How to Trap Photons? Storing Single-Photon Quantum States in Collective Atomic Excitations *

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We show that it is possible to "store" quantum states of single-photon fields by mapping them onto *collective* meta-stable states of an optically dense, coherently driven medium inside an optical resonator. An adiabatic technique is suggested which allows to transfer non-classical correlations from traveling-wave single-photon wave-packets into atomic states and vise versa with nearly 100% efficiency. In contrast to previous approaches involving single atoms, the present technique does not require the strong coupling regime corresponding to high-Q micro-cavities. Instead, intracavity Electromagnetically Induced Transparency is used to achieve a strong coupling between the cavity mode and the atoms.

I. INTRODUCTION

Nearly fifteen years ago Marlan Scully and his co-workers envisioned that coherence effects in atoms can be used to correlate quantum fluctuations in lasers [1]. Since then the concepts of atomic coherence and interference were extended and applied to many areas of quantum optics and beyond [2]. Examples include electromagnetically induced transparency (EIT) [3], lasing without inversion (LWI) [4], quenching of spontaneous emission [5], sensitive spectroscopy in coherent media [6,7], and the enhancement of linear and nonlinear susceptibilities [8,9].

The present contribution is stimulated by recent experiments, in which Electromagnetically Induced Transparency has been used to dramatically reduce the group velocity of light pulses in a coherently driven, optically dense ensemble of atoms [10–12]. This slow-down and the associated group delay can be viewed as a temporary storage of light energy in the atomic medium and its subsequent release. The slowly traveling light pulses propagate, under ideal conditions, without losses and distortion.

The present paper demonstrates that it is possible to use closely related ideas to "store" and preserve *quantum states* of free-space light fields over a very long time interval. Processes of this kind open up interesting prospectives for quantum information processing without the usual "strong coupling" requirement of cavity QED.

An important class of schemes for quantum communication and computing in based on an elementary process in which single quanta of excitation are transfered back and forth between an atom and photon-number states of the radiation field [13]. This is achieved within the framework of cavity QED by an adiabatic rotation of dark states [14] wherein a single atom is strongly coupled to the mode of a high-Q micro-cavity. Based on this technique, excitations can be transferred from an atom in one cavity to a different atom in a second cavity, resulting in an entanglement of a pair of atoms separated by a long distance [15–18]. Also sources for single-photon wave-packets referred to as photon guns [19] or turnstile devices [20] were suggested and methods for entanglement engineering of single-photon wave-packets proposed [21]. Furthermore, adiabatic passage of this kind can be used as the basis for an elementary quantum logic gate [22]. Experimental realizations of these ideas are however quite challenging, as the excitation rate determined by the vacuum Rabi-frequency (atom-cavity coupling constant) must exceed the decay out of the cavity. Despite an exciting progress towards the realization of such a strong-coupling regime, extreme technological challenges remain [23].

The present proposal suggests an alternative root towards the solution of these problems. Specifically, we show here that it is possible to map the quantum states of *traveling light waves* onto *collective* meta-stable states of optically dense, coherently driven media inside an optical resonator. In particular, we suggest and analyze an adiabatic transfer method which allows one to transfer non-classical states of light fields into atomic Zeeman sub-levels and vise versa

*This paper is dedicated to Marlan O. Scully on the occasion of his 60th birthday. We are grateful to him for introducing us to this exciting field and for his continuous inspiration and encouragement.

with nearly 100% efficiency. This process is based on the effect of intracavity electromagnetically induced transparency, suggested in [24]. In contrast to single-atom approaches, the technique described here, does not require the usual strong-coupling regime of cavity QED. The key mechanism which allows us to avoid this stringent requirement is the use of an optically dense many-atom system. In such a system single photons couple to *collective excitations* associated with EIT, and the corresponding coupling strength exceeds that of an individual atom by the square root of the number of atoms.

Before proceeding we also note that a transfer of photon squeezing to a partial spin squeezing of an ensemble of atoms has been suggested and demonstrated in [25] and [26]. Here spin squeezed states are generated when an initially unexcited vapor absorbs non-classical light beams. In this case the transfer of non-classical correlations from light to atoms is however incomplete due to dissipation. For instance, only 50% of spin squeezing can be achieved by this method. Furthermore the process is irreversible. The present paper, in contrast, suggests a general method, by which non-classical excitations can be completely transfered to or from the media. In the ideal limit no dissipation or decoherence is present.

II. INTRACAVITY EIT WITH QUANTUM FIELDS

The adiabatic transfer and storage mechanisms proposed in the present paper are based on intracavity EIT [24]. We therefore first review the properties of intracavity EIT with special emphasis on the interaction of the combined cavity–atomic system with few-photon quantum fields. Recently this approach has also been applied to the treatment of a "photon blockade" in a cavity EIT setup [27]. In different context, similar ideas were used to describe dark states in Bose-Einstein Condensates [28].

Consider a system consisting of a single-mode cavity containing N identical three-level atoms as shown in Fig. 1. Assume that one of the two optically allowed transitions is coupled by a cavity mode, whereas the other is coupled by a field in a coherent state. We will show later on that the coherent field remains essentially unaffected by the interaction. Therefore it can be represented by a time-dependent c-number Rabi-frequency $\Omega(t)$. The dynamics of this system is described by the interaction Hamiltonian:

$$H = \hbar g \sum_{i=1}^{N} \hat{a}\sigma_{ab}^{i} + \hbar\Omega(t)\mathrm{e}^{-i\nu t}\sum_{i=1}^{N}\sigma_{ac}^{i} + \mathrm{h.c.}. \tag{1}$$

Here $\sigma_{\mu\nu}^{i} = |\mu\rangle_{ii}\langle\nu|$ is the flip operator of the ith atom between states $|\mu\rangle$ and $|\nu\rangle$. g is the coupling constant between the atoms and the field mode (vacuum Rabi-frequency) which for simplicity is assumed to be equal for all atoms. In view of the symmetry of the coupling, it is convenient to introduce collective atomic operators $\sigma_{ab} = \sum_{i=1}^{N}\sigma_{ab}^{i}$ and $\sigma_{ac} = \sum_{i=1}^{N}\sigma_{ac}^{i}$. These operators couple symmetric, Dicke-like states which we denote as

$$|b\rangle \equiv |b_1...b_N\rangle, \tag{2}$$

$$|a\rangle \equiv \sum_{i=1}^{N}\frac{1}{\sqrt{N}}|b_1...a_i...b_N\rangle, \tag{3}$$

$$|c\rangle \equiv \sum_{i=1}^{N}\frac{1}{\sqrt{N}}|b_1...c_i...b_N\rangle, \tag{4}$$

$$|aa\rangle \equiv \sum_{i\neq j=1}^{N}\frac{1}{\sqrt{2N(N-1)}}|b_1...a_i...a_j....b_N\rangle, \tag{5}$$

$$|ac\rangle \equiv \sum_{i\neq j=1}^{N}\frac{1}{\sqrt{N(N-1)}}|b_1...a_i...c_j..b_N\rangle, \text{ etc.} \tag{6}$$

Quantum and classical fields cause transitions between these states as indicated in Fig. 1.

