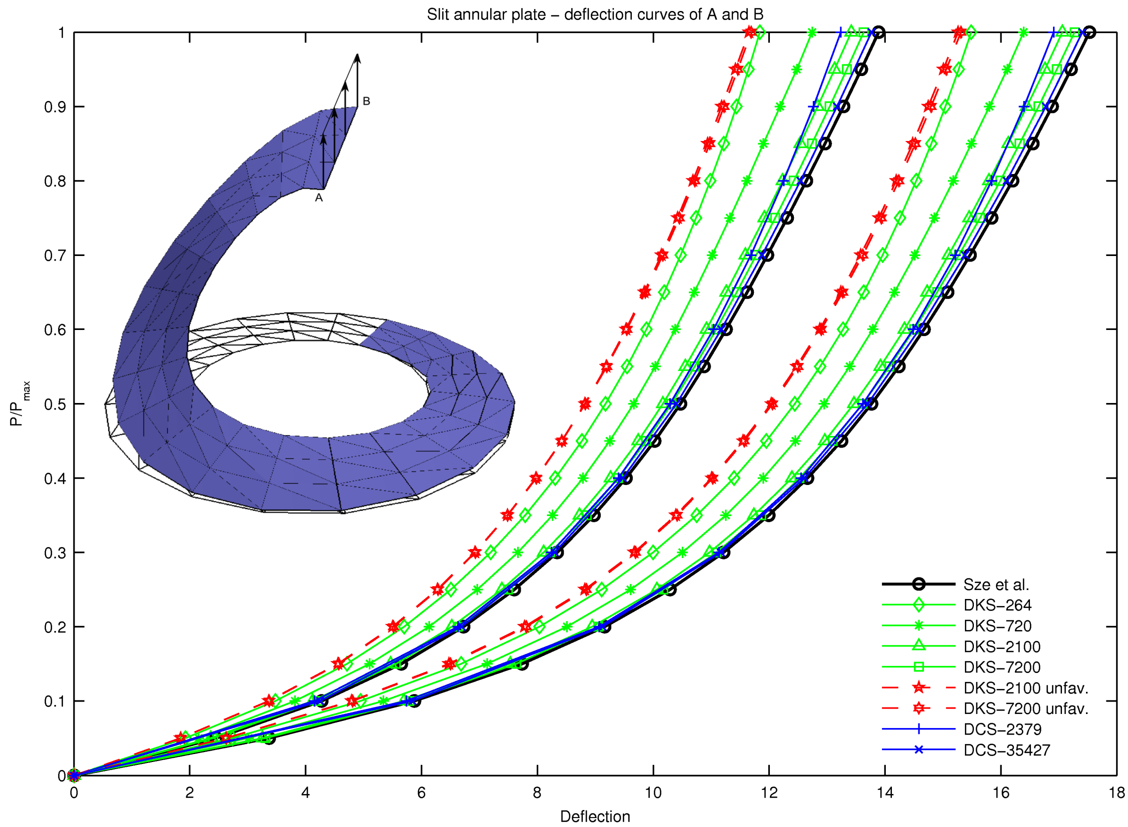


Figure 5. Slit annular plate and its benchmark results.

The slit annular plate and its benchmark results are presented in Fig. 5. One end of the plate is fully clamped, the other is subjected to a distributed vertical load. Vertical deflections of the corner vertices A and B of the loaded end were measured. As we can see the DKS model provides good results even for coarser favourable meshes. Results produced with finer meshes are comparable to those provided by the DCS with a similar number of DOF (blue lines in the plots). The curves converge to the reference solution provided by Sze et al. or into its neighbourhood. Unfavourable meshes act stiffer and judging by how close they are to each other, there is little or no convergence to the reference solution.

5.2 Pinched hemisphere

In this test a shell in the form of a rotated 72° circular arc (hemisphere with 18° cutout) is pinched in one direction and pulled in another. The model and its benchmark results are presented in Fig. 6. The deflections of the pulled vertex A and the the pushed vertex B were measured. The results are similar to those of the slit annular plate: favourable meshes perform reasonably well and comparable to the results of the DCS with a similar number of DOF. Unfavourable meshes are stiffer, and in this example it can be clearly seen that they are stiffer even for small deformations.

