



Fraunhofer Institut
Techno- und
Wirtschaftsmathematik

G. Hanselmann, A. Sarishvili

Heterogeneous redundancy in
software quality prediction using
a hybrid Bayesian approach

© Fraunhofer-Institut für Techno- und Wirtschaftsmathematik ITWM 2007

ISSN 1434-9973

Bericht 125 (2007)

Alle Rechte vorbehalten. Ohne ausdrückliche schriftliche Genehmigung des Herausgebers ist es nicht gestattet, das Buch oder Teile daraus in irgendeiner Form durch Fotokopie, Mikrofilm oder andere Verfahren zu reproduzieren oder in eine für Maschinen, insbesondere Datenverarbeitungsanlagen, verwendbare Sprache zu übertragen. Dasselbe gilt für das Recht der öffentlichen Wiedergabe.

Warennamen werden ohne Gewährleistung der freien Verwendbarkeit benutzt.

Die Veröffentlichungen in der Berichtsreihe des Fraunhofer ITWM können bezogen werden über:

Fraunhofer-Institut für Techno- und
Wirtschaftsmathematik ITWM
Fraunhofer-Platz 1

67663 Kaiserslautern
Germany

Telefon: +49(0)631/3 1600-0
Telefax: +49(0)631/3 1600-1099
E-Mail: info@itwm.fraunhofer.de
Internet: www.itwm.fraunhofer.de

Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.



Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001



Fraunhofer Institut
Techno- und
Wirtschaftsmathematik

Heterogeneous Redundancy in Software Quality Prediction using a Hybrid Bayesian Approach

Gerrit Hanselmann and Alex Sarishvili

September 19, 2007

Abstract

With the ever-increasing significance of software in our everyday lives, it is vital to afford reliable software quality estimates. Typically, quantitative software quality analyses rely on either statistical fault prediction methods (FPMs) or stochastic software reliability growth models (SRGMs). Adopting solely FPMs or SRGMs, though, may result in biased predictions that do not account for uncertainty in the distinct prediction methods; thus rendering the prediction less reliable. This paper identifies flaws of the individual prediction methods and suggests a hybrid prediction approach that combines FPMs and SRGMs. We adopt FPMs for initially estimating the expected number of failures for finite failure SRGMs. Initial parameter estimates yield more accurate reliability predictions until sufficient failures are observed that enable stable parameter estimates in SRGMs. Being at the equilibrium level of FPM and SRGM predictions we suggest combining the competing prediction methods with respect to the principle of heterogeneous redundancy. That is, we propose using the individual methods separately and combining their predictions. In this paper we suggest Bayesian model averaging (BMA) for combining the different methods. The hybrid approach allows early reliability estimates and encourages higher confidence in software quality predictions.

Keywords—Reliability Prediction, Fault Prediction, Non-homogeneous Poisson Process, Bayesian Model Averaging

MSC: 62F15, 62H12, 62C10

1 Introduction

Conventionally, quality prediction approaches use exclusively fault prediction methods (FPMs) or software reliability growth models (SRGMs). FPMs use software metrics and fault data from previous software releases or similar software projects to predict the number of faults of the software. SRGMs treat the software as a black-box and gather failure data throughout functional black-box testing; thereby, assuming software is exercised along its operational profile [1]. The observed failures are used to calibrate a stochastic model employed for predicting the reliability of the software.

Even though the competing models can complement each other, combining these two approaches is hardly implemented, if at all. Intuitively, FPMs render useful for initially estimating the number of finally expected failures of finite failure SRGMs, as long as not enough failures have been observed during testing. This combination enables early usage of SRGMs, even before testing has commenced. On the other hand, with more and more failures collected during testing the predictive performance of SRGMs is gradually improving. Yet SRGMs are always heavily dependent on the testing process and the failures collected during testing. This innate dependence causes biased predictions if the model assumptions are not properly met. FPMs are not as much dependent on testing. Considering only the software under test¹ the FPM approach is independent of testing. Only software measures are used to predict the number of faults. However, there is not the one software complexity measure that can be directly related to the number of faults expected. There is always a lot of uncertainty involved in building FPM models. Other sources for bias in the FPM models are the issue of transferability of a fault prediction model. Adopting a FPM model demands closeness of the current to the historical software projects, and even software development processes. SRGMs ignore any software measures or required closeness to former projects they are only dependent on the testing process.

The distinct software quality prediction approaches are quite different in use and in the data they draw on. Thus the individual prediction methods are prone to different kinds of errors. With the hybrid approach we suggest combining FPMs and SRGMs on different levels. We aim for:

- Enabling SRGM predictions from the very start of testing
- Supporting a priori model selection for SRGMs
- Providing higher confidence in reliability predictions
- Reducing bias inherent in the individual prediction methods
- Introducing more flexibility for adapting the prediction process to concrete projects.

¹Faults from previous or similar software projects may as well be found during testing.

To meet these aims we unveiled flaws of the distinct quality prediction approaches and identified coupling points. Drawing on these coupling points we suggest initial parameter estimates for SRGMs using FPMs. We propose a stopping rule for these initial parameter estimates when the prediction performance reaches a certain level. Confidence bounds are used to compare FPM with SRGM prediction performance. Having passed the stopping point for initial parameter estimates we propose model combination on the principle of heterogeneous redundancy. That is, using the individual prediction methods separately and combining their predictions. Since the approaches are entirely different, draw on different data, and are prone to different kinds of errors this combination is expected to yield more trustworthy reliability predictions. With the combination we are in average superior to solely employing the individual approaches, for we gain at least their prediction accuracy but are provided with a second independent prediction. This encourages higher confidence in the quality predictions. We use Bayesian model averaging (BMA) for combining the different FPM and SRGM models. Since neither FPM nor SRGM has one single optimal model for their predictions but in both distinct methods it is not possible to select the single best model a priori BMA does not only account for the bias innate in the distinct methods but also for the bias of the different models within the individual methods.

Table 1 lists the acronyms used throughout this paper. Section 2 provides a survey of existing quality prediction approaches and touches on their deficiencies. Section 3 is the main chapter and introduces the hybrid approach. It presents the initial parameter estimates, the stopping rule, and the model combination. Section 4 presents the conclusions and future work.

Table 1: Acronyms

SRG	software reliability growth
SRGM	software reliability growth model
FPM	fault prediction model
BMA	bayesian model averaging
(E)NHPP	(enhanced) non-homogeneous poisson process
ML	maximum likelihood
RSM	response surface model

2 Software quality prediction

2.1 Software Reliability Growth Models

Reliability is a key quality characteristic of software, which reflects the users view on software and favors trade-offs in release planning. Software reliability is the probability of failure-free operation for a specified period of time in a specified environment [2]. In order to predict software reliability we need to model the failure process of software. During testing and debugging we usually have reliability growth due to the (perfect) removal of faults causing the experienced failures.

Reliability prediction usually involves testing the software, and collecting the times of its failures², t_i . Using t_i we can calibrate the SRGMs and predict software's reliability. A comprehensive overview on SRGMs can be found in [3–6].

A large number of SRGMs have been developed over the years. Among the first were the binomial model of Jelinski and Moranda [7] and the Goel-Okumoto model [8] that describes the failure process as a NHPP (Non-Homogeneous Poisson Process). Since then, many other models have been suggested. For NHPP models Gokhale and Trivedi proposed a unifying framework, enhanced (E)NHPP, which also captures test-coverage [9]. However, the models rely on rather crucial assumptions. Software should be exercised along its operational profile, repairs should be effected instantly and without introducing new faults, software failures are independent, and so on. Beyond these issues, SRGMs can be employed only very late in the software development cycle, a priori model selection is not possible, and the models perform only well if sufficient failures have been observed. It is not possible, though, to pre-determine whether enough failure data has been collected. Having only insufficient failure data obstructs finding maximum likelihood estimates for the model parameters if it is at all possible. Even being provided with sufficient failure data maximum likelihood estimates are sometimes instable [10, 11]. With the hybrid approach we propose early parameter estimates using fault prediction models to enable early usage of SRGMs and to circumvent the problems of instable parameter estimates.