Under conditions of two-photon resonance, i.e. when the energy difference between levels c and b equals the energy difference per photon of the two fields, i.e. when $\omega_{cb} = \nu - \nu_c$, ν and ν_c being the frequencies of the classical drive field and the cavity mode, the interaction Hamiltonian (1) has families of "dark" eigenstates with zero eigenvalues. These states decouple from both quantum and classical fields by interference. For example, the dark state (Fig.1b) involving at most one cavity photon corresponds to

$$|D,1\rangle = -i\frac{\Omega|b,1\rangle - g\sqrt{N}|c,0\rangle}{\sqrt{\Omega^2+g^2N}} = -i\cos\theta(t)\,|b,1\rangle + i\sin\theta(t)\,|c,0\rangle, \tag{7}$$

where we have introduced the mixing angle $\theta(t) = \arctan[g\sqrt{N}/\Omega(t)]$. This state has a form analogous to that of the usual dark state formed by a pair of coherent classical fields. In particular, we note that in the limit $g\sqrt{N} \gg \Omega$ the state $|D,1\rangle$ corresponds nearly identically to the state $|c,0\rangle$. In this case a single-photon excitation is, in essence, shared among the atoms.

Let us now discuss the principle of intracavity EIT as introduced in Ref. [24]. To this end we include dissipation and decays into the analysis. Three important mechanisms corresponding to such dissipation should be distinguished. First of all, we note that the states of the type given by Eq. (7) are immune against decay from the excited atomic levels, as they contain no component of such states. The dark state however is sensitive to the decay of the lower level coherence between levels b and c. This decay (γ_{bc}) sets the ultimate upper limit on the lifetime of the dark state $|D\rangle$. Finally, there is the effect of the finite Q-value of the cavity. A bare-cavity decay with a rate γ leads to a decay of the dark state $|D,1\rangle$ with the effective rate

$$\frac{\gamma_D}{2} = \frac{\gamma}{2}\cos^2\theta(t). \tag{8}$$

Thus for $\cos^2\theta \ll 1$, i.e. for $g\sqrt{N} \gg \Omega$ the effect of the cavity decay is substantially reduced. In this limit, a superposition given by Eq.(7) contains only a very small ($\sim \Omega/g\sqrt{N}$) component of the single-photon state $|b,1\rangle$. This increases the lifetime of the combined atom-cavity system and is the essential feature of intracavity EIT.

Before concluding we note another interesting property of intracavity EIT, which is important for our present purposes. By changing the Rabi-frequency of the classical driving field $\Omega(t)$, i.e. by varying the mixing angle $\theta(t)$, one can change the coupling of the cavity-dark state to the environment. In what follows we show that this will allow us to effectively load the cavity system with an excitation resulting from an incoming photon wave packet and to subsequently release this energy into a desired photon packet after some storage period.

III. MANIPULATION OF SINGLE-PHOTON EXCITATION BY ADIABATIC FOLLOWING

A. coupling of cavity-dark state to free-field modes

We now discuss the problem of transferring a single-photon state of the free field to a single-photon cavity dark state and vice versa. We will show that these processes can be achieved by *adiabatically rotating* the cavity dark state in a specific way. We consider an effective one-dimensional model with a Fabry-Perot type cavity as shown in Fig. 2. The z-axis is parallel to the propagation of the input and outgoing modes. $z=0$ characterizes the position of the partially transmitting input mirror of the cavity. The other mirror of the cavity is assumed to be 100% reflecting.

To model the input-output processes we introduce a continuum of free-space modes with field operators $\hat{b}_k$ which are coupled to the selected cavity mode with a coupling constant κ. For simplicity we assume that the coupling constant is the same for all relevant modes. This interaction is described by the following effective Hamiltonian

$$V_{\rm cav-free} = \hbar\sum_k \kappa\hat{a}^\dagger\hat{b}_k + {\rm h.c.}. \tag{9}$$

We consider an input field in a general single-photon state $|\Psi_{\rm in}(t)\rangle = \sum_k \xi_k^{\rm in}(t)|1_k\rangle$ with $\xi_k^{\rm in}(t) = \xi_k^{\rm in}(t_0)\,{\rm e}^{-i\omega_k(t-t_0)}$. Here $|1_k\rangle$ stands for $|0,\ldots,1_k,\ldots,0\rangle$ and $\sum_k|\xi_k^{\rm in}|^2 = 1$. In what follows we describe these fields by an envelope "wave function" $\Phi_{\rm in}(z,t)$ defined by:

$$\Phi_{\rm in}(z,t) = \sum_k \langle 0_k|\,\hat{b}_k\,{\rm e}^{ikz}\,|\Psi_{\rm in}(t)\rangle. \tag{10}$$

In a continuum limit we have $\xi_k(t)\to\xi(\omega_k,t)$ and $\sum_k \to (L/2\pi)\int{\rm d}k$ where L is the quantization length. Hence

$$\Phi_{\rm in}(z,t) = \frac{L}{2\pi c}\int{\rm d}\omega_k\,\xi^{\rm in}(\omega_k,t)\,{\rm e}^{ikz}. \tag{11}$$