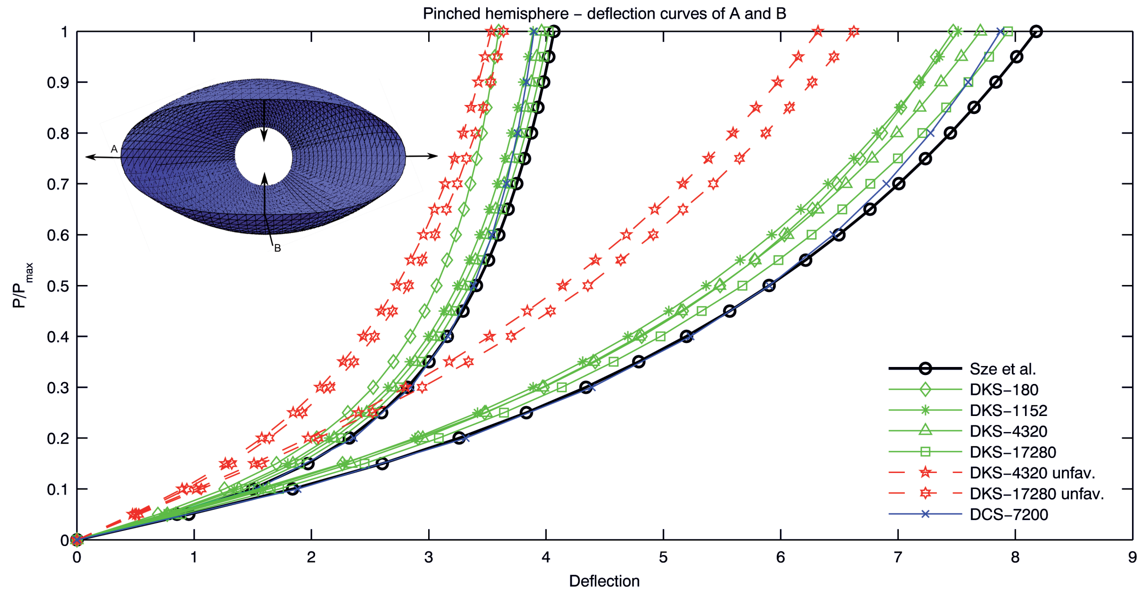


Figure 6. Pinched hemisphere and its benchmark results.

5.3 Cylinder radial pull-out

Fig. 7 presents a cylinder being pulled out by radial forces applied at vertex A and its opposite on the other side. Once the deformation reaches a certain point the middle part of the cylinder buckles inwards. Vertical deflection of vertex A and horizontal (radial) deflections of vertices B (middle) and C (edge) were measured. The difference between B and C deflections shows the amount of buckling. First thing we notice looking at the plots: even the coarsest favourable mesh produces good results before the buckling occurs. After that coarse meshes simply cannot reproduces high local deformations and show quantitatively wrong results (qualitatively they still follow the reference solution). A test with a very fine favourable mesh produces results which almost coincide with the reference solution, successfully validating the model. Unfavourable meshes act stiffer just as in the other examples.

5.4 Eigenfrequencies

The goal of this research is the simulation of large non-linear deformations. However for the sake of model validation several experiments were undertaken to show how well the linearisation of the DKS corresponds to the linear beam and plate theory. Three examples are presented here: simply supported Fig. 9(a) and cantilever Fig. 9(b) beams, modeled with a thin triangle stripe, and a simply supported plate Fig. 9(c).

The resulting eigenfrequencies are given in Tables 1, 2 and 3, compared with the exact solution provided by the corresponding shear-free continuous vibration theory. Both coarse and fine beam models produce satisfactory results. The frequency deviation for the simply supported beam with 10 segments is less than 4%, for the beam with 100 segments it doesn't exceed 0.5%. The frequency deviation for the cantilever beam with 10 segments is about 15%, for the beam with 100 segments it is about 1.5%. The results of the plate, just as those of static benchmarks, were influenced by the triangulation. The frequency deviation for the tested favourable meshes is about 5-15%. For the unfavourable meshes it reaches 30%.

6 CONCLUSIONS

In the work presented, we derived low order discrete shell models of Cosserat (DCS) and Kirchhoff type defined on *triangular* meshes by using geometric finite difference approximations of the basic fundamental forms entering into the deformation energy of the shell configuration. In particular, we deduced a rotation–