In practice many of the presented assumptions are often violated. So, software is commonly tested systematically with the purpose of finding faults. Also, there is often a correlation of successive failures [12] which comes along with testing for finding bugs. These violations lead to flawed reliability predictions. With the hybrid approach we intend to provide a back up for the predictions in order

²Collecting failure data in terms of grouped failure per time interval or collecting inter-failure times is also possible

to reduce the bias caused by violating the assumptions.

2.2 Fault Prediction Models

Faults inherent in software may cause failures while running the software. So estimates on the number of faults are quite interesting in measuring software's quality. For estimating this number of faults, fault prediction models search for attributes of the software or of the software development process to extract causes for faults. Based on these attributes a fault prediction model can be built. Diverse software metrics have been developed that in some way affect the number of faults and, thus, can be employed in fault prediction models³.

Using the collected metrics and faults from previous releases of similar software the fault prediction models can be built. Starting with simple univariate prediction models based solely on lines of code measures [13] up to complex prediction models like response surface models, multivariate adaptive regression splines (MARS), neuronal networks, and the like, there exists a broad variety of fault prediction models. A critical overview is provided in Fenton and Neil [14] further references are [15, 16].

Fault prediction models suffer from their uncertain or even lacking transferability. Having created a fault prediction model based on metrics and faults from historical projects it is not sure whether this model is applicable for a current project. Fenton and Neil [14] found that regression modeling alone is inadequate for software fault prediction. In the hybrid approach we want to combine fault prediction with reliability prediction that is solely based on data from the current project. Fault prediction is supposed to provide an independent back up for reliability prediction, and vice versa.

³Typically size measures like lines of code are strongly correlated to faults.

3 Hybrid Reliability Prediction

As listed, sole use of FPMs or SRGMs is tainted with much uncertainties and tends to yield predictions that are sometimes not satisfactory. The individual techniques rely on many assumptions that impede their adoption in specific projects or render their results inaccurate without any further information about their prediction accuracy. If data of one of the individual prediction techniques is flawed all models within this prediction method suffer from these data. The hybrid approach that incorporates two distinct prediction techniques based on different data and assumptions remedies these deficiencies.

Intuitively, the notion of combining different prediction techniques seems straightforward. Yet, only little research has been conducted on this topic. So far, combining FPMs and SRGMs targets at alleviating the prediction incapability of SRGMs at the very start of testing when only little failures have been observed. With ENHPP Gokhale and Trivedi [9] stated the possibility of initially estimating the number of expected failures of SRGMs using FPMs. Xie et al. [17] proposed a method for early estimating the fault detection rate based on former projects. They make rather strict assumptions on the similarity of the involved projects, though. Musa [4] proposed the fault exposure ratio for providing early parameter estimates for SRGMs. This was further investigated by Malaiya et al., e.g. [18]. A different approach for failure estimates using FPMs was made by Nagappan et al. [19,20]. They proposed early estimation of post-release failures based on historical failures and product and process metrics.

Besides these approaches to support SRGMs using FPMs there are approaches that state superior predictions for combining different models of a single prediction method. Lyu et al. [21] proposed a linear combination of different SRGMs to obtain improved results. They found that the combination, even in its simplest format, appeared to provide more accurate predictions than the individual SRGMs alone [22]. The combination of solely SRG models, though, is prone to the same kind of errors that afflict individual SRGMs, e. g. since individual SRGMs depend on testing so does their combination. Apparently, homogeneous redundancy improves reliability predictions, for it moderates the flaws of the individual SRG models. However, it does not address the deficiencies within the prediction technique itself. Generally in forecasting, model combination has attracted much attention. A summary can be found in [23].

The main challenge for the hybrid approach is the identification of appropriate coupling techniques of FPMs with SRGMs. Fig. 1 depicts the combination and coupling respectively of FPMs and SRGMs.

We identified four main coupling ideas:

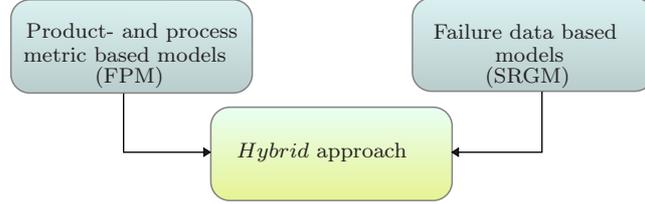


Figure 1 Overview of the Hybrid approach

- Specific SRG model parameter estimation via FPM model
- SRG model class estimation via FPM model
- SRG model fault intensity function class estimation via FPM model
- On principle of heterogeneous redundancy based coupling technique.

In this paper we concentrate on the combination of the first and fourth coupling, which addresses the parameter estimation problem, the problem of estimating conditional confidence bounds, and the problem of combining distinct modeling approaches.

3.1 Initial Parameter Estimates

With the hybrid approach we resume research on providing early parameter estimates for SRGMs. FPMs can be used to estimate the number of expected failures of SRGMs given infinite testing time. That is, we identify a suitable fault prediction model, f_{FPM} , based on software measures, X , and fault data from previous or similar software

$$\hat{a} = f_{FPM}(X, \theta) + \epsilon. \quad (1)$$

Here \hat{a} is the estimated expected number of failures given infinite testing time. We use as the function approximator the so called response surface model with second order interactions,

$$f_{FPM}(X, \theta) = \theta_0 + \sum_{i=1}^N \theta_i X_i + \sum_{k < j} \theta_{kj} X_k X_j, \quad (2)$$

with parameter vector $\theta \in R^{N+m+1}$ and the number of summands in the second term of equation 2 equals $m = \binom{N}{2}$. $X \in R^N$ is the vector of software measures available, and $\epsilon \sim N(0, I\sigma^2)$ is the independently identically distributed term of random errors with zero expectation and homoscedastic variance covariance matrix $I\sigma^2$.

We estimate the SRGM with a modified ENHPP model in the following manner. Let $N(t)$ be the accumulated number of detected failures until time t , which is

a random variable with Poisson distribution, i. e.

$$P(N(t) = k) = \frac{[m(t)]^k}{k!} e^{-m(t)}, k = 0, 1, \dots, \infty. \quad (3)$$

The NHPP expected number of failures is expressed as

$$m(t) = \hat{a}Kc(t).$$

Here $K \in [0, 1]$ is the efficiency of failure detection in case of exercising the respective fault site, and $c(t)$ is the test coverage function. Because the test coverage function $c(t)$ is hardly to be determined by the testing crew we have decided to estimate the coverage growth by the logistic function

$$c(t) = \frac{1}{1 + u \exp(-vt)}.$$

Then we estimate the parameters u and v by maximum likelihood. Estimating the expected number of failures using FPM enables reliability predictions with only little failure data, or even before testing has commenced. Furthermore, we circumvent the problem of instable parameter estimates and the problem of instable reliability growth trends [10, 11]. For the coverage function, $c(t)$, it is as well possible to use other coverage functions like the exponential coverage function which is used in the Goel-Okumoto [8] model. This and other coverage functions are listed in [9].