The normalization condition $(L/2\pi c)\int{\rm d}\omega_k|\xi^{\rm in}(\omega_k,t)|^2 = 1$ of the Fourier coefficients implies the normalization of the input wave-function

$$\int\frac{{\rm d}z}{L}\,|\Phi_{\rm in}(z,t)|^2 = 1. \tag{12}$$

B. input-output problem

When the single-photon wave-packet interacts with the combined system of cavity mode and atoms, the general state can be written in the form:

$$|\Psi(t)\rangle = b(t)\big|b,1,0_k\big\rangle + c(t)\big|c,0,0_k\big\rangle + a(t)|a,0,0_k\rangle + \sum_k \xi_k(t)\big|b,0,1_k\big\rangle, \tag{13}$$

where, for example, $|b,1,0_k\rangle$ denotes the state corresponding to the atomic system in the collective state $|b\rangle$, the cavity mode in the single-photon state and there are no photons in the outside modes. We now assume that the bare frequency of the cavity mode coincides with the $a-b$ transition frequency of the atoms as well as the carrier frequency of the input wave packet, i.e. $\nu_c = \omega_{ab} \equiv \omega_a - \omega_b = \omega_0$. Furthermore we assume that the classical driving field is tuned to resonance with the $a-c$ transition, i.e. $\nu = \omega_{ac}$. This also implies that the system is in perfect two-photon resonance. Under these conditions we can make a transformation into a frame rotating with optical frequencies. The following equations of motion describe the evolution of the slowly-varying state amplitudes:

$$\dot a(t) = -\frac{\gamma_a}{2}\,a(t) - ig\sqrt{N}b(t) - i\Omega c(t), \tag{14}$$

$$\dot b(t) = -ig\sqrt{N}a(t) - i\kappa\sum_k \xi_k(t), \tag{15}$$

$$\dot c(t) = -\frac{\gamma_c}{2}\,c(t) - i\Omega a(t), \tag{16}$$

$$\dot\xi_k(t) = -i\Delta_k\xi_k(t) - i\kappa b(t), \tag{17}$$

where $\Delta_k = \omega_k - \omega_0 = kc - \omega_0$ is the detuning of the free-field modes from the cavity resonance, and $\omega_0 = \nu_c = \omega_{ab}$. In order to model the decay processes such as spontaneous emission and the finite lifetime of the state c (and ultimately the dark state) we use an open system approach and introduce decay rates γ_a and γ_c out of the system.

We note the enhancement of the coupling of atoms with the cavity mode by a factor $\sqrt{N}$ due to collective effects. At the same time, however, no such enhancement of the decay rates γ_a and γ_c takes place as the decays affect the atoms individually. In the following we assume that γ_c is sufficiently small. In this case it can be ignored during the time required for the input and the output processes. γ_c will be taken into account however for the storage time interval.

To describe the adiabatic transfer we proceed by introducing a basis of dark and bright states, $|D\rangle$ and $|B\rangle$ [29]:

$$|D\rangle = -i\cos\theta(t)\,\big|b,1,0_k\big\rangle + i\sin\theta(t)\,\big|c,0,0_k\big\rangle, \tag{18}$$

$$|B\rangle = \sin\theta(t)\,\big|b,1,0_k\big\rangle + \cos\theta(t)\,\big|c,0,0_k\big\rangle, \tag{19}$$

where $\tan\theta(t) = g\sqrt{N}/\Omega(t)$. The evolution equations can be re-written in terms of corresponding probability amplitudes as

$$\dot a(t) = -\frac{\gamma_a}{2}\,a(t) - i\Omega_0(t)\,B(t), \tag{20}$$

$$\dot B(t) = -i\dot\theta(t)\,D(t) - i\Omega_0\,a(t) - i\kappa\sin\theta(t)\sum_k \xi_k(t), \tag{21}$$

$$\dot D(t) = -i\dot\theta(t)\,B(t) + \kappa\cos\theta(t)\sum_k \xi_k(t), \tag{22}$$

$$\dot\xi_k(t) = -i\Delta_k\xi_k(t) - i\kappa\sin\theta(t)B(t) - \kappa\cos\theta(t)D(t). \tag{23}$$

Here $\Omega_0(t) = \sqrt{g^2N + \Omega^2(t)}$, and the terms proportional to $\dot\theta$ describe the non-adiabatic coupling between the bright and dark state. We now adiabatically eliminate the excited state, which is possible if the characteristic time T of the process is sufficiently large compared to the radiative lifetime of the excited state ($\gamma_a T \gg 1$). In a second step we adiabatically eliminate the bright-state amplitude and disregard non-adiabatic corrections. The conditions under which such an elimination is justified will be discussed later. We finally arrive at

$$\dot D(t) = \kappa\cos\theta(t)\sum_k \xi_k(t), \tag{24}$$

$$\dot\xi_k(t) = -i\Delta_k\,\xi_k(t) - \kappa\cos\theta(t)\,D(t). \tag{25}$$

One immediately recognizes from these equations, that the total probability of finding the system in a free-field single photon state or in the cavity-dark state is conserved

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(|D(t)|^2 + \sum_k |\xi_k(t)|^2\right) = 0. \tag{26}$$

Thus under adiabatic conditions there is only an exchange of probability between the free-field states and the cavity dark state.

Formally integrating Eq.(25) leads to

$$\xi(\omega_k, t) = \xi^{\rm in}(\omega_k, t_0)\,\mathrm{e}^{-i\Delta_k(t-t_0)} - \kappa \int_{t_0}^t \mathrm{d}\tau\, \cos\theta(\tau) D(\tau)\,\mathrm{e}^{-i\Delta_k(t-\tau)} \tag{27}$$

and therefore

$$\begin{aligned}\dot{D}(t) = &\frac{\kappa L}{2\pi c}\,\cos\theta \int \mathrm{d}\omega_k\, \xi^{\rm in}(\omega_k, t_0)\,\mathrm{e}^{-i\Delta_k(t-t_0)} \\ &-\kappa^2\cos\theta(t)\int_{t_0}^t \mathrm{d}\tau \cos\theta(\tau) D(\tau) \frac{L}{2\pi c}\int \mathrm{d}\omega_k \mathrm{e}^{-i\Delta_k(t-\tau)}.\end{aligned} \tag{28}$$

In the first term we can identify the wave function of the input photon at $z = 0$. Furthermore in the Markov-limit $\int \mathrm{d}\omega_k\, \mathrm{e}^{-i\Delta_k(t-\tau)} \to 2\pi\delta(t-\tau)$. Thus we find

$$\dot{D}(t) = \sqrt{\gamma\frac{c}{L}}\cos\theta(t)\Phi_{\rm in}(0,t) - \frac{\gamma}{2}\cos^2\theta(t) D(t) \tag{29}$$

where we have introduced the empty-cavity decay rate $\gamma = \kappa^2 L/c$. If t_0 is a time sufficiently before any excitation of the cavity system takes place, i.e. if $\Phi_{\rm in}(0,t) = 0$ for all $t \le t_0$, the solution of (29) can be written as

$$D(t) = \sqrt{\gamma\frac{c}{L}}\int_{t_0}^t \mathrm{d}\tau\, \cos\theta(\tau)\, \Phi_{\rm in}(0,\tau)\, \exp\left\{-\frac{\gamma}{2}\int_\tau^t \mathrm{d}\tau'\, \cos^2\theta(\tau')\right\}. \tag{30}$$