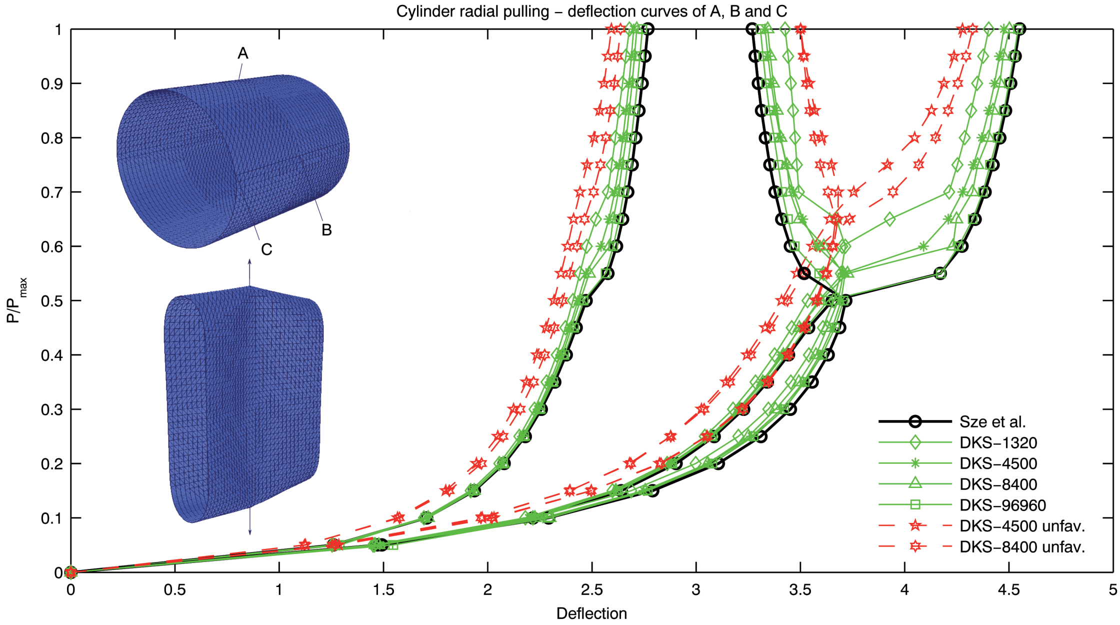
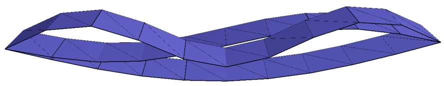
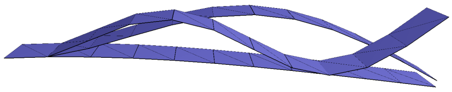


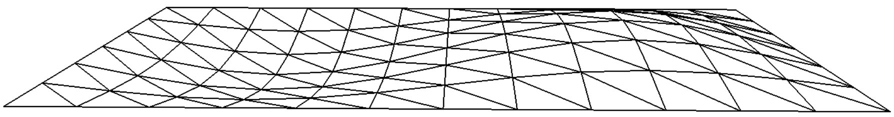
Figure 7. Cylinder radial pull-out and its benchmark results.



(a) Simply supported beam, DKS 10 segments



(b) Cantilever beam, DKS 10 segments



(c) Simply supported plate, second mode shape, DKS 6x12 segments

Figure 8. Free vibration mode shapes with DKS.

Table 1. Simply supported beam free vibration eigenfrequencies.

DKS 10 segments	DKS 100 segments	Euler-Bernoulli continuous
4.71	4.55	4.53
18.40	18.22	18.13
39.72	40.98	40.81

Table 2. Cantilever beam free vibration eigenfrequencies.

DKS 10 segments	DKS 100 segments	Euler-Bernoulli continuous
1.40	1.59	1.61
8.77	9.97	10.12
24.17	27.92	28.34

free approximation (DKS) of our discrete Kirchhoff model with the triangle vertex positions as degrees of freedom. Both models behave physically plausible already on very coarse meshes, and show good convergence properties on regular meshes. Moreover, from the theoretical side, this deduction provides

Table 3. Simply supported plate free vibration eigenfrequencies.

DKS 6x12 segments		DKS 12x24 segments		Kirchhoff-Love
favourable	unfavourable	favourable	unfavourable	continuous
25.91	29.32	24.43	27.72	22.67
42.06	48.15	39.44	45.59	36.27
66.43	74.03	63.50	72.81	58.94
80.33	84.11	80.65	87.91	77.08
97.33	102.97	96.22	108.84	90.69
97.55	118.89	96.23	110.75	90.69

a common geometric framework for several existing models. However, our numerical tests show that the simulation quality can be deteriorated with an unfavourable triangulation, causing the shell to show up to about 20% higher bending stiffness than the reference solution, stiffness increase arising probably due to membrane locking. This is however not a critical flaw. It is much easier to make a favourable mesh than an unfavourable, and chaotic triangulations in our experiments were explcitely designed to produce bad results, showing the limits of the model. A typical triangulation produced by CAD or FEM software would be much more favourable for the DKS model.

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