3.2 Stopping Rule for Initial Parameter Estimates

From different publications mentioned above it is clear that at the beginning of testing the SRG models have poor predictive performance. Therefore at initial state of testing the hybrid model, with the initial parameter estimates, seems to be the only possibility to achieve appropriate estimates of the failure content of the software. At this stage combining FPMs and SRGMs on the heterogeneous redundancy principle is not reasonable. However, with increasing testing duration the SRG model performance improves due to the fact that more and more data is available for the maximum likelihood (ML) estimation of the parameters in the model. Conversely, the FPMs do not depend on the current failure finding process, and therefore their performance is constant over the entire testing period. Having increasing model performance of the SRGMs and constant performance of the FPMs it becomes evident that we need to define a stopping rule for the initial parameter estimates. This stopping rule is also the starting point for combining FPMs and SRGMs on the principle of heterogeneous redundancy. When the predictions of SRGMs are of equal performance as the predictions of FPMs this combination assures improved prediction accuracy. Figure 2 displays the notion of the testing time dependent coupling and combination implemented in the hybrid approach.

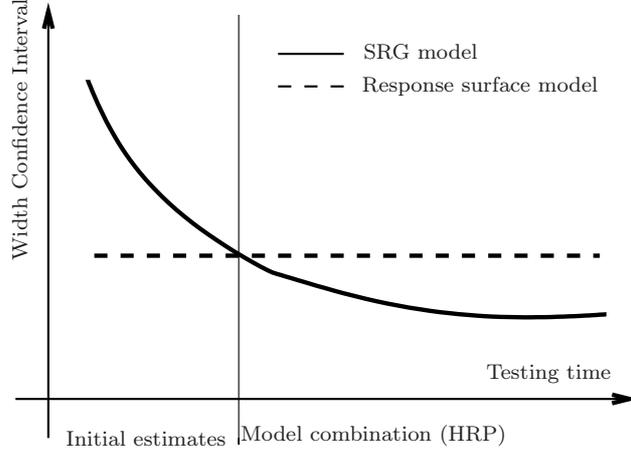


Figure 2 Stopping rule for initial parameter estimates

It is convenient to take the width of the estimated confidence interval of the interesting model parameter as model performance measure, e. g. the confidence interval of the expected number of failures in the code by infinite time of testing. The confidence interval for \hat{a} from the response surface model (see formula 2) for a particular vector of software complexity measure, X , can be estimated by

$$B_{RSM}(X) = \left[\hat{a} \pm t_{n-2; \alpha/2} s \sqrt{\frac{1}{N} + \frac{(X - \bar{X})^2}{(n-1)s_x}} \right] \quad (4)$$

where $t_{n-2; \alpha/2}$ is the $\alpha/2$ quantile of the student's t-distribution

$$s^2 = \frac{1}{N-2} \sum_{i=1}^N (a_i - \hat{a}_i)^2, s_x^2 = \sum_{i=1}^N (X_i - \bar{X})^2.$$

On the other side we need the confidence interval of the parameter \tilde{a} of the NHPP model, which is estimated by the maximization of the corresponding ML equation. A standard result says that the ML estimator has a limiting normal distribution

$$\tilde{a} \xrightarrow{n \rightarrow \infty} \mathcal{N}(\tilde{a}, I(\tilde{a})^{-1})$$

where

$$I(\tilde{a}) = - \sum_{i=1}^n \frac{\partial^2 \ln(L_i)}{\partial \tilde{a}^2}$$

is the local Fisher information. The logarithmic ML equation with respect to the model described in formula 3, is expressed as follows

$$\ln(L_i) = \ln \left[\frac{[m(t_i) - m(t_{i-1})]^{f_i}}{f_i!} e^{[m(t_i) - m(t_{i-1})]} \right],$$

for any $i = 1, \dots, n$, where f_1, \dots, f_n are the numbers⁴ of detected failures respective to the time intervals $(t_0 = 0, t_1), (t_1, t_2), \dots, (t_{n-1}, t_n)$. With this result we can construct the asymptotic variance of the ML estimator of the mentioned parameter, which is the inverse of the local Fisher information

$$\text{var}(\tilde{a}) = \frac{1}{I(\tilde{a})}.$$

With this knowledge we can compute the confidence interval of the SRG model, which is described as follows

$$B_{SRG}(t) = \left[\tilde{a} \pm Z_{\alpha/2} \sqrt{\text{var}(\tilde{a})} \right], \quad (5)$$

where $Z_{\alpha/2}$ is the $100(1 + \alpha)/2$ percentile of the standard normal distribution.

Using these confidence intervals we can estimate the breaking time (BT) (vertical line on Fig. 2), where we change the hybrid model from initial parameter estimates to the heterogeneous redundancy principle based model. Setting the y-axis of Fig. 2 as the width of the confidence intervals of the two models. Break time is the time t for which the following holds

$$|B_{SRG}(t)| = |B_{RSM}(X_c)| \quad (6)$$

that is the time t under the condition that $|B_{SRG}(t)| = |B_{RSM}(X_c)|$, where X_c is the vector of software complexity measure for the current software under testing. The confidence interval widths can be computed by the formulas 4 and 5.

3.3 Model combination on principle of heterogeneous redundancy

Being at the equilibrium level of FPMs and SRGMs, that is having passed the breaking time (BT) (see equation 6) both prediction methods yield reasonable quality predictions. With the hybrid approach we suggest combining these predictions to account for the uncertainties innate in the individual methods. The notion of using redundant approaches to gain higher trust in systems is pervasive in fault tolerant system. Thus using different FPM or SRG models is straightforward and alleviates problems as model selection and the like. For SRGMs this approach has been successfully employed by Lyu et al. [21]. Yet, since all the models within either FPM or the SRG method suffer from the same kind of errors, e.g. all draw on the same data, homogeneous redundancy will not suffice. Thus we suggest combining FPM and SRGM predictions.

Model combination is often considered an obstacle in finding the true model [24]. We argue, that in case of software quality prediction there is no such

⁴We collected grouped failure data.

one true model that generates the fault data or describes the failure process of software. Conversely, using multiple approaches and combining them on principal of heterogeneous redundancy yields more trustworthy predictions, for the individual predictions are not suppressed but are used as information for building a more accurate model.

Combining two distinct prediction approaches, however, demands that these approaches predict the same variable. For simplicity we assume that a fault predicted with FPMs causes finally a failure. This assumption holds when the FPMs are calibrated with faults that were observed as failures, i.e. failures found during testing and after release are used to calibrate FPM models. Thus with FPM (see equation 1) we can compute the number of expected failures, \hat{a} . On the site of SRGMs we have the mean value function, $\mu(t)$, the number of expected failures up to time t . Thus, with $t \rightarrow \infty$ we obtain the eventually expected number of failures. Let F be the expected number of failures then we have $F = \hat{a}$ on site of the FPM models and $F = \lim_{t \rightarrow \infty} \mu(t)$ on site of SRGMs.

For the combination of these prediction results we chose Bayesian Model Averaging (BMA) [25]. BMA provides a coherent mechanism to account for different model uncertainties. It averages over the different predictions, weighted by the posterior model probability. Let $M_{FPM_1}, \dots, M_{FPM_I}$ be the fault prediction models and $M_{SRGM_{I+1}}, \dots, M_{SRGM_k}$ be the reliability prediction models. Then we have M_1, \dots, M_k different models that predict the number of failures, F . The posterior distribution given data, D , is

$$p(F|D) = \sum_{i=1}^k p(F|M_i, D)p(M_i|D), \quad (7)$$

where $p(F|M_i, D)$ is the posterior distribution of F under model M_i and $p(M_i|D)$ is the posterior probability of model M_i given D . Since we have different kinds of data for fault and reliability prediction models, equation 7 forms to

$$\begin{aligned} p(F|RGD, FPD) &= \sum_{j=1}^I p(F|M_{FPM_j}, FPD)p(M_j|FPD) \\ &+ \sum_{j=I+1}^k p(F|M_{SRGM_j}, RGD)p(M_j|RGD), \end{aligned} \quad (8)$$

where FPD and RGD stand for the metrics, i.e. the data for the fault prediction models, and the failure data, i.e. the data for the SRGMs, respectively. Thereby it is indifferent whether we compute the posterior probability $p(M|D)$ for FPMs or for SRGMs. The posterior probabilities of the k considered models have to add up to 1

$$\sum_{i=1}^k p(M_i|D) = 1.$$

The BMA point prediction of the number of experienced failures is

$$\hat{F}_{BMA} = \sum_{i=1}^k \hat{F}_i p(M_i|D),$$

where \hat{F}_i is the estimated number of faults of model i and the posterior probability for model M_i is given by

$$p(M_i|D) = \frac{p(D|M_i)p(M_i)}{\sum_{j=1}^k p(D|M_j)p(M_j)},$$

with

$$p(D|M_i) = \int p(D|\theta_i, M_i)p(\theta_i|M_i)d\theta_i. \quad (9)$$

Equation 9 is the integrated likelihood of model M_i where θ_i is the vector of parameters of model M_i , $p(\theta_i|M_i)$ is the prior density of θ_i under model M_i , $p(D|\theta_i, M_i)$ is the likelihood, and $p(M_i)$ is the prior probability that M_i is the true model [25].