Substituting Eq. (30) into Eq. (27) leads to the input-output relation

$$\begin{aligned}\Phi_{\rm out}(0,t) = &\,\Phi_{\rm in}(0,t) \\ &-\gamma\, \cos\theta(t)\int_{t_0}^t \mathrm{d}\tau\, \cos\theta(\tau)\, \Phi_{\rm in}(0,\tau)\, \exp\left\{-\frac{\gamma}{2}\int_\tau^t \mathrm{d}\tau'\, \cos^2\theta(\tau')\right\}.\end{aligned} \tag{31}$$

Before proceeding let us consider the conditions for the adiabatic elimination of the bright-state amplitude. For this we substitute the formal integral (27) into Eqs.(14- 16) and take the Markov-limit. We then find that adiabatic following occurs when

$$\Omega_0^2 \gg \gamma\gamma_a, \quad \Omega_0^2 \gg \frac{\gamma_a}{T}, \quad \Omega_0^2 \gg \sqrt{\frac{\gamma}{T}}\gamma_a. \tag{32}$$

We note that these conditions also enshure that spontaneous Raman scattering in other than the cavity mode are negligible. Since the characteristic input-pulse length and thus the characteristic times T have to be larger or equal to the cavity decay time γ^{-1}, the first condition is the most stringent one.

It is important to note that in order to ensure adiabaticity it is sufficient that

$$g^2 N \gg \gamma\gamma_a. \tag{33}$$

This condition should be contrasted to the corresponding condition of adiabatic transfer with a single atom. The single-atom case requires a strong-coupling regime corresponding (at least) to $g^2 \ge \gamma\gamma_a$ [13]. The latter is very difficult to realize experimentally.

Let us now discuss the implications of Eqs.(30) and (31). If $\cos\theta$ is constant in time, the atoms simply cause a change of the cavity decay rate, according to $\gamma \to \gamma\cos^2\theta$, Eq.(8). Hence, by increasing the atom density and therefore decreasing $\cos\theta$, the effective lifetime of the cavity mode can be increased. This is however of no help if we are interested in "storing" a photon wave packet. When the effective Q-value of the cavity is increased, the resonances of the combined atom-cavity system become extremely narrow and the outgoing wave packet is smeared out in time. Furthermore there is an increasing component corresponding to the input field directly reflected from the input mirror. Clearly the transfer of photons from an input pulse into the cavity deteriorates significantly when the pulse length becomes shorter than the effective cavity decay time. This is illustrated in Fig. 3, where we have shown the input and output wave functions for different values of the effective cavity decay. The input wave function is a hyperbolic secant pulse.

We now describe a method which allows one to capture and to subsequently release a single-photon state of the light field. In order to achieve this, we utilize techniques of adiabatic transfer [14]. To motivate the analysis carried out below we note that the state $|D,1\rangle$, Eq. (7) couples to the free-field light modes only due to the admixture of the state $|b,1\rangle$. As can be seen from Eq.(24) the coupling of the dark state to the free-field light modes depends on the cosine of the mixing angle θ. When the Rabi-frequency of the classical field Ω is large, $\cos\theta$ is large and there is a strong coupling between cavity-dark state and free field. In this case the free-field photons can leak in an out of the cavity. However, when Ω is small this leakage is effectively stopped. Therefore, by first accumulating the field in a cavity mode and then adiabatically switching off the driving field, an initial free-space wave packet can be stored in a long-lived atom-like dark state. The latter can be released by simply reversing the process, i.e. by an (adiabatic) increase of the Rabi-frequency of the driving field. These two processes will now be discussed in detail.

C. optimization of input: quantum impedance matching

In this section we show how to optimize the time dependence of $\cos\theta(t)$ such that the dark-state amplitude will asymptotically come close to unity. It is clear at hand that this is only possible for a bandwidth of the incoming wave function which is less or at most equal to the bare-cavity bandwidth, i.e. for a wave-packet which is longer than the bare-cavity decay time. Also the time when the adiabatic transfer starts must coincide with the arrival time of the photon wave-packet.

In order to achieve a maximum transfer of free-field photons into cavity photons, the outgoing field components should be minimized. This can be done for example by using the destructive interference of the directly reflected and the circulating components. A necessary condition for complete destructive interference can be obtained by differentiating the input-output relation Eq. (31) and setting $\Phi_{\rm out} = \dot{\Phi}_{\rm out} = 0$. This yields

$$-\frac{\mathrm{d}}{\mathrm{d}t}\ln\,\cos\theta(t) + \frac{\mathrm{d}}{\mathrm{d}t}\ln\Phi_{\rm in}(t) = \frac{\gamma}{2}\cos^2\theta(t). \tag{34}$$

This equation has a simple physical interpretation. The first term on the l.h.s. is the amplitude loss rate of the photon field inside the cavity. When the rotation angle θ is increased by decreasing the Rabi-frequency of the classical driving field, the atoms will absorb photons from the cavity mode to re-establish the dark state by a Raman transition from $|b\rangle$ to $|c\rangle$.

The term on the right-hand side is the effective amplitude decay rate due to cavity losses. Thus if $\Phi_{\rm in}$ would be constant, Eq.(34) constitutes, what in classical systems is known as *impedance matching condition* [30]. Under conditions of impedance matching, there is complete destructive interference of the directly reflected part of the incoming wave and the circulating field leaking out through the input mirror. The classical impedance-matching condition needs to be modified when the input field is time-dependent, as the circulating field "sees" a slightly changed input field after a cavity-round trip. This then leads to the second term on the l.h.s. of Eq.(34). An intuitive derivation of this term as well as a simple physical explanation of the quantum impedance matching condition is given in the Appendix.