The integrated likelihood from equation 9 for the FP model can be estimated in the following way. For simplicity we reject from the equations the conditional information on models. Let $g(\theta_i) = \log(p(D|\theta_i)p(\theta_i))$ and let $\tilde{\theta}_i = \arg \max_{\theta \in \Theta_i} g(\theta)$. Then after Taylor series expansion truncated at the second term we obtain:

$$g(\theta_i) \approx g(\tilde{\theta}_i) + 1/2(\theta_i - \tilde{\theta}_i)^T g''(\tilde{\theta}_i)(\theta_i - \tilde{\theta}_i).$$

It follows

$$\begin{aligned} p(D|M_i) &= \int e^{g(\theta_i)} d\theta_i \\ &= e^{g(\tilde{\theta}_i)} \int e^{1/2(\theta_i - \tilde{\theta}_i)^T g''(\tilde{\theta}_i)(\theta_i - \tilde{\theta}_i)} d\theta_i. \end{aligned}$$

By recognizing the integrand as proportional to the multivariate normal density and using the Laplace method for integrals we obtain:

$$p(D|M_i) = e^{g(\tilde{\theta}_i)} (2\pi)^{d_i/2} |A|^{-1/2}, \quad (10)$$

where d_i is the number of parameters in the model M_i , and $A = -g''(\tilde{\theta}_i)$. It can be shown that for large N : $\tilde{\theta}_i \approx \hat{\theta}_i$, where $\hat{\theta}_i$ is the MLE, and $A \approx NI$. Thereby I is the expected Fisher information matrix. It is a $d \times d$ matrix whose (l, j) elements are given by:

$$I_{lj} = -E \left[\sum_{l=1}^N \frac{\partial^2 \log(p(a_l|\theta))}{\partial \theta_l \partial \theta_j} \Big|_{\theta=\hat{\theta}_i} \right]. \quad (11)$$

If we take the logarithm of equation 10 we obtain

$$\begin{aligned}\log(p(D|M_i)) &= \log p(D|\hat{\theta}_i) + \log p(\hat{\theta}_i) + d/2 \log(2\pi) \\ &- d/2 \log(N) - 1/2 \log |I| + \mathcal{O}(N^{-1/2}).\end{aligned}$$

Now if we retain only the terms which do not vanish for $N \rightarrow \infty$ we get the following final formula

$$\log(p(D|M_i)) = \log p(D|\hat{\theta}_i) - d/2 \log(N) + \mathcal{O}(1). \quad (12)$$

Combining FPMs and SRGMs we have two types of model classes in equation 9. For FPM we have chosen response surface models, and for SRGMs NHPP based models with different types of Poisson process intensity functions. The log likelihood function in the Fisher information matrix presented in equation 11 for the response surface models and under normality assumption, i. e. $a_1, \dots, a_N \sim \mathcal{N}(F_{FPM}(M, \theta), 1)$ are

$$\log(p(a|\theta)) = -N \log \sqrt{2\pi} - 0.5 \sum_{i=1}^N (a_i - f_{FPM}(M, \theta))^2.$$

The log likelihood function for the models based on the Poisson process can be expressed as follows, we observe a non-homogeneous Poisson process $f = f_1, \dots, f_n$, then the log likelihood is given by:

$$\begin{aligned}\log(p(f|\theta)) &= \sum_{i=1}^n f_i \log[m(t_i) - m(t_{i-1})] \\ &- [m(t_i) - m(t_{i-1})] - \log(f_i!),\end{aligned}$$

where f_i is the number of detected failures in the interval $t_i - t_{i-1}$, $m(t_i)$ is the expectation function of the process, and the $t_i, i = 1, \dots, n$ are time interval boundaries in which the number of failures have been counted.

4 Conclusion

In software industry is a strong demand for early and accurate software quality predictions that can be trusted. Combining two entirely distinct prediction approaches, the resulting hybrid approach meets this demand. Using redundancy is a wide-spread method in fault tolerant systems to improve dependability of such systems. This notion is also straightforward in software quality prediction. Being equipped with two independent quality predictions decisions whether releasing or further testing software can be made with more confidence. The hybrid approach uses initial parameter estimates for providing early reliability predictions and provides a tool to decide how long this initial parameter estimates are useful.

Before FPM and SRGM predictions have reached this point combining them in accord with the redundancy principle would degrade prediction performance. Yet, as we have defined a criterion to decide how long initial parameter estimates are beneficial and when fault prediction and reliability prediction models are of equal performance we know when to adopt FPM and SRGM predictions with respect to the principle of heterogeneous redundancy. With model combination using Bayesian model averaging (BMA) we not only fortify the predictions with a second independent one, but provide a combined model that accounts for the uncertainty in the distinct modeling approaches. Since we adopt FPM and SRGM prediction independently we achieve at least their prediction accuracy but tend to be superior to the individual predictions and are in any case equipped with a second independent prediction. Furthermore, using the stopping point rule we can, even without model combination, put more trust in one method than in the other.

Reviving the last comment, next steps are using BMA together with the stopping rule algorithm to account for the increasing modeling performance of SRGMs over testing time. Also we want to employ Monte Carlo Markov Chains for calculating the posterior probability of a model to overcome the constant error factor in equation 12.

References

- [1] J. Musa, "Operational profiles in software-reliability engineering," *IEEE Transactions on Software Engineering*, vol. 10(2), pp. 14–32, 1993.
- [2] Standard Glossary of Software Engineering Terminology, Std-729-1991 ed., ANSI/IEEE, 1991.
- [3] M. R. Lyu, *Handbook of Software Reliability Engineering*. McGraw-Hill, 1995.
- [4] J. D. Musa, A. Iannino, and K. Okumoto, *Software Reliability Measurement Prediction Application*. McGraw-Hill, 1987.
- [5] P. B. Lakey and A. M. Neufelder, *System and Software Reliability Assurance Notebook*, 1997.
- [6] H. Pham, *Software Reliability*. Springer, 2000.
- [7] Z. Jelinski and P. Moranda, *Software reliability research*, W. Freiberger, Ed. Academic Press, 1972.
- [8] A. L. Goel and K. Okumoto, "Time-dependent error-detection rate model for software reliability and other performance measures," *IEEE Transactions on Reliability*, vol. 28(3), pp. 206–211, 1979.
- [9] S. S. Gokhale and K. Trivedi, "A time/structure based software reliability model," *Annals of Software Engineering*, vol. 8, pp. 85–121, 1999.
- [10] G. J. Knafl, "Solving maximum likelihood equations for two-parameter software reliability models using grouped data," in *Third International Symposium on Software Reliability Engineering*, 1992.
- [11] G. J. Knafl and J. Morgan, "Solving ml equations for 2-parameter poisson-process models for ungrouped software failure data," *IEEE Transactions on Reliability*, vol. 45, pp. 42–53, 1996.
- [12] K. G. Popstojanova and K. Trivedi, "Effects of failure correlation on software in operation," in *Dependable Computing, 2000. Proceedings. 2000 Pacific Rim International Symposium on*, 2000.
- [13] F. Akiyama, "An example of software systems debugging," *Information Processing*, vol. 71, pp. 353–379, 1971.
- [14] N. E. Fenton and M. Neil, "A critique of software defect prediction models," *IEEE Transactions on Software Engineering*, vol. 25(5), pp. 675–689, 1999.
- [15] R. Neumann and S. Bibi, "Buildind fault prediciton models from abstract cognitive complexity metrics - analyzing and interpreting fault related influences," in *IWMS/MetriKon 2004*, 2004.
- [16] T. M. Khoshgoftaar, J. C. Munson, B. Bhattacharya, and G. Richardson, "Predictive modeling techniques of software quality from software measures," *IEEE Transactions on Software Engineering*, vol. 18(11), pp. 979–987, 1992.
- [17] M. Xie, G. Hong, and C. Wohlin, "A practical method for the estimation of software reliability growth in the early stage of testing," in *ISSRE '97: Proceedings of the Eighth International Symposium on Software Reliability Engineering (ISSRE '97)*, 1997.

- [18] Y. Malaiya, A. von Mayrhauser, and P. Srimani, "An examination of fault exposure ratio," *IEEE Transactions on Software Engineering*, vol. 19(11), pp. 1087–1094, 1993.
- [19] N. Nagappan, L. Williams, M. Vouk, and J. Osborne, "Early estimation of software quality using in-process testing metrics: A controlled case study," in *International Conference on Software Engineering (ICSE 2005)*, 2005.
- [20] N. Nagappan, T. Ball, and B. Murphy, "Using historical in-process and product metrics for early estimation of software failures," in *17th International Symposium on Software Reliability Engineering (ISSRE'06)*, 2006.
- [21] M. R. Lyu and A. Nikora, "Using software reliability models more effectively," *IEEE Software*, pp. 4–52, 1992.
- [22] M. R. Lyu, *Encyclopedia of Software Engineering*. John Wiley & Sons, 2002, ch. Software Reliability Theory.
- [23] R. T. Clemen, "Combining forecasts: A review and annotated bibliography," *Journal of Forecasting*, vol. 5, pp. 559–583, 1989.
- [24] R. L. Winkler, "Combining forecasts: A philosophical basis and some current issues," *Journal of Forecasting*, vol. 5, pp. 605–609, 1989.
- [25] J. A. Hoeting, D. Madigan, A. E. Raftery, and C. T. Volinsky, "Bayesian model averaging: A tutorial," *Statistical Science*, vol. 14, pp. 382–417, 1999.

Published reports of the Fraunhofer ITWM

The PDF-files of the following reports are available under:

www.itwm.fraunhofer.de/de/zentral__berichte/berichte

1. D. Hietel, K. Steiner, J. Struckmeier
A Finite - Volume Particle Method for Compressible Flows
(19 pages, 1998)
2. M. Feldmann, S. Seibold
Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 pages, 1998)
3. Y. Ben-Haim, S. Seibold
Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 pages, 1998)
4. F.-Th. Lentens, N. Siedow
Three-dimensional Radiative Heat Transfer in Glass Cooling Processes
(23 pages, 1998)
5. A. Klar, R. Wegener
A hierarchy of models for multilane vehicular traffic
Part I: Modeling
(23 pages, 1998)
Part II: Numerical and stochastic investigations
(17 pages, 1998)
6. A. Klar, N. Siedow
Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes
(24 pages, 1998)
7. I. Choquet
Heterogeneous catalysis modelling and numerical simulation in rarified gas flows
Part I: Coverage locally at equilibrium
(24 pages, 1998)
8. J. Ohser, B. Steinbach, C. Lang
Efficient Texture Analysis of Binary Images
(17 pages, 1998)
9. J. Orlik
Homogenization for viscoelasticity of the integral type with aging and shrinkage
(20 pages, 1998)
10. J. Mohring
Helmholtz Resonators with Large Aperture
(21 pages, 1998)
11. H. W. Hamacher, A. Schöbel
On Center Cycles in Grid Graphs
(15 pages, 1998)
12. H. W. Hamacher, K.-H. Küfer
Inverse radiation therapy planning - a multiple objective optimisation approach
(14 pages, 1999)
13. C. Lang, J. Ohser, R. Hilfer
On the Analysis of Spatial Binary Images
(20 pages, 1999)
14. M. Junk
On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes
(24 pages, 1999)
15. M. Junk, S. V. Raghurame Rao
A new discrete velocity method for Navier-Stokes equations
(20 pages, 1999)
16. H. Neunzert
Mathematics as a Key to Key Technologies
(39 pages (4 PDF-Files), 1999)
17. J. Ohser, K. Sandau
Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem
(18 pages, 1999)
18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel
Solving nonconvex planar location problems by finite dominating sets
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 pages, 2000)
19. A. Becker
A Review on Image Distortion Measures
Keywords: Distortion measure, human visual system
(26 pages, 2000)
20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn
Polyhedral Properties of the Uncapacitated Multiple Allocation Hub Location Problem
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 pages, 2000)
21. H. W. Hamacher, A. Schöbel
Design of Zone Tariff Systems in Public Transportation
(30 pages, 2001)
22. D. Hietel, M. Junk, R. Keck, D. Teleaga
The Finite-Volume-Particle Method for Conservation Laws
(16 pages, 2001)
23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel
Location Software and Interface with GIS and Supply Chain Management
Keywords: facility location, software development, geographical information systems, supply chain management
(48 pages, 2001)
24. H. W. Hamacher, S. A. Tjandra
Mathematical Modelling of Evacuation Problems: A State of Art
(44 pages, 2001)
25. J. Kuhnert, S. Tiwari
Grid free method for solving the Poisson equation
Keywords: Poisson equation, Least squares method, Grid free method
(19 pages, 2001)
26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier
Simulation of the fiber spinning process
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD
(19 pages, 2001)
27. A. Zemitis
On interaction of a liquid film with an obstacle
Keywords: impinging jets, liquid film, models, numerical solution, shape
(22 pages, 2001)
28. I. Ginzburg, K. Steiner
Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models
(22 pages, 2001)
29. H. Neunzert
»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«
Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrrskalalanalyse, Strömungsmechanik
(18 pages, 2001)
30. J. Kuhnert, S. Tiwari
Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation
AMS subject classification: 76D05, 76M28
(25 pages, 2001)
31. R. Korn, M. Krekel
Optimal Portfolios with Fixed Consumption or Income Streams
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems
(23 pages, 2002)
32. M. Krekel
Optimal portfolios with a loan dependent credit spread
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics
(25 pages, 2002)
33. J. Ohser, W. Nagel, K. Schladitz
The Euler number of discretized sets – on the choice of adjacency in homogeneous lattices
Keywords: image analysis, Euler number, neighborhood relationships, cuboidal lattice
(32 pages, 2002)