We now illustrate the remarkable performance of the adiabatic transfer mechanism under conditions of quantum impedance matching. Since Eq.(34) depends explicitly on the pulse shape, let us specify a particular form of the input pulse. Consider, for example, the case of a normalized hyperbolic secant input pulse

$$\Phi_1(t) = \Phi_{\rm in}^{(1)}(z=0,t) = \sqrt{\frac{L}{cT}}\,\mathrm{sech}\,\left[\frac{2t}{T}\right]. \tag{35}$$

The quantum impedance matching condition leads to the nonlinear first-order differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\cos\theta(t) + \frac{\gamma}{2}\cos^3\theta(t) + \frac{2}{T}\tanh\left[2t/T\right]\cos\theta(t) = 0. \tag{36}$$

Eq.(36) can be solved analytically and we are looking for solutions with the asymptotic behavior $\cos\theta \to 0$ for $t \to \infty$. One of such solutions corresponds to

$$\cos\theta(t) = \sqrt{\frac{2}{\gamma T}}\,\frac{\operatorname{sech}[2t/T]}{\sqrt{1+\tanh[2t/T]}}. \tag{37}$$

The specific form of the mixing angle given by the above equation can be achieved, provided that the single-photon pulse is long enough ($\gamma T \geq 4$), by changing the Rabi-frequency of the classical driving field according to:

$$\Omega(t) = g\sqrt{N}\frac{\operatorname{sech}(2t/T)}{\sqrt{[1+\tanh(2t/T)][\tanh(2t/T)+\gamma T/2-1]}}. \tag{38}$$

With this choice for the driving field one finds that the dark-state population corresponding to an input field Φ_1 evolves according to:

$$|D(t)|^2 = \frac{1+\tanh[2t/T]}{2}. \tag{39}$$

Clearly the population of the dark state approaches unity as $t \to \infty$. This is illustrated in Fig. 4.

An obvious *disadvantage* of the quantum impedance matching condition Eq.(34) is its explicit dependence on the shape of the input pulses $\Phi_{\rm in}$. We will now show that the asymptotic population of the dark state is, in fact, not very sensitive to the actual shape. To illustrate this, we have plotted in Fig. 4 the time dependence of the dark-state population for a Gaussian input field

$$\Phi_2(t) = \Phi_{\rm in}^{(2)}(z=0,t) = \sqrt{\frac{L}{cT}}\left(\frac{2}{\pi}\right)^{1/4}\exp\Bigl\{-\frac{t^2}{T^2}\Bigr\} \tag{40}$$

as well as for a hyper-Gaussian wave function

$$\Phi_3(t) = \Phi_{\rm in}^{(3)}(z=0,t) = \sqrt{\frac{L}{cT}}\left(\frac{\Gamma\left[\frac{5}{4}\right]}{2^{3/4}}\right)^{1/2}\exp\Bigl\{-\frac{t^4}{T^4}\Bigr\}. \tag{41}$$

With these initial pulses we use the “incorrect” mixing angle, Eq.(37), chosen to optimize the input for a hyperbolic secant pulse. By numerically integrating the equations of motion, we find the asymptotic values of the dark-state amplitudes are in these cases $D \to 0.9942$ and $D \to 0.9778$ respectively. This indicates that there is only a modest dependence upon the actual shape of the input pulse for a given function $\cos\theta(t)$.

It should be noted that an exact timing of the arrival time is essential. A small delay δt in the arrival time of the pulses leads to a decrease of the asymptotic amplitude of the dark state proportional to δt^2.

In the above discussion we have assumed that the external control field is at all time in a coherent state and have represented it by its coherent amplitude $\Omega(t)$. This assumption is only valid if the drive field remains unaffected by the interaction with the ensemble of atoms even when its intensity is turned to zero. This is however the case here, since although $\Omega(t) \to 0$, the ratio of $\Omega(t)$ to the effective Rabi-frequency of the field mode $g\sqrt{\langle n(t)\rangle}$ is always much larger than unity. In fact in the case of impedance matching one finds the asymptotic behavior $\Omega(t)/g\sqrt{\langle n(t)\rangle} \to \sqrt{N}$.

D. output

In order to release the stored photon into free-field photons at some later time t_1, one can simply reverse the adiabatic rotation of the mixing angle. The resulting wave-packet will not necessarily have the same pulse form as the original one. The latter aspect is not essential for the purposes of quantum information processing. It is however important that the output wave-packet is generated in a well defined way and corresponds, in the ideal limit, to a single-photon Fock state.

For a time t_1 large enough, such that $\Phi_{\rm in}(0,t) = 0$ for all $t > t_1$, and for $\cos\theta(t_1) = 0$ we find from the input-output relation

$$\Phi_{\rm out}(t) = -\sqrt{\frac{\gamma L}{c}}\,D(t_1)\cos\theta(t)\,\exp\left\{-\frac{\gamma}{2}\int_{t_1}^{t}{\rm d}\tau\,\cos^2\theta(\tau)\right\}. \tag{42}$$

Thus the shape of the output wave-packet is entirely determined by the function $\cos\theta(t)$. For the time-reversal of Eq.(37) a hyperbolic secant output pulse is generated. This is illustrated in Fig. 5. If the dark-state decay during the unloading period is again neglected, the amplitude of the output wave function depends on the dark state amplitude at the release time only. One easily verifies that the total number of photons in $\Phi_{\rm out}$ is given by

$$\frac{c}{L}\int_{t_1}^{\infty} {\rm d}t\, |\Phi_{\rm out}(t)|^2 = |D(t_1)|^2 . \tag{43}$$

The ultimate fidelity of the storage is determined by the decay of the collective dark state during the storage time. Under reasonable conditions the dark-state decay can be neglected during the loading and unloading periods. Hence we only need to determine how $D(t_1)$ (at the time of the release) differs from $D(t_0)$ (at the time of arrival), where $t_1 - t_0$ is the storage time. If we take into account a decay out of the atomic level $|c\rangle$ with a single-atom decay rate γ_c, we find the simple result

$$D(t_1) = D(t_0)\, \exp\left\{-\frac{\gamma_c}{2}(t_1 - t_0)\right\}. \tag{44}$$

It is worth noting that the decay of the collective dark state is identical to the *single-atom* decay. This may seem as a surprise on first glance, since the coupling strength to the cavity mode is enhanced by a factor $\sqrt{N}$. One should bear in mind however that the decay affects only those atoms which are in state c and that in the collective dark state each atom has only a probability of $1/N$ to be in that state.

IV. TRANSFER AND STORAGE OF NON-CLASSICAL SUPERPOSITION STATES

A convenient way of encoding quantum information in photons is to use the analogy between spin-1/2 systems and polarization states. We therefore include polarization of the quantum field and study the interaction of superpositions of polarization states with the intracavity EIT system.

Let us consider a quantum field consisting of a right (σ_+) and left (σ_-) circularly polarized components interacting with a multi-state system shown in Fig. 6a. The system is driven by a classical driving field of different polarization and frequency characterized by the time-dependent Rabi-frequency Ω.