34. I. Ginzburg, K. Steiner
Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting
Keywords: Lattice Boltzmann models; free-surface phenomena; interface boundary conditions; filling processes; injection molding; volume of fluid method; interface boundary conditions; advection-schemes; up-wind-schemes (54 pages, 2002)
35. M. Günther, A. Klar, T. Materne, R. Wegener
Multivalued fundamental diagrams and stop and go waves for continuum traffic equations
Keywords: traffic flow, macroscopic equations, kinetic derivation, multivalued fundamental diagram, stop and go waves, phase transitions (25 pages, 2002)
36. S. Feldmann, P. Lang, D. Prätzel-Wolters
Parameter influence on the zeros of network determinants
Keywords: Networks, Equicofactor matrix polynomials, Realization theory, Matrix perturbation theory (30 pages, 2002)
37. K. Koch, J. Ohser, K. Schladitz
Spectral theory for random closed sets and estimating the covariance via frequency space
Keywords: Random set, Bartlett spectrum, fast Fourier transform, power spectrum (28 pages, 2002)
38. D. d'Humières, I. Ginzburg
Multi-reflection boundary conditions for lattice Boltzmann models
Keywords: lattice Boltzmann equation, boundary conditions, bounce-back rule, Navier-Stokes equation (72 pages, 2002)
39. R. Korn
Elementare Finanzmathematik
Keywords: Finanzmathematik, Aktien, Optionen, Portfolio-Optimierung, Börse, Lehrerweiterbildung, Mathematikunterricht (98 pages, 2002)
40. J. Kallrath, M. C. Müller, S. Nickel
Batch Presorting Problems: Models and Complexity Results
Keywords: Complexity theory, Integer programming, Assignment, Logistics (19 pages, 2002)
41. J. Linn
On the frame-invariant description of the phase space of the Folgar-Tucker equation
Key words: fiber orientation, Folgar-Tucker equation, injection molding (5 pages, 2003)
42. T. Hanne, S. Nickel
A Multi-Objective Evolutionary Algorithm for Scheduling and Inspection Planning in Software Development Projects
Key words: multiple objective programming, project management and scheduling, software development, evolutionary algorithms, efficient set (29 pages, 2003)
43. T. Bortfeld, K.-H. Küfer, M. Monz, A. Scherrer, C. Thieke, H. Trinkaus
Intensity-Modulated Radiotherapy - A Large Scale Multi-Criteria Programming Problem
Keywords: multiple criteria optimization, representative systems of Pareto solutions, adaptive triangulation, clustering and disaggregation techniques, visualization of Pareto solutions, medical physics, external beam radiotherapy planning, intensity modulated radiotherapy (31 pages, 2003)
44. T. Halfmann, T. Wichmann
Overview of Symbolic Methods in Industrial Analog Circuit Design
Keywords: CAD, automated analog circuit design, symbolic analysis, computer algebra, behavioral modeling, system simulation, circuit sizing, macro modeling, differential-algebraic equations, index (17 pages, 2003)
45. S. E. Mikhailov, J. Orlik
Asymptotic Homogenisation in Strength and Fatigue Durability Analysis of Composites
Keywords: multiscale structures, asymptotic homogenization, strength, fatigue, singularity, non-local conditions (14 pages, 2003)
46. P. Domínguez-Marín, P. Hansen, N. Mladenović, S. Nickel
Heuristic Procedures for Solving the Discrete Ordered Median Problem
Keywords: genetic algorithms, variable neighborhood search, discrete facility location (31 pages, 2003)
47. N. Boland, P. Domínguez-Marín, S. Nickel, J. Puerto
Exact Procedures for Solving the Discrete Ordered Median Problem
Keywords: discrete location, Integer programming (41 pages, 2003)
48. S. Feldmann, P. Lang
Padé-like reduction of stable discrete linear systems preserving their stability
Keywords: Discrete linear systems, model reduction, stability, Hankel matrix, Stein equation (16 pages, 2003)
49. J. Kallrath, S. Nickel
A Polynomial Case of the Batch Presorting Problem
Keywords: batch presorting problem, online optimization, competitive analysis, polynomial algorithms, logistics (17 pages, 2003)
50. T. Hanne, H. L. Trinkaus
knowCube for MCDM – Visual and Interactive Support for Multicriteria Decision Making
Key words: Multicriteria decision making, knowledge management, decision support systems, visual interfaces, interactive navigation, real-life applications. (26 pages, 2003)
51. O. Iliev, V. Laptev
On Numerical Simulation of Flow Through Oil Filters
Keywords: oil filters, coupled flow in plain and porous media, Navier-Stokes, Brinkman, numerical simulation (8 pages, 2003)
52. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva
On a Multigrid Adaptive Refinement Solver for Saturated Non-Newtonian Flow in Porous Media
Keywords: Nonlinear multigrid, adaptive refinement, non-Newtonian flow in porous media (17 pages, 2003)
53. S. Kruse
On the Pricing of Forward Starting Options under Stochastic Volatility
Keywords: Option pricing, forward starting options, Heston model, stochastic volatility, cliquet options (11 pages, 2003)
54. O. Iliev, D. Stoyanov
Multigrid – adaptive local refinement solver for incompressible flows
Keywords: Navier-Stokes equations, incompressible flow, projection-type splitting, SIMPLE, multigrid methods, adaptive local refinement, lid-driven flow in a cavity (37 pages, 2003)
55. V. Starikovicus
The multiphase flow and heat transfer in porous media
Keywords: Two-phase flow in porous media, various formulations, global pressure, multiphase mixture model, numerical simulation (30 pages, 2003)
56. P. Lang, A. Sarishvili, A. Wirsen
Blocked neural networks for knowledge extraction in the software development process
Keywords: Blocked Neural Networks, Nonlinear Regression, Knowledge Extraction, Code Inspection (21 pages, 2003)
57. H. Knaf, P. Lang, S. Zeiser
Diagnosis aiding in Regulation Thermography using Fuzzy Logic
Keywords: fuzzy logic, knowledge representation, expert system (22 pages, 2003)
58. M. T. Melo, S. Nickel, F. Saldanha da Gama
Largescale models for dynamic multi-commodity capacitated facility location
Keywords: supply chain management, strategic planning, dynamic location, modeling (40 pages, 2003)
59. J. Orlik
Homogenization for contact problems with periodically rough surfaces
Keywords: asymptotic homogenization, contact problems (28 pages, 2004)
60. A. Scherrer, K.-H. Küfer, M. Monz, F. Alonso, T. Bortfeld
IMRT planning on adaptive volume structures – a significant advance of computational complexity
Keywords: Intensity-modulated radiation therapy (IMRT), inverse treatment planning, adaptive volume structures, hierarchical clustering, local refinement, adaptive clustering, convex programming, mesh generation, multi-grid methods (24 pages, 2004)
61. D. Kehrwald
Parallel lattice Boltzmann simulation of complex flows
Keywords: Lattice Boltzmann methods, parallel computing, microstructure simulation, virtual material design, pseudo-plastic fluids, liquid composite moulding (12 pages, 2004)
62. O. Iliev, J. Linn, M. Moog, D. Niedziela, V. Starikovicus
On the Performance of Certain Iterative Solvers for Coupled Systems Arising in Discretization of Non-Newtonian Flow Equations
Keywords: Performance of iterative solvers, Preconditioners, Non-Newtonian flow (17 pages, 2004)
63. R. Ciegis, O. Iliev, S. Rief, K. Steiner
On Modelling and Simulation of Different Regimes for Liquid Polymer Moulding
Keywords: Liquid Polymer Moulding, Modelling, Simulation, Infiltration, Front Propagation, non-Newtonian flow in porous media (43 pages, 2004)