We assume that initially all population is in the lower state $|b\rangle$ coupled by both σ_+ and σ_- components. We consider here the interaction of such atomic ensemble with a single photon wave-packets of the type

$$|\Psi_{\rm in}(t)\rangle = \sum_k \xi_{+k}^{\rm in}(t)|1_{+k}\rangle|0_{-k}\rangle + \sum_k \xi_{-k}^{\rm in}(t)|0_{+k}\rangle|1_{-k}\rangle. \tag{45}$$

$|\Psi_{\rm in}\rangle$ is an eigenstate of the photon number operator $\hat{n} \equiv \hat{n}_+ + \hat{n}_-$ with eigenvalue unity, i.e. $\sum_k \Big(|\xi_{+k}|^2 + |\xi_{-k}|^2\Big) = 1$. Since polarization states are distinguishable one immediately recognizes that the interaction of atoms and cavity separates into two families of states, which do not couple to each other. This is illustrated in Fig. 6b. Thus the state vector of the interacting system can be written as

$$|\Psi(t)\rangle = |\Psi_+(t)\rangle\,|0_-\rangle + |\Psi_-(t)\rangle\,|0_+\rangle, \tag{46}$$

$$|\Psi_+(t)\rangle = b_+(t)\big|b, 1_+, 0_{+k}\big\rangle + c_+(t)\big|c_+, 0_+, 0_{+k}\big\rangle + a_+(t)|a_+, 0_+, 0_{+k}\big\rangle + \sum_k \xi_{+k}(t)\big|b, 0_+, 1_{+k}\big\rangle, \tag{47}$$

$$|\Psi_-(t)\rangle = b_-(t)\big|b, 1_-, 0_{-k}\big\rangle + c_-(t)\big|c_-, 0_-, 0_{-k}\big\rangle + a_-(t)|a_-, 0_-, 0_{-k}\big\rangle + \sum_k \xi_{-k}(t)\big|b, 0_-, 1_{-k}\big\rangle. \tag{48}$$

The equations of motion for the state amplitudes separate into two sets, identical to Eqs.(14-17). We thus can proceed in exactly the same way as in the previous section. In particular we introduce the dark states

$$|D_+\rangle = \frac{\Omega|b, 1_+, 0_-\rangle - g\sqrt{N}|c_+, 0_+, 0_-\rangle}{\sqrt{\Omega^2 + g^2 N}}, \tag{49}$$

$$|D_-\rangle = \frac{\Omega|b, 0_+, 1_-\rangle - g\sqrt{N}|c_+, 0_+, 0_-\rangle}{\sqrt{\Omega^2 + g^2 N}}, \tag{50}$$

where $0_\pm$ and $1_\pm$ denote the cavity-mode excitation and we have dropped the free-field component for simplicity. In the adiabatic limit the total number of excitations in both sub-systems is constant, i.e.

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(|D_+(t)|^2+\sum_k|\xi_{+k}(t)|^2\right)=0, \tag{51}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(|D_-(t)|^2+\sum_k|\xi_{-k}(t)|^2\right)=0. \tag{52}$$

Let us now consider the case when the initial wave packet is in a coherent superposition of two polarization states with identical envelopes, i.e.

$$\xi_{+k}^{in}(t)=\alpha\,\xi_k^{in}(t),\qquad \xi_{-k}^{in}(t)=\beta\,\xi_k^{in}(t). \tag{53}$$

In this case the adiabatic following technique described above can be performed for both polarizations in parallel yielding, apart from overall constants, an identical evolution of the dark state amplitudes $|D_\pm\rangle$. The general state of a free field (45) can therefore be transfered back and forth to a collective atomic state

$$|\Psi_{\rm in}\rangle\longleftrightarrow\Big[\alpha|c_+\rangle+\beta|c_-\rangle\Big]|0_+,0_-\rangle. \tag{54}$$

We note in particular that the relative phase between the left- and right-circularly polarized input wave packets is mapped onto the relative phase between the collective atomic states $|c_+\rangle$ and $|c_-\rangle$. Hence quantum mechanical superposition states can be "stored" in collective atomic excitations.

Before concluding we remark that much more general field states can be transfered onto the atoms. Consider for instance an entangled state composed of two single-photon states of different polarization. Of particular interest are maximally entangled superpositions such as $\sim|0_+,0_-\rangle+|1_+,1_-\rangle$. An input state of this form contains a zero- and a two-photon component. Using the adiabatic techniques of the present paper it is also possible to transfer states of this kind onto collective atomic states. The theoretical description of the interaction is however more involved, as it requires invoking higher-order dark states. In particular, for mapping such entangled two-photon states onto atoms, two additional dark states play an important role:

$$|D_0\rangle=|b,0_+,0_-\rangle, \tag{55}$$

$$|D_2\rangle=\frac{\Omega^2|b,1_+,1_-\rangle-g\sqrt{N}\Omega(|c_+,0_+,1_-\rangle+|c_-,1_+,0_-\rangle)+g^2\sqrt{N(N-1)}|c_+,c_-,0_+,0_-\rangle}{\sqrt{\Omega^4+2g^2N\Omega^2+g^4N(N-1)}}. \tag{56}$$

It is obvious at the intuitive level that, in the ideal limit, an adiabatic transfer will yield atomic states of the type $\sim(|b\rangle+|c_+,c_-\rangle)|0_+,0_-\rangle$. At the same time we note that due to a different functional form of the doubly excited dark state, and due to a cross-coupling between different channels of excitation, the conditions for generating such states can be somewhat different from those described in previous sections. The specific conditions as well as applications to quantum information processing will be discussed in detail elsewhere.

V. SUMMARY

In conclusion we suggested a new technique for mapping quantum states of the radiation field onto collective atomic excitations. Our approach utilizes intracavity electromagnetically induced transparency and therefore does not require the usual strong-coupling condition of cavity QED. By adiabatically rotating the dark state(s) of a system consisting of a large number of multi-level atoms interacting with a single cavity mode, quantum impedance matching of this cavity can be achieved for an input single-photon wave-packet. In this case the quantum state of the radiation field can be transferred with nearly 100% efficiency to a non-decaying, meta-stable state of the atoms. The quantum states of the field can therefore be "stored" in long-lived atomic superpositions. Reversing the adiabatic rotation the stored state can be transformed back into a well defined output wave-packet.

In addition to rather direct applications for quantum memory registers, extension of these ideas to quantum networks and entanglement distribution are obvious. If the input photon wave-packet of the system is entangled with some other system, this entanglement is transferred to the collective atomic state. The storage mechanism also allows to reshape the output wavepacket with respect to the input in an (almost) arbitrary way. Furthermore applications of these ideas to elementary logic gates are likely. We therefore anticipate important applications in different areas of quantum information processing such as quantum communication and quantum computing.

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APPENDIX

The impedance matching condition (34) can be given a simple physical explanation. For this we consider the Fabry-Perot cavity as shown in Fig. 2. The lossless input mirror has an amplitude reflectivity and transmission of R and T, satisfying the usual relations $R^*T + RT^* = 0$ and $|R|^2 + |T|^2 = 1$. Without loss of generality we set $R^* = R$ and $T = i|T|$. Input and output field strength and the circulating field component are denoted by $E_{\rm in}$, $E_{\rm out}$ and E_c.

If the carrier frequency of the input field coincides with the cavity resonance one has the following relations between the three field components

$$E_c(t) = TE_{\rm in}(t) + R\zeta E_c(t-\tau_c), \tag{57}$$

$$E_{\rm out}(t) = T\zeta E_c(t-\tau_c) + RE_{\rm in}(t). \tag{58}$$

ζ denotes the amplitude losses in a single round-trip and we have denoted the round-trip time as τ_c. Substituting (57) into (58) yields

$$E_{\rm out}(t) = RE_{\rm in}(t) + \frac{T}{R}\big[E_c(t) - TE_{\rm in}(t)\big] = \frac{1}{R}E_{\rm in}(t) + \frac{T}{R}E_c(t). \tag{59}$$

The resonator set-up is called impedance matched, if the first and second term in Eq.(59) interfere destructively. To find a condition for such a destructive interference, we have to determine the circulating field in terms of the input field. Since the round-trip time τ_c is short compared to the characteristic time of changes in the input field, we may set $E_c(t-\tau_c) \approx E_c(t) - \tau_c \dot{E}_c(t)$. We do keep the first time derivative here, as it will lead to a modification of the impedance matching condition for a time-dependent input field. We thus obtain from (57) the differential equation

$$\dot{E}_c(t) = -\eta\, E_c(t) + \frac{T}{R\zeta\tau_c}E_{\rm in}(t) \tag{60}$$

where $\eta = (1-R\zeta)/(R\zeta\tau_c)$. Eq. (60) has the simple solution

$$E_c(t) = \frac{T}{R\zeta\tau_c}\int_0^\infty {\rm d}\tau E_{\rm in}(t-\tau)\,{\rm e}^{-\eta\tau}. \tag{61}$$

For small internal losses and a reflectivity of the input mirror near unity we have $R \approx 1-\gamma\tau_0/2$, $T^2 = R^2 - 1 \approx -\gamma\tau_0$ and $\zeta \approx 1-\gamma_{\rm int}\tau_c/2$. Here γ is the empty-cavity decay rate, τ_0 is the empty-cavity round-trip time, and we have introduced the effective decay rate of the circulating field due to internal losses $\gamma_{\rm int}$. In this limit $\eta \approx (\gamma/2)(\tau_0/\tau_c) + \gamma_{\rm int}/2$. Thus we eventually obtain for the output field

$$E_{\rm out}(t) = \frac{1}{R}E_{\rm in}(t) - \frac{1}{R}\gamma\frac{\tau_0}{\tau_c}\int_0^\infty {\rm d}\tau\, E_{\rm in}(t-\tau)\,{\rm e}^{-\eta\tau}. \tag{62}$$

Setting $E_{\rm out} = 0$, multiplying with R, and differentiating yields

$$0 = \dot{E}_{\rm in} - \gamma\frac{\tau_0}{\tau_c}E_{\rm in} + \eta E_{\rm in}, \tag{63}$$

which can be brought into the form

$$\frac{\gamma_{\rm int}}{2} + \frac{\rm d}{{\rm d}t}\ln E_{\rm in}(t) = \frac{\gamma}{2}\frac{\tau_0}{\tau_c}. \tag{64}$$

This is the generalized impedance matching condition for a single-sided Fabry-Perot cavity with internal losses ($\gamma_{\rm int}$), a round-trip time τ_c, and a time-dependent input field. We will now show that for the system discussed in the present paper $\tau_0/\tau_c = \cos^2\theta(t)$ and $\gamma_{\rm int} = -2\frac{\rm d}{{\rm d}t}\ln\cos\theta(t)$.

In order to determine the round-trip time we note, that the large linear dispersion of the EIT medium in our system leads to a strong group delay. The group velocity of a weakly excited, propagating field mode interacting with N Λ-type atoms is given by

$$v_{\rm gr} = \frac{c}{1+\dfrac{g^2N}{\Omega^2(t)}} = \frac{c}{1+\tan^2\theta(t)} = c\,\cos^2\theta(t), \tag{65}$$

where $\Omega(t)$ is the Rabi-frequency of the classical driving field and g describes the atom-field coupling strength. Thus

$$\frac{\tau_0}{\tau_c} = \cos^2\theta(t). \tag{66}$$

In order to determine the internal photon losses in the system, we consider the equation of motion for the probability to find a single photon inside the cavity, which is identical to the probability to find the system in state $|b,1,0_k\rangle$. Under adiabatic conditions, the system is always in the dark state $|D\rangle$, thus the 1-photon probability reads

$$p_1(t) = \Bigl|\bigl\langle b,1,0_k\bigl|D(t)\bigr\rangle\Bigr|^2 = \cos^2\theta(t). \tag{67}$$

Differentiating this expression with respect to time yields

$$\gamma_{\rm int} \equiv -\frac{\rm d}{{\rm d}t}\ln p_1(t) = -2\frac{\rm d}{{\rm d}t}\ln\cos\theta(t) \tag{68}$$

With this, Eq.(64) goes over into the quantum impedance condition Eq.(34).

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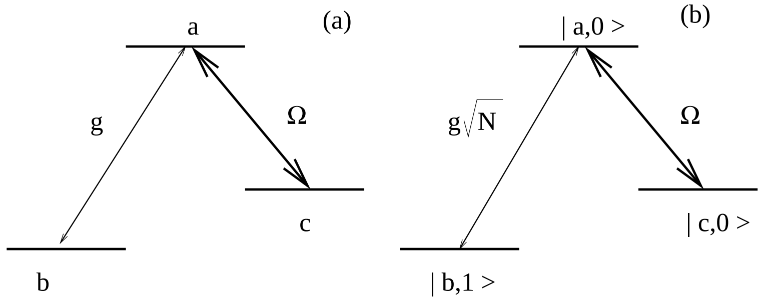

FIG. 1. (a) Three-level atoms interacting with quantum field and driven by classical field with Rabi frequency $\Omega(t)$. g is the coupling constant between quantum field and atoms. (b) Interaction of singly excited mode with N 3-level atoms in the basis of collective states.