64. T. Hanne, H. Neu
Simulating Human Resources in Software Development Processes
Keywords: Human resource modeling, software process, productivity, human factors, learning curve (14 pages, 2004)
65. O. Iliev, A. Mikelic, P. Popov
Fluid structure interaction problems in deformable porous media: Toward permeability of deformable porous media
Keywords: fluid-structure interaction, deformable porous media, upscaling, linear elasticity, stokes, finite elements (28 pages, 2004)
66. F. Gaspar, O. Iliev, F. Lisbona, A. Naumovich, P. Vabishchevich
On numerical solution of 1-D poroelasticity equations in a multilayered domain
Keywords: poroelasticity, multilayered material, finite volume discretization, MAC type grid (41 pages, 2004)
67. J. Ohser, K. Schladitz, K. Koch, M. Nöthe
Diffraction by image processing and its application in materials science
Keywords: porous microstructure, image analysis, random set, fast Fourier transform, power spectrum, Bartlett spectrum (13 pages, 2004)
68. H. Neunzert
Mathematics as a Technology: Challenges for the next 10 Years
Keywords: applied mathematics, technology, modelling, simulation, visualization, optimization, glass processing, spinning processes, fiber-fluid interaction, turbulence effects, topological optimization, multicriteria optimization, Uncertainty and Risk, financial mathematics, Malliavin calculus, Monte-Carlo methods, virtual material design, filtration, bio-informatics, system biology (29 pages, 2004)
69. R. Ewing, O. Iliev, R. Lazarov, A. Naumovich
On convergence of certain finite difference discretizations for 1D poroelasticity interface problems
Keywords: poroelasticity, multilayered material, finite volume discretizations, MAC type grid, error estimates (26 pages, 2004)
70. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva
On Efficient Simulation of Non-Newtonian Flow in Saturated Porous Media with a Multigrid Adaptive Refinement Solver
Keywords: Nonlinear multigrid, adaptive refinement, non-Newtonian in porous media (25 pages, 2004)
71. J. Kalcsics, S. Nickel, M. Schröder
Towards a Unified Territory Design Approach – Applications, Algorithms and GIS Integration
Keywords: territory design, political districting, sales territory alignment, optimization algorithms, Geographical Information Systems (40 pages, 2005)
72. K. Schladitz, S. Peters, D. Reinelt-Bitzer, A. Wiegmann, J. Ohser
Design of acoustic trim based on geometric modeling and flow simulation for non-woven
Keywords: random system of fibers, Poisson line process, flow resistivity, acoustic absorption, Lattice-Boltzmann method, non-woven (21 pages, 2005)
73. V. Rutka, A. Wiegmann
Explicit Jump Immersed Interface Method for virtual material design of the effective elastic moduli of composite materials
Keywords: virtual material design, explicit jump immersed interface method, effective elastic moduli, composite materials (22 pages, 2005)
74. T. Hanne
Eine Übersicht zum Scheduling von Baustellen
Keywords: Projektplanung, Scheduling, Bauplanung, Bauindustrie (32 pages, 2005)
75. J. Linn
The Folgar-Tucker Model as a Differential Algebraic System for Fiber Orientation Calculation
Keywords: fiber orientation, Folgar-Tucker model, invariants, algebraic constraints, phase space, trace stability (15 pages, 2005)
76. M. Speckert, K. Dreßler, H. Mauch, A. Lion, G. J. Wierda
Simulation eines neuartigen Prüfsystems für Achserprobungen durch MKS-Modellierung einschließlich Regelung
Keywords: virtual test rig, suspension testing, multibody simulation, modeling hexapod test rig, optimization of test rig configuration (20 pages, 2005)
77. K.-H. Küfer, M. Monz, A. Scherrer, P. Süß, F. Alonso, A. S. A. Sultan, Th. Bortfeld, D. Craft, Chr. Thieke
Multicriteria optimization in intensity modulated radiotherapy planning
Keywords: multicriteria optimization, extreme solutions, real-time decision making, adaptive approximation schemes, clustering methods, IMRT planning, reverse engineering (51 pages, 2005)
78. S. Amstutz, H. Andrä
A new algorithm for topology optimization using a level-set method
Keywords: shape optimization, topology optimization, topological sensitivity, level-set (22 pages, 2005)
79. N. Ettrich
Generation of surface elevation models for urban drainage simulation
Keywords: Flooding, simulation, urban elevation models, laser scanning (22 pages, 2005)
80. H. Andrä, J. Linn, I. Matei, I. Shklyar, K. Steiner, E. Teichmann
OPTCAST – Entwicklung adäquater Strukturoptimierungsverfahren für Gießereien Technischer Bericht (KURZFASSUNG)
Keywords: Topologieoptimierung, Level-Set-Methode, Gießprozesssimulation, Gießtechnische Restriktionen, CAE-Kette zur Strukturoptimierung (77 pages, 2005)
81. N. Marheineke, R. Wegener
Fiber Dynamics in Turbulent Flows Part I: General Modeling Framework
Keywords: fiber-fluid interaction; Cosserat rod; turbulence modeling; Kolmogorov's energy spectrum; double-velocity correlations; differentiable Gaussian fields (20 pages, 2005)
- Part II: Specific Taylor Drag**
Keywords: flexible fibers; $k-\epsilon$ turbulence model; fiber-turbulence interaction scales; air drag; random Gaussian aerodynamic force; white noise; stochastic differential equations; ARMA process (18 pages, 2005)
82. C. H. Lampert, O. Wirjadi
An Optimal Non-Orthogonal Separation of the Anisotropic Gaussian Convolution Filter
Keywords: Anisotropic Gaussian filter, linear filtering, orientation space, nD image processing, separable filters (25 pages, 2005)
83. H. Andrä, D. Stoyanov
Error indicators in the parallel finite element solver for linear elasticity DDFEM
Keywords: linear elasticity, finite element method, hierarchical shape functions, domain decomposition, parallel implementation, a posteriori error estimates (21 pages, 2006)
84. M. Schröder, I. Solchenbach
Optimization of Transfer Quality in Regional Public Transit
Keywords: public transit, transfer quality, quadratic assignment problem (16 pages, 2006)
85. A. Naumovich, F. J. Gaspar
On a multigrid solver for the three-dimensional Biot poroelasticity system in multilayered domains
Keywords: poroelasticity, interface problem, multigrid, operator-dependent prolongation (11 pages, 2006)
86. S. Panda, R. Wegener, N. Marheineke
Slender Body Theory for the Dynamics of Curved Viscous Fibers
Keywords: curved viscous fibers; fluid dynamics; Navier-Stokes equations; free boundary value problem; asymptotic expansions; slender body theory (14 pages, 2006)
87. E. Ivanov, H. Andrä, A. Kudryavtsev
Domain Decomposition Approach for Automatic Parallel Generation of Tetrahedral Grids
Key words: Grid Generation, Unstructured Grid, Delaunay Triangulation, Parallel Programming, Domain Decomposition, Load Balancing (18 pages, 2006)
88. S. Tiwari, S. Antonov, D. Hietel, J. Kuhnert, R. Wegener
A Meshfree Method for Simulations of Interactions between Fluids and Flexible Structures
Key words: Meshfree Method, FPM, Fluid Structure Interaction, Sheet of Paper, Dynamical Coupling (16 pages, 2006)
89. R. Ciegis, O. Iliev, V. Starikovicius, K. Steiner
Numerical Algorithms for Solving Problems of Multiphase Flows in Porous Media
Keywords: nonlinear algorithms, finite-volume method, software tools, porous media, flows (16 pages, 2006)
90. D. Niedziela, O. Iliev, A. Latz
On 3D Numerical Simulations of Viscoelastic Fluids
Keywords: non-Newtonian fluids, anisotropic viscosity, integral constitutive equation (18 pages, 2006)