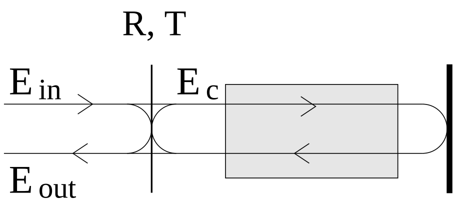

FIG. 2. Cavity set-up. R and T are amplitude reflectivity and transmittivity of input mirror. $E_{\rm in}$, $E_{\rm out}$ and E_c denote input, output and circulating field components.

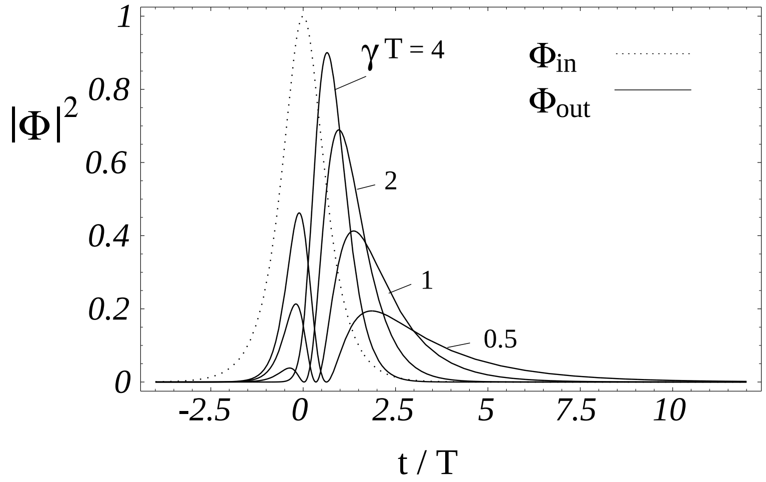

FIG. 3. Shape of input and output single-photon wave functions of a Fabry-Perot-type resonator for different cavity decay rates. Decreasing of cavity width leads to delocalized output wave function and increasing component reflected at $t = 0$. T characterizes the time unit.

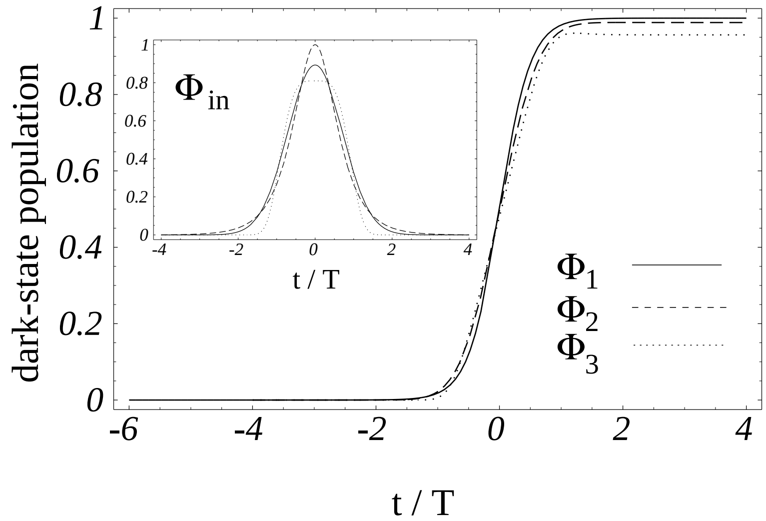

FIG. 4. Population of dark state $|D(t)|^2$ for hyperbolic secant (Φ_1), Gaussian (Φ_2), and hyper-Gaussian (Φ_3) input. $\cos\theta(t)$ is optimized for quantum impedance matching of Φ_1. $\gamma T = 4$. Shape of input wave functions shown in inset. T characterizes the time unit.

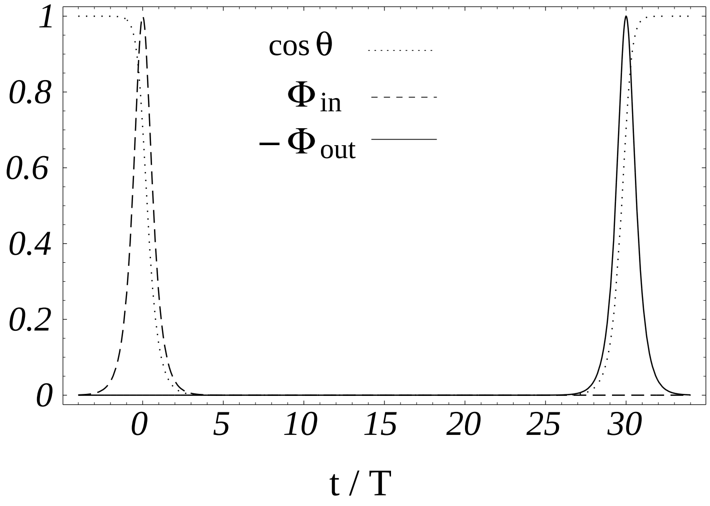

FIG. 5. Input and output wave functions for hyperbolic secant input wave packet Φ_1, $\gamma T = 4$ and optimized $\cos\theta(t)$. At $t \approx 30T$ $\cos\theta(t)$ is time reversed to release photon wave packet. T characterizes the time unit.

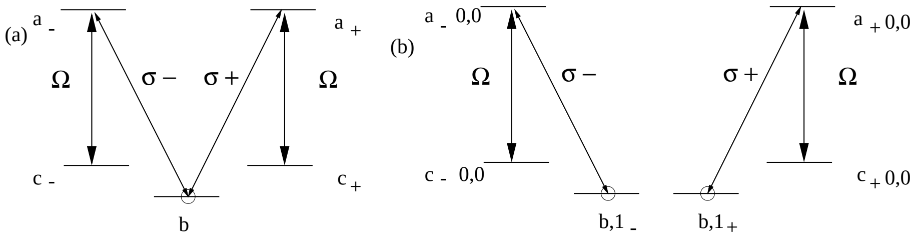

FIG. 6. (a) Prototype of a multi-state atom for storing polarization states of quantum field. (b) Interaction of single-photon wave-packets of different polarizations with collective excitations.