91. A. Winterfeld
Application of general semi-infinite Programming to Lapidary Cutting Problems
Keywords: large scale optimization, nonlinear programming, general semi-infinite optimization, design centering, clustering
(26 pages, 2006)
92. J. Orlik, A. Ostrovska
Space-Time Finite Element Approximation and Numerical Solution of Hereditary Linear Viscoelasticity Problems
Keywords: hereditary viscoelasticity; kern approximation by interpolation; space-time finite element approximation, stability and a priori estimate
(24 pages, 2006)
93. V. Rutka, A. Wiegmann, H. Andrä
EJIM for Calculation of effective Elastic Moduli in 3D Linear Elasticity
Keywords: Elliptic PDE, linear elasticity, irregular domain, finite differences, fast solvers, effective elastic moduli
(24 pages, 2006)
94. A. Wiegmann, A. Zemitis
EJ-HEAT: A Fast Explicit Jump Harmonic Averaging Solver for the Effective Heat Conductivity of Composite Materials
Keywords: Stationary heat equation, effective thermal conductivity, explicit jump, discontinuous coefficients, virtual material design, microstructure simulation, EJ-HEAT
(21 pages, 2006)
95. A. Naumovich
On a finite volume discretization of the three-dimensional Biot poroelasticity system in multilayered domains
Keywords: Biot poroelasticity system, interface problems, finite volume discretization, finite difference method
(21 pages, 2006)
96. M. Krekel, J. Wenzel
A unified approach to Credit Default Swap-tion and Constant Maturity Credit Default Swap valuation
Keywords: LIBOR market model, credit risk, Credit Default Swaption, Constant Maturity Credit Default Swap-method
(43 pages, 2006)
97. A. Dreyer
Interval Methods for Analog Circuits
Keywords: interval arithmetic, analog circuits, tolerance analysis, parametric linear systems, frequency response, symbolic analysis, CAD, computer algebra
(36 pages, 2006)
98. N. Weigel, S. Weihe, G. Bitsch, K. Dreßler
Usage of Simulation for Design and Optimization of Testing
Keywords: Vehicle test rigs, MBS, control, hydraulics, testing philosophy
(14 pages, 2006)
99. H. Lang, G. Bitsch, K. Dreßler, M. Speckert
Comparison of the solutions of the elastic and elastoplastic boundary value problems
Keywords: Elastic BVP, elastoplastic BVP, variational inequalities, rate-independency, hysteresis, linear kinematic hardening, stop- and play-operator
(21 pages, 2006)
100. M. Speckert, K. Dreßler, H. Mauch
MBS Simulation of a hexapod based suspension test rig
Keywords: Test rig, MBS simulation, suspension, hydraulics, controlling, design optimization
(12 pages, 2006)
101. S. Azizi Sultan, K.-H. Küfer
A dynamic algorithm for beam orientations in multicriteria IMRT planning
Keywords: radiotherapy planning, beam orientation optimization, dynamic approach, evolutionary algorithm, global optimization
(14 pages, 2006)
102. T. Götz, A. Klar, N. Marheineke, R. Wegener
A Stochastic Model for the Fiber Lay-down Process in the Nonwoven Production
Keywords: fiber dynamics, stochastic Hamiltonian system, stochastic averaging
(17 pages, 2006)
103. Ph. Süß, K.-H. Küfer
Balancing control and simplicity: a variable aggregation method in intensity modulated radiation therapy planning
Keywords: IMRT planning, variable aggregation, clustering methods
(22 pages, 2006)
104. A. Beaudry, G. Laporte, T. Melo, S. Nickel
Dynamic transportation of patients in hospitals
Keywords: in-house hospital transportation, dial-a-ride, dynamic mode, tabu search
(37 pages, 2006)
105. Th. Hanne
Applying multiobjective evolutionary algorithms in industrial projects
Keywords: multiobjective evolutionary algorithms, discrete optimization, continuous optimization, electronic circuit design, semi-infinite programming, scheduling
(18 pages, 2006)
106. J. Franke, S. Halim
Wild bootstrap tests for comparing signals and images
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression
(13 pages, 2007)
107. Z. Drezner, S. Nickel
Solving the ordered one-median problem in the plane
Keywords: planar location, global optimization, ordered median, big triangle small triangle method, bounds, numerical experiments
(21 pages, 2007)
108. Th. Götz, A. Klar, A. Unterreiter, R. Wegener
Numerical evidence for the non-existing of solutions of the equations describing rotational fiber spinning
Keywords: rotational fiber spinning, viscous fibers, boundary value problem, existence of solutions
(11 pages, 2007)
109. Ph. Süß, K.-H. Küfer
Smooth intensity maps and the Bortfeld-Boyer sequencer
Keywords: probabilistic analysis, intensity modulated radiotherapy treatment (IMRT), IMRT plan application, step-and-shoot sequencing
(8 pages, 2007)
110. E. Ivanov, O. Gluchshenko, H. Andrä, A. Kudryavtsev
Parallel software tool for decomposing and meshing of 3d structures
Keywords: a-priori domain decomposition, unstructured grid, Delaunay mesh generation
(14 pages, 2007)
111. O. Iliev, R. Lazarov, J. Willems
Numerical study of two-grid preconditioners for 1d elliptic problems with highly oscillating discontinuous coefficients
Keywords: two-grid algorithm, oscillating coefficients, preconditioner
(20 pages, 2007)
112. L. Bonilla, T. Götz, A. Klar, N. Marheineke, R. Wegener
Hydrodynamic limit of the Fokker-Planck equation describing fiber lay-down processes
Keywords: stochastic differential equations, Fokker-Planck equation, asymptotic expansion, Ornstein-Uhlenbeck process
(17 pages, 2007)
113. S. Rief
Modeling and simulation of the pressing section of a paper machine
Keywords: paper machine, computational fluid dynamics, porous media
(41 pages, 2007)
114. R. Ciegis, O. Iliev, Z. Lakdawala
On parallel numerical algorithms for simulating industrial filtration problems
Keywords: Navier-Stokes-Brinkmann equations, finite volume discretization method, SIMPLE, parallel computing, data decomposition method
(24 pages, 2007)
115. N. Marheineke, R. Wegener
Dynamics of curved viscous fibers with surface tension
Keywords: Slender body theory, curved viscous bers with surface tension, free boundary value problem
(25 pages, 2007)
116. S. Feth, J. Franke, M. Speckert
Resampling-Methoden zur mse-Korrektur und Anwendungen in der Betriebsfestigkeit
Keywords: Weibull, Bootstrap, Maximum-Likelihood, Betriebsfestigkeit
(16 pages, 2007)
117. H. Knaf
Kernel Fisher discriminant functions – a concise and rigorous introduction
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression
(30 pages, 2007)
118. O. Iliev, I. Rybak
On numerical upscaling for flow in heterogeneous porous media
Keywords: numerical upscaling, heterogeneous porous media, single phase flow, Darcy's law, multiscale problem, effective permeability, multipoint flux approximation, anisotropy
(17 pages, 2007)
119. O. Iliev, I. Rybak
On approximation property of multipoint flux approximation method
Keywords: Multipoint flux approximation, finite volume method, elliptic equation, discontinuous tensor coefficients, anisotropy
(15 pages, 2007)
120. O. Iliev, I. Rybak, J. Willems
On upscaling heat conductivity for a class of industrial problems
Keywords: Multiscale problems, effective heat conductivity, numerical upscaling, domain decomposition
(15 pages, 2007)

121. R. Ewing, O. Iliev, R. Lazarov, I. Rybak
On two-level preconditioners for flow in porous media
Keywords: Multiscale problem, Darcy's law, single phase flow, anisotropic heterogeneous porous media, numerical upscaling, multigrid, domain decomposition, efficient preconditioner
(18 pages, 2007)

122. M. Brickenstein, A. Dreyer
POLYBORI: A Gröbner basis framework for Boolean polynomials
Keywords: Gröbner basis, formal verification, Boolean polynomials, algebraic cryptanalysis, satisfiability
(23 pages, 2007)

123. O. Wirjadi
Survey of 3d image segmentation methods
Keywords: image processing, 3d, image segmentation, binarization
(20 pages, 2007)

124. S. Zeytun, A. Gupta
A Comparative Study of the Vasicek and the CIR Model of the Short Rate
Keywords: interest rates, Vasicek model, CIR-model, calibration, parameter estimation
(17 pages, 2007)

125. G. Hanselmann, A. Sarishvili
Heterogeneous redundancy in software quality prediction using a hybrid Bayesian approach
Keywords: reliability prediction, fault prediction, non-homogeneous poisson process, Bayesian model averaging
(17 pages, 2007)

Status quo: September 